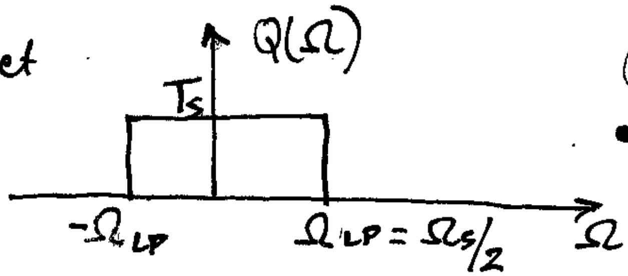
ELE 431 S

LECTURE 3

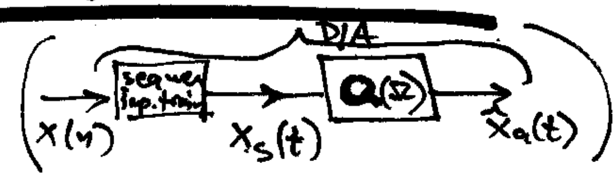
IDEAL SAMPLING

- RECONSRUCTION USING IDEAL LOW PASS FILTER. (D/A conversion or D/C)

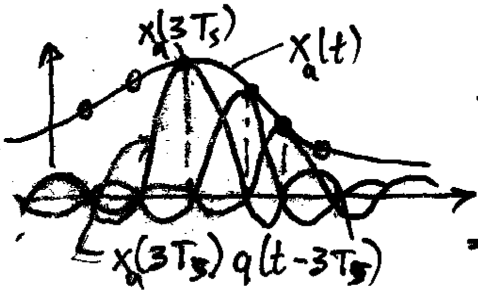
Let



(Assuming $\Omega_s \geq 2\Omega_0$)



$$\hat{X}_a(\Omega) = X_s(\Omega)Q(\Omega) \Rightarrow \hat{x}_a(t) = x_s(t) * q(t) = \left[\sum_{k=-\infty}^{\infty} x_a(kT_s)\,\delta(t-kT_s)\right] * \frac{T_s \sin[\Omega_{LP} t]}{\pi t} \Rightarrow$$



$$\Rightarrow \hat{x}_a(t) = \sum_{k=-\infty}^{\infty} x_a(kT_s)\,\delta(t-kT_s) * \frac{\sin[\Omega_s t/2]}{\Omega_s t/2} \Rightarrow$$

$$\Rightarrow \hat{x}_a(t) = \sum_{k=-\infty}^{\infty} x_a(kT_s)\,\frac{\sin[\Omega_s (t-kT_s)/2]}{\Omega_s (t-kT_s)/2} = x_a(t)$$

THE SAMPLING THEOREM

If the highest frequency contained in a signal $x_a(t)$ is Ω_0 and the signal is uniformly sampled at a rate $\Omega_s \geq 2\Omega_0$, then, $x_a(t)$ can be exactly recovered from its sample values using the interpolation ~~formula~~ (function)

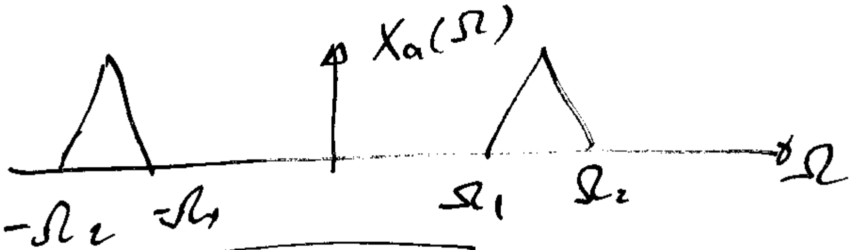
$$q(t) = \frac{\sin[\Omega_s t/2]}{\Omega_s t/2}$$

and then, $x_a(t) = \sum_{k=-\infty}^{\infty} x_a(kT_s)\, q(t-kT_s)$, where $\{x_a(kT_s)\}$ are the samples of $x_a(t)$. taken with rate $T_s = \frac{2\pi}{\Omega_s}$.

* In practice we should take $\Omega_s > 2\Omega_0$ (i.e. consider a sinusoid sampled at $\Omega_s = 2\Omega_0$ at its zero values!!!)

A useful application of aliasing
(bandpass version of the sampling theorem)

Consider a bandpass narrowband signal



Assume $\boxed{\Omega_2 - \Omega_1 \ll \Omega_1}$

By sampling the signal at a rate $\boxed{\begin{array}{l}\Omega_s \geq 2(\Omega_2-\Omega_1) \\ \Omega_s < 4(\Omega_2-\Omega_1)\end{array}}$
but $\Omega_s < \Omega_1$

then a $\overset{\text{low pass}}{\text{replica}}$ of the signal will appear in the range $0 < \Omega < \frac{\Omega_s}{2}$

__Note:__ $\Omega_s = 2(\Omega_2 - \Omega_1)$ is allowed provided that
Ω_2 is a multiple of $\Omega_2 - \Omega_1$

Ex. $\Omega_2 = 3(\Omega_2 - \Omega_1)$