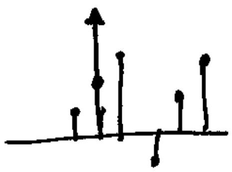
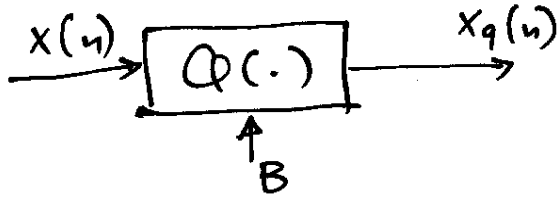
QUANTIZATION AND CODING

Quantization: The process of converting a discrete time signal to digital

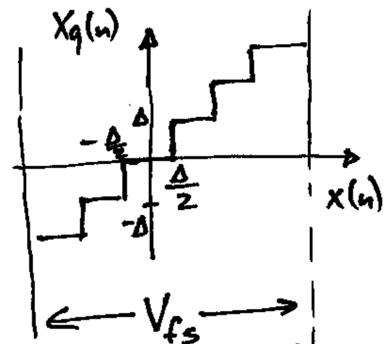
* Allow 2^B quantization levels where each discrete sample is represented by B bits
* Non-invertible process





Uniform quantizer: All levels are equally spaced

Rounting: Assign $x(n)$ to the nearest quantization level

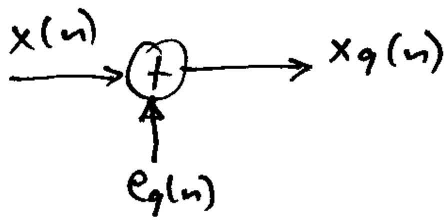


<u>Then:</u> Quantization step: $\Delta = \dfrac{V_{fs}}{2^B}$ LSB level (least significant bit) (full scale dynamic range)

Quantization error: (random signal) <u>noise like</u> $e_q(n) = x_q(n) - x(n) \sim \text{Uniform}\left(-\frac{\Delta}{2}, \frac{\Delta}{2}\right)$

$\left(\text{mean error} = 0 \;,\; \underset{(\text{power})}{\text{variance}} = \dfrac{\Delta^2}{12}\right)$

Alternative representation:



* <u>ANALYSIS</u> : Assuming dynamic range of signal is less than V_{fs}.

$$\text{SQNR (Signal to quant. noise ratio)} = 10\log_{10}\frac{\text{Power signal}}{\text{Power}(e_q(n))} = 10\log_{10}\frac{P_s}{\frac{\Delta^2}{12}}$$

$$= 10\log_{10}\frac{12 P_s}{V_{fs}^2}\cdot 2^{2B} = 2B\cdot 10\log_{10}2 + 10\log_{10}12 + 10\log_{10}\frac{P_s}{V_{fs}^2}$$

$$= 6.02B + 10.8 + 10\log_{10}\frac{P_s}{V_{fs}^2}$$

Assuming $P_s \approx V_{fs}^2 \longrightarrow \boxed{\text{SQNR} \cong 6.02\,B + 10.8}$ dB

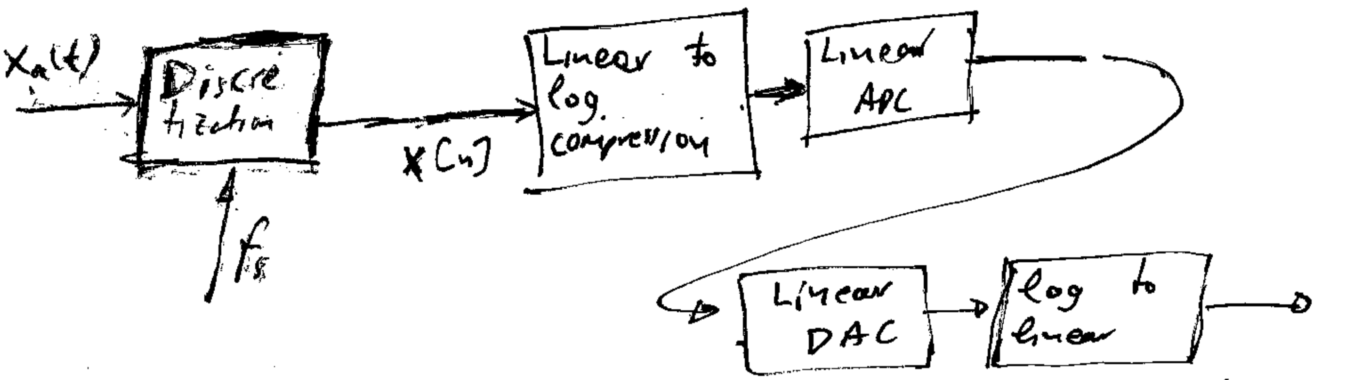
Discussion:

$$SNR / \text{input value} = \frac{|X(\omega)|^2}{\sigma_e^2}$$

- Thus the smaller the input (signal) value the higher the effect of quantization noise

- Real life signals exhibit a higher percentage of low values compared to high values.

- In order to reduce the average quantization noise / value and make the effect of noise constant over the dynamic range of signal — Non-uniform quantizer must be employed.



μ-law compressor $\quad \frac{V_{out}}{V_{in}} = \frac{\ln\left(1+\mu \frac{V_{in}}{V_{max}}\right)}{\ln(1+\mu)}$

$\mu > 0$: compression constant

Examples: Assume that we use uniform Quantizer

(1.) Let $x[n] = A\cos[\omega n + \theta]$

Then Let

$$V_{fs} = 2A \quad , \quad P_x = \frac{A^2}{2}$$

$$SQNR = 6.02\,B + 10.8 + 10\log_{10}\frac{\frac{A^2}{2}}{(2A)^2} = 6.02\,B + 10.8 + 10\log_{10}\frac{A^2}{8A^2}$$

$$= 6.02\,B + 10.8 + 10\log_{10}\frac{1}{8} = 6.02B + 10.8 - 9.03$$

$$= 6.02\,B + 1.77 \text{ dB}$$

(2) Let $x(n) \sim U\left[-\frac{V_{fs}}{2}, \frac{V_{fs}}{2}\right]$, $P_x = \sigma_x^2 = \frac{V_{fs}^2}{12}$

Then: $$SQNR = 6.02\,B + 10.8 + 10\log_{10}\frac{\frac{V_{fs}^2}{12}}{V_{fs}^2} =$$

$$= 6.02\,B + 10.8 + 10\log_{10}\frac{1}{12} =$$

$$= 6.02\,B + 10.8 - 10.8 = 6.02\,B$$