LECTURE 8 Sahon 8.5, 8.6

SAMPLING THE DTFT of aperiodic discrete signals.

- Let $X(\omega)=\sum_{n=-\infty}^{\infty} x(n)\, e^{-j\omega n}$

- Evaluate $X(\omega)$ at $\omega=\frac{2\pi}{N}k$, $k=0,\ldots,N-1$ (N indep. frequencies)

i.e., $X(k)=X\left(\omega=2\pi k/N\right)=\sum_{n=-\infty}^{\infty} x(n)\, e^{-j\frac{2\pi kn}{N}}$ (easy to be calculated with digital computers)

Discrete-periodic (per. N)

⇓

There is a periodic sequence $\tilde{x}(n)$ with DFS $X(k)$, i.e.,

$$\tilde{x}(n)=\frac{1}{N}\sum_{k=0}^{N-1} X(k)e^{j\frac{2\pi kn}{N}}=\frac{1}{N}\sum_{k=0}^{N-1}\sum_{m=-\infty}^{\infty} x(m)\, e^{-j\frac{2\pi mk}{N}} e^{j\frac{2\pi kn}{N}}=$$

$$=\frac{1}{N}\sum_{m=-\infty}^{\infty} x(m)\underbrace{\sum_{k=0}^{N-1} e^{-j\frac{2\pi k(m-n)}{N}}}_{=\begin{cases}N, & m-n=lN,\ l=0,\pm1,\ldots\\ 0, & \text{otherwise}\end{cases}} \Rightarrow \boxed{\tilde{x}(n)=\sum_{l=-\infty}^{\infty} x(n+lN)}$$

* Time aliasing if $x(n)$ longer than N samples.

The N-DFT of aperiodic sequences is a transformation between N-points (one period) of $\tilde{x}(n)=\sum_{l=-\infty}^{\infty} x(n+lN)$ and N-points (one period) of $X(k)=X(\omega=\frac{2\pi k}{N})$

N-DFT pair.

$$X(k)=\sum_{n=\langle N\rangle}\tilde{x}(n)\, e^{-j\frac{2\pi kn}{N}}$$

$$\tilde{x}(n)=\frac{1}{N}\sum_{k=\langle N\rangle} X(k)\, e^{j\frac{2\pi kn}{N}}$$

forward DFT: if $x(n)$ of length $\le N$

inverse DFT

$k,n\in\langle N\rangle$

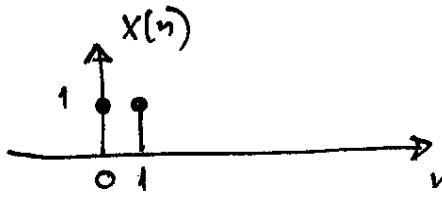
$$X(k)=\sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi kn}{N}}$$

$$x(n)=\frac{1}{N}\sum_{k=0}^{N-1} X(k)\, e^{j\frac{2\pi kn}{N}}$$

$\langle N\rangle \equiv$ period of length N, usually $\langle N\rangle=0,1,\ldots,N-1$

* Observe similarity with DTFS (However, in DFT N is chosen by us)

Ex. Let



$$X(\omega) = 2\cos\frac{\omega}{2}\, e^{-j\omega/2} = 1 + e^{-j\omega}$$ 18

Evaluate: 1-DFT, 2-DFT, 3-DFT, 8-DFT

A) Working in frequency domain.

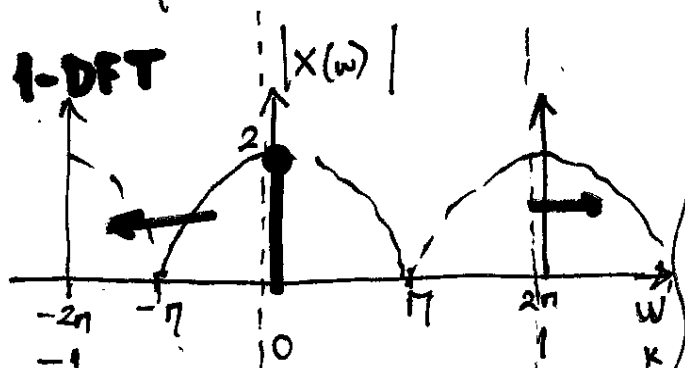
$$|X(k)| = X\left(\omega = \frac{2\pi k}{N}\right), \quad k = 0, \ldots, N-1$$

