ELE 431 S

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Lecture 8

DESIGN OF FIR FILTERS

→ Computer aided frequency sampling method

Let $h(n)$, $n=0,1,\dots,M-1$ be the impulse response of an FIR filter

Then, $$H(\omega)=\sum_{n=0}^{M-1}h(n)e^{-j\omega n}=\sum_{n=0}^{M-1}\Big[\underbrace{\frac{1}{M}\sum_{k=0}^{M-1}H(k)e^{j\frac{2\pi kn}{M}}}_{\text{IDFT}}\Big]e^{-j\omega n}$$

$$=\frac{1}{M}\sum_{k=0}^{M-1}H(k)\left[\sum_{n=0}^{M-1}e^{j\frac{2\pi kn}{M}}e^{-j\omega n}\right]=\frac{1}{M}\sum_{k=0}^{M-1}H(k)\left[\sum_{n=0}^{M-1}\left(e^{j\frac{2\pi k}{M}}e^{-j\omega}\right)^n\right]$$

$$=\frac{1}{M}\sum_{k=0}^{M-1}H(k)\,\frac{1-\left(e^{j\frac{2\pi k}{M}}e^{-j\omega}\right)^M}{1-e^{j\frac{2\pi k}{M}}e^{-j\omega}}=\frac{1}{M}\sum_{k=0}^{M-1}H(k)\,\frac{1-e^{-j\omega M}}{1-e^{j\frac{2\pi k}{M}}e^{-j\omega}}$$

$$=\underbrace{\frac{1-e^{-j\omega M}}{M}\sum_{k=0}^{M-1}\frac{H(k)}{1-e^{j\frac{2\pi k}{M}}e^{-j\omega}}}_{A}=\frac{\sin\frac{\omega M}{2}}{M}\,2je^{-j\frac{\omega M}{2}}\cdot\sum_{k=0}^{M-1}\frac{H(k)}{2je^{-j\left(\frac{\omega}{2}-\frac{\pi k}{M}\right)}\sin\left[\frac{\omega}{2}-\frac{\pi k}{M}\right]}$$

Finally

$$\boxed{H(\omega)=\left[\frac{\sin\frac{\omega M}{2}}{M}\cdot\sum_{k=0}^{M-1}\frac{H(k)\,e^{j\frac{\pi k}{M}}}{\sin\left[\frac{\omega}{2}-\frac{\pi k}{M}\right]}\right]\cdot e^{-j\omega(M-1)/2}}$$

Thus, given $H(k)=H\left(\omega=\frac{2\pi k}{M}\right)$ $k=0,1,\dots,M-1$, this formula can interpolate samples at any frequency in the range $|\omega|\le\pi$

DESIGN IDEA: { Specify the filter in terms of samples of $H(\omega)$. Then interpolate using the above formula. }

ie. fill the gaps

DESIGN PROCEDURE:

Specify $H_d(\omega)$ of a causal stable filter

Obtain samples $H(k) = H_d(\omega)\Big|_{\omega = \frac{2\pi k}{M}}$, $k = 0, 1, \ldots, M-1$

Using the interpolation formula estimate $H(\omega)$, $|\omega| \leq \pi$

Is $H(\omega)$ close to $H_d(\omega)$ within given specifications?
i.e., is $\varepsilon^2 = \frac{1}{2\pi}\int_\Delta |H_d(\omega) - H(\omega)|^2 d\omega$ sufficiently small?
(Δ: stopband or transition band or any other interval.

