Realization of FIR Filter using Frequency - Sampling Structure

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Recall : In a previous lecture, you saw how the frequency-sampling method could be used to DESIGN FIR filters, i.e., given filter specifications, how to design a filter that best approximates these specs.

Goal : In this tutorial, we look at a related problem : realization of FIR filter, i.e., given an impulse response, how do we implement it? It turns out that the frequency-sampling concept can also be applied in this context.

Method :

* Given an FIR filter with impulse response

$$h[n] = \begin{cases} h[0],\ h[1],\ \ldots,\ h[N-1] \\ 0 \quad , \text{ for } n<0 \text{ and } n \geq N \end{cases} \tag{1}$$

$$\Big\updownarrow \text{ N-DFT}$$

$$H[k] \ ,\ k = 0, 1, \cdots, N-1 \tag{2}$$

* Consider the transfer function :

$$H(z) = \sum_{n=0}^{N-1} h[n] z^{-n} = \sum_{n=0}^{N-1} \Bigg[\underbrace{\frac{1}{N} \sum_{k=0}^{N-1} H[k] e^{\frac{jkn2\pi}{N}}}_{\text{IDFT}} \Bigg] z^{-n}$$

②

$$= \frac{1}{N} \sum_{n=0}^{N-1} H(k) \overbrace{\left[\sum_{n=0}^{N-1} \left(e^{j\frac{2\pi k}{N}} z^{-1} \right)^{n} \right]}^{\text{geometric series}}$$

$$= \underbrace{\left[\frac{1}{N} \left(1 - z^{-N} \right) \right]}_{Ⓐ} \underbrace{\left[\sum_{n=0}^{N-1} \frac{H(k)}{1 - e^{j 2\pi k/N} z^{-1}} \right]}_{Ⓑ} \qquad (3)$$

* Eq (3) suggests a cascade implementation of 2 parts :

  Ⓐ a comb filter

  Ⓑ parallel network or bank of resonators.

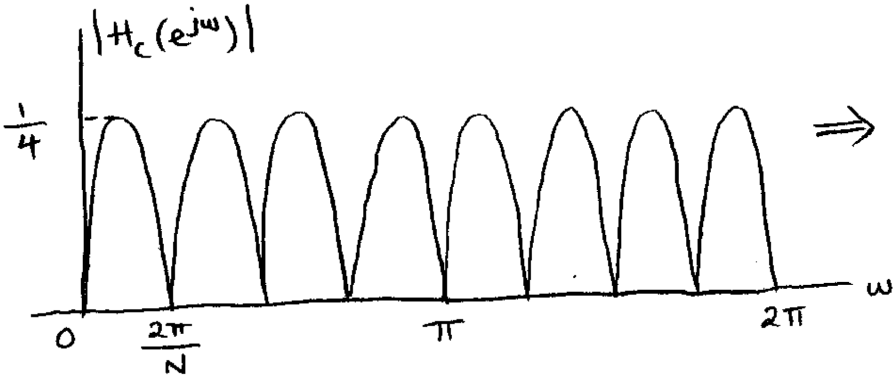
* <u>Comb filter</u> :

$$\frac{1 - z^{-N}}{N} = H_c(z) \text{ is basically an FIR filter} \qquad (4)$$

Also the freq. response :

$$H_c(e^{jw}) = \frac{1 - e^{-jNw}}{N} = \frac{2j}{N} e^{-jNw/2} \sin(Nw/2) \qquad (5)$$

For $N = 8$, the mag. response $|H_c(e^{jw})|$:



⇒ "comb"

* **Resonators** :

$$H_{R,k}(z) = \frac{H(k)}{1 - e^{j2\pi k/N} z^{-1}} \qquad (6)$$

is known as a resonator or a one-pole filter, with pole on the unit circle, at frequency $\omega_k = 2\pi k/N$.

$\Rightarrow$ B in (3) is a bank of N resonators

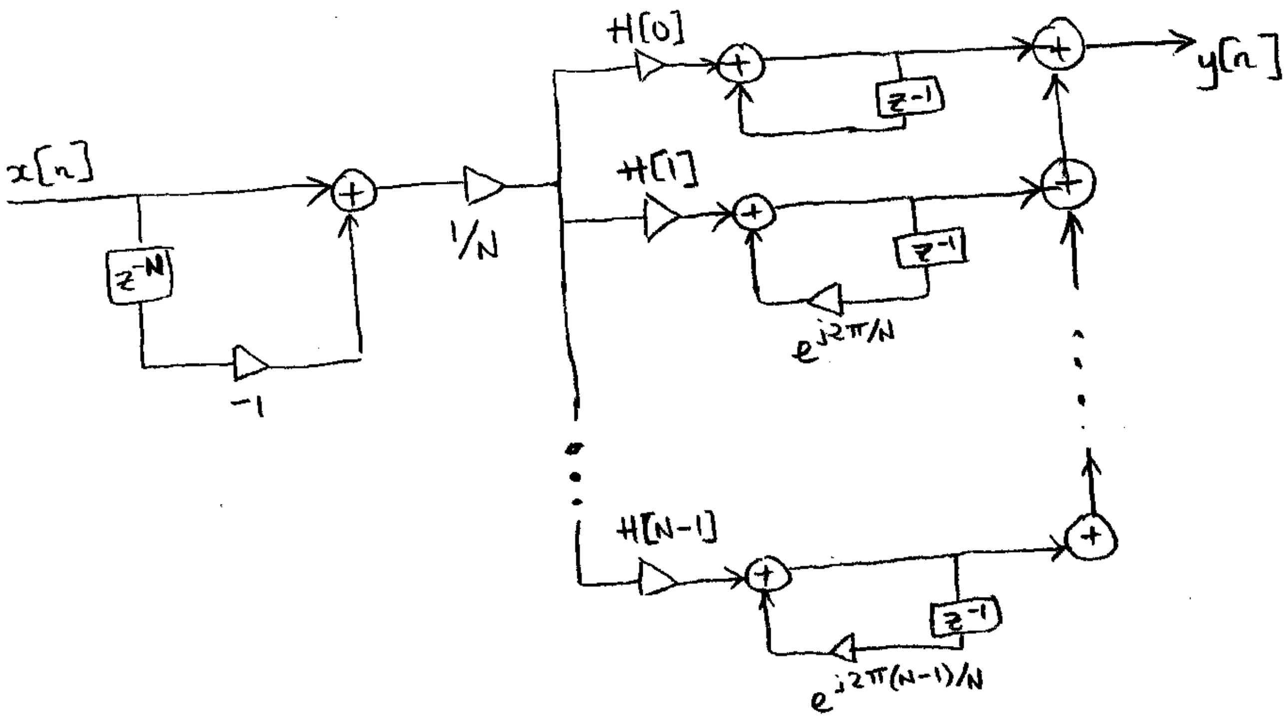
$\Rightarrow$ recursive implementation.