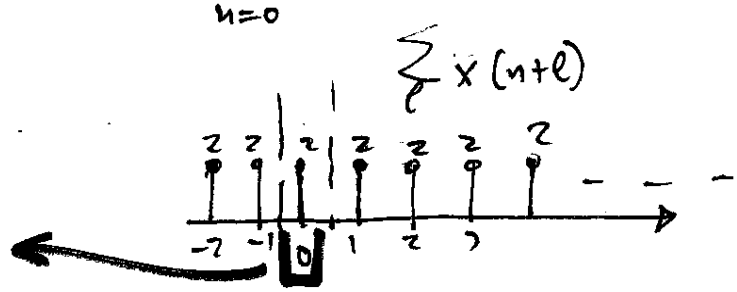
B) Time domain $\xrightarrow{N\text{-DFT}}$ Frequency domain

Form $\tilde{x}(n) = \sum_{\ell=-\infty}^{\infty} x(n+\ell N)$

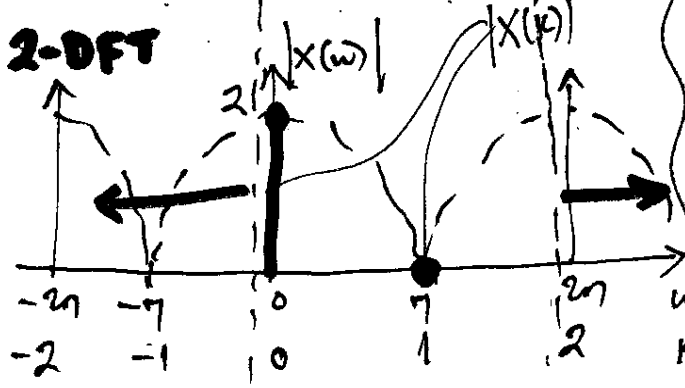
Then $X(k) = \sum_{n=0}^{N-1} \tilde{x}(n)\, e^{-j\frac{2\pi k n}{N}}$ $k = 0, 1, 2, \ldots, N-1$

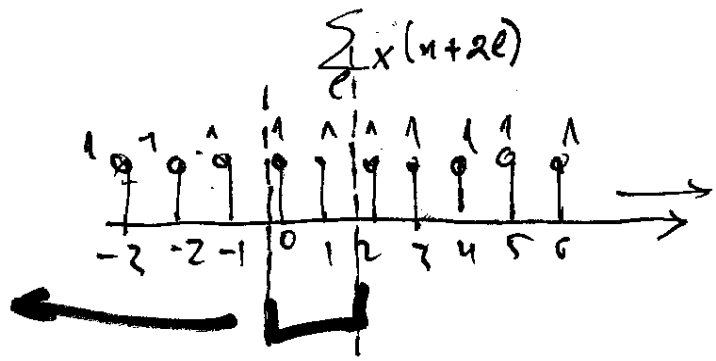
1-DFT



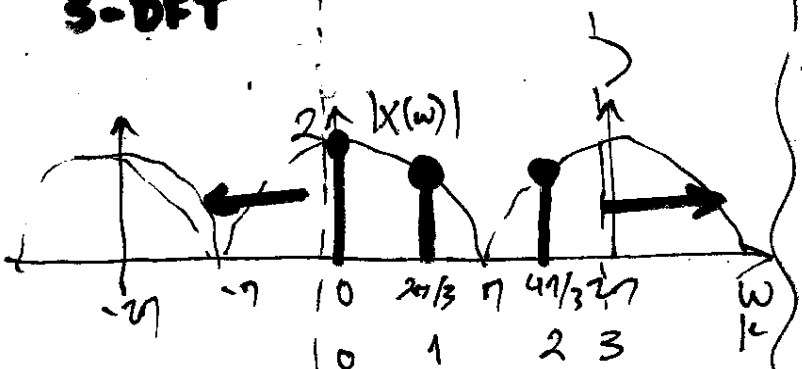


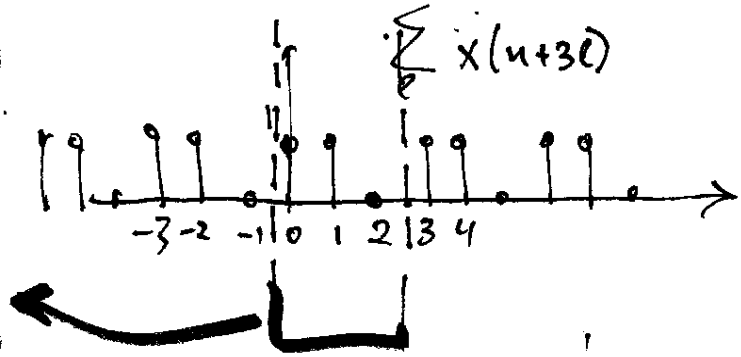
2-DFT





3-DFT





8-DFT