NO

Reassign values to $H(k)$ for particular values of k
For ex.: "Gradient search optimization technique

$$H_{new}(k) = H_{old}(k) - \mu \frac{\partial \varepsilon^2}{\partial H(k)}$$

μ: step size

or "apply more "ad-hoc" procedures"

YES

Obtain:

$$h(n) = \frac{1}{M}\sum_{k=0}^{M-1} H(k) e^{+j\frac{2\pi k n}{M}}$$

$n = 0, 1, \ldots, M-1$

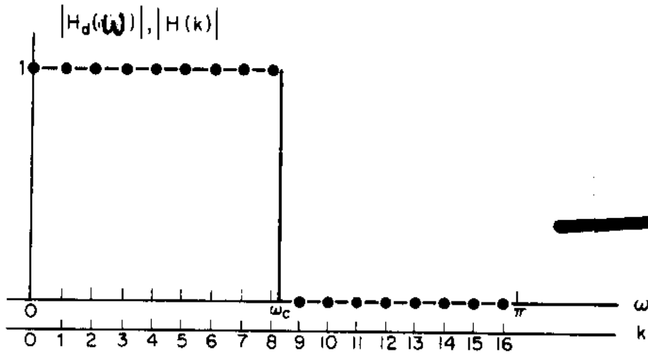
Implement $h(n)$

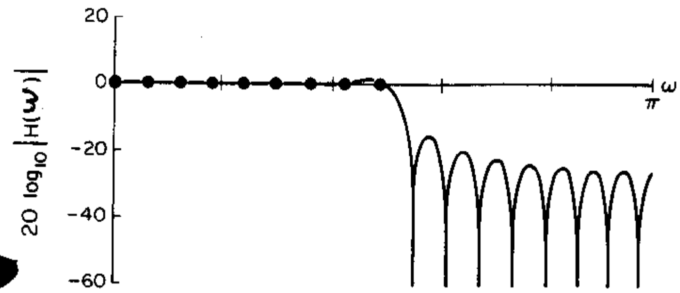
* Lack of flexibility in specifying ω_s and ω_p since samples of $H(k)$ are at integer location multiples of $\frac{2\pi}{M}$
By increasing M better accuracy is possible

Ex Design ideal low pass filter with $\omega_c = \frac{\pi}{2}$ and linear phase, i.e. $|H_d(\omega)| = 1$

$\angle H_d(\omega) = \omega \frac{M-1}{2}$ $\quad |\omega| \le \pi/2$

1)





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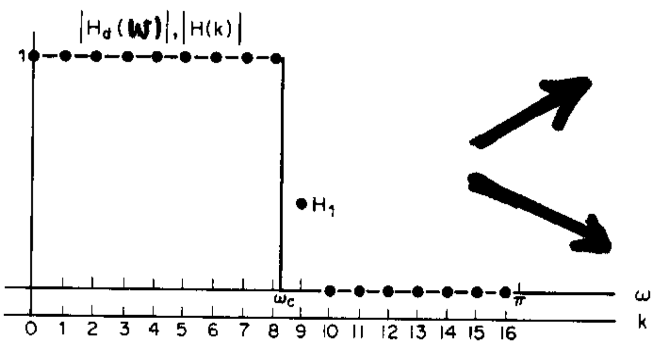

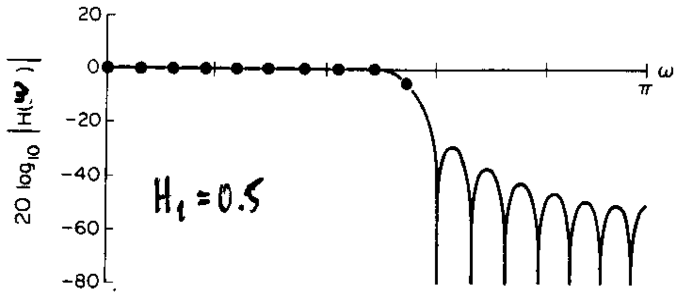

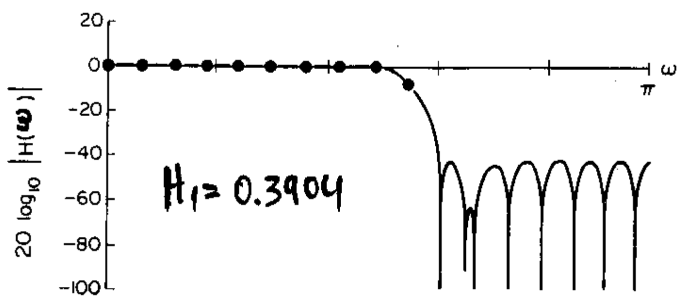

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