ch.7

DESIGN OF DIGITAL FILTERS

General Case: Design of LTI systems using finite precision arithmetic

step 1: Specify desired properties of digital filter

step 2: Approximate specifications using a causal discrete time system (filter)

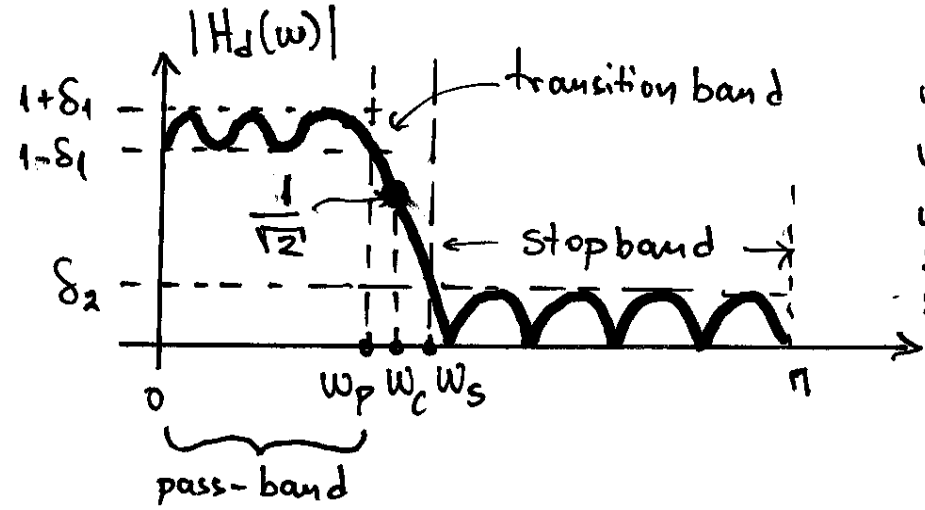
step 3: Realize using finite precision arithmetic

In this part of the course we will concentrate on step 2. Step 1 depends on the application. Step 3 will be examined later.

✱ The sampling frequency plays no role in design $\Rightarrow T_s = 1$

step 1: A set of specifications can be given in form of a tolerance scheme. Let us call the specif. $H_d(\omega)$, i.e., desired response

Ex:



ω_c: cut off freq.
ω_p: Passband freq.
ω_s: stopband "
δ_1: passband ripple
δ_2: stopband "

Phase specification: Usually the only constraints imposed are those for stability and causality requirements, i.e., all poles of $H_d(z)$ be inside unit circle and $h_d(n) \geq 0$ for all n

Design of Linear phase filters (FIR) using windows

(1)

step 1. Specify $H_d(\omega)$

step 2. Determine $h_d(n)$

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(\omega)\, e^{j\omega n}\, e^{-j\omega k_0}\, d\omega, \quad n = 0, 1, \dots, \infty$$

linear phase (delay in time domain)!

step 3. Truncate $h_d[n]$ to yield an M coefficient FIR Filter that:

$$\boxed{h(n) = h_d(n) \cdot w(n)}$$

where $w(n) = \begin{cases} \neq 0, & n = 0, 1, \dots, M-1 \\ 0, & \text{otherwise} \end{cases}$: window of length M

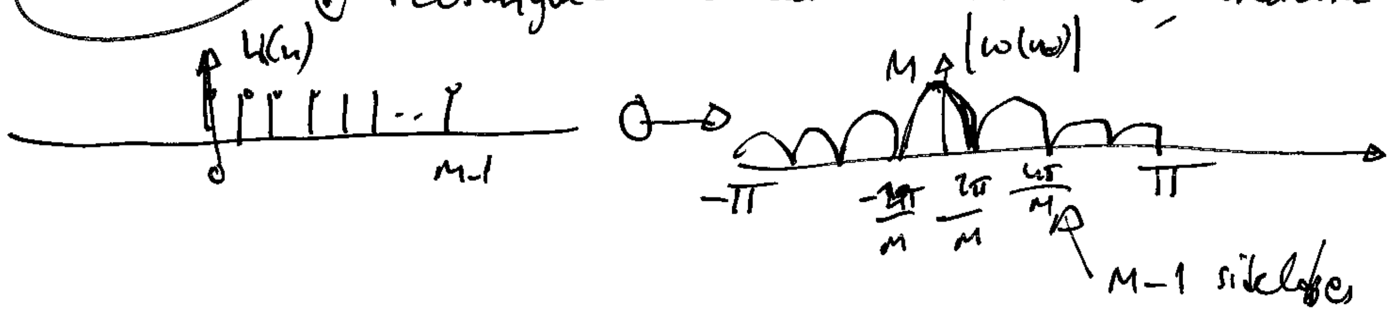
This implies that $H(\omega) = \text{DTFT}(h(n)) = W(\omega) * H_d(\omega) =$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} W(\nu) H(\omega - \nu)\, d\nu$$

Complex periodic convolution

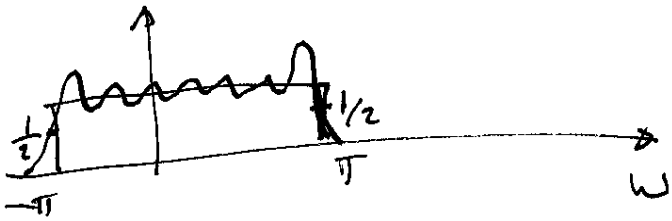
WINDOWS:

(1) Rectangular window $W(\omega) = \begin{cases} 1, & n = 0, \dots, M-1 \\ 0, & \text{otherwise} \end{cases}$



2

* The narrower the main lobe of $W(\omega)$ the more $H_d(\omega) \simeq H(\omega)$
* $H_d(\omega) * W(\omega)$ has the effect of smoothing $H_d(\omega)$



Gibbs phenomenon

* The sidelobes create undesired oscillating variations
* If M increases the sidelobe width decreases but the area underneath remains the same so the effect of sidelobes remains the same (Gibbs phenomenon)
* Rectangular window has significant sidelobes (13 dB below main lobe)

In general we want $\rightarrow$ w(n) as short as possible to minimize computations.
narrow main lobe
low magnitude sidelobes

So many other windows have been proposed with lower sidelobes that the rectangular window. $\longrightarrow$ wider main lobe

(there is a trade off between, width of main lobe and size of sidelobes)

For fixed M the rectangular window gives the best MSE (mean square error) that is:

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} |H_d(\omega) - H(\omega)|^2 = \text{minimum}$$

Window	Peak amplitude of sidelobes	Width of main lobe
Rectangular	-13 dB	$\frac{4\pi}{M}$ $\left(2\times\frac{2\pi}{M}\right)$
Bartlett (triangular)	-25 dB	$\frac{8\pi}{M}$
Blackman	-57 dB	$\frac{12\pi}{M}$
Hamming	-41 dB	$\frac{8\pi}{M}$
Hanning	-57 dB	$\frac{8\pi}{M}$

BASIC CONCLUSION:

Through the choice of the window shape and duration we can exercise some control over the design process.
(tradeoff between sidelobe amplitude that affects the stopband attenuation and main lobe width that affects the length of the transition region)

* However, lack of precise control of critical frequencies ω_s, ω_p.

PRACTICAL LIMITATION: Difficulty in determining $h_d(n)$ in step 2

So in practice we sample $H_d(\omega)$ with M points and then apply M-IDFT

(which creates time aliasing)

the higher M the less time aliasing