* **Pole-zero cancellation** :

The poles at $e^{j2\pi k/N}$ due to B in (3) are cancelled by the zeros due to A, the comb filter, which are the roots of

$$1 - z^{-N} = 0 \qquad (7)$$

* **Implementation** : From (3), we have a cascade of :



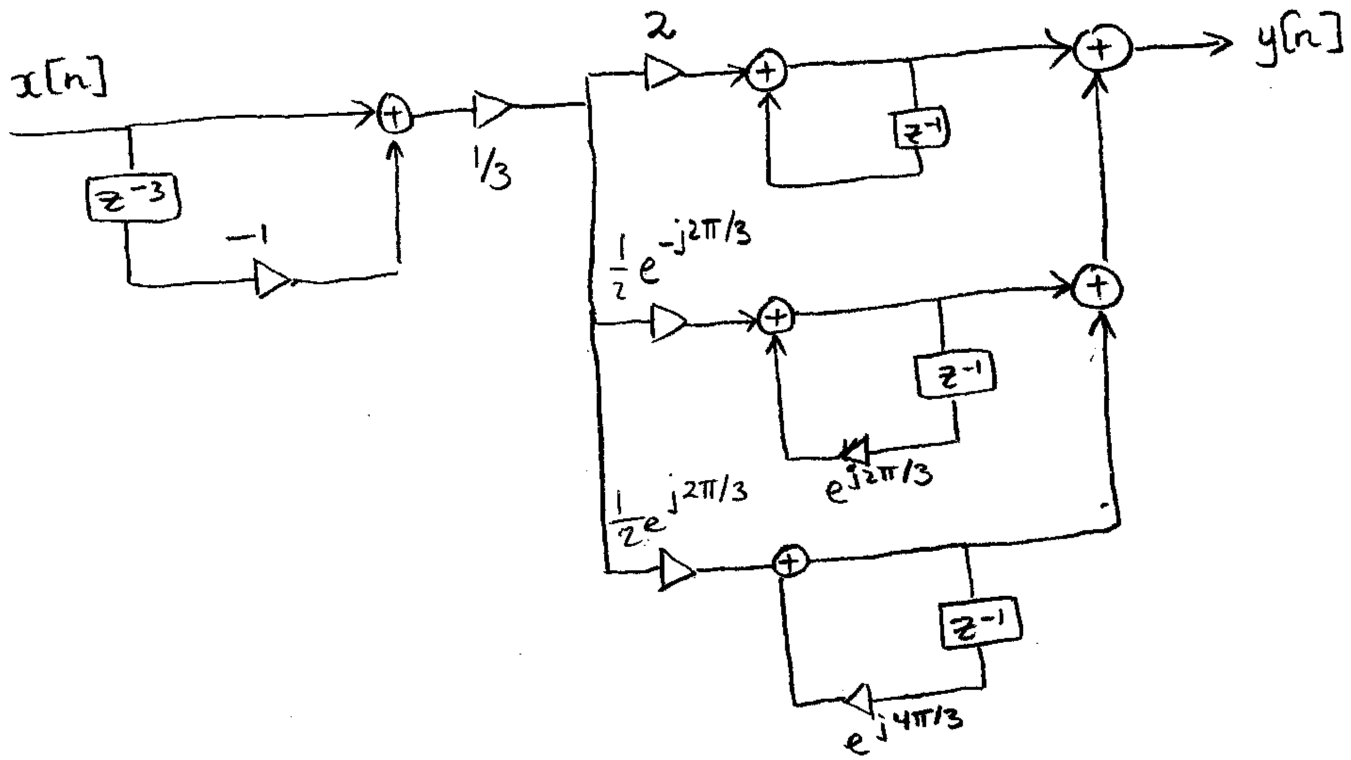
Example : Given an FIR filter with impulse response

$$h[n] = \begin{cases} 1/2 & , n = 0, 2 \\ 1 & , n = 1 \\ 0 & , \text{otherwise} \end{cases}$$

* Here $N = 3$; compute 3-DFT :

$$H[k] = \left\{ 2, \frac{e^{-j2\pi/3}}{2}, \frac{e^{j2\pi/3}}{2} \right\}$$

$$\Rightarrow \quad H(z) = \frac{1}{3}\left(1 - z^{-3}\right)\left[\frac{2}{1-z^{-1}} + \frac{\frac{e^{-j2\pi/3}}{2}}{1-e^{j2\pi/3}z^{-1}} + \frac{\frac{e^{j2\pi/3}}{2}}{1-e^{j4\pi/3}z^{-1}}\right] \tag{8}$$



Problem : The above example requires the use of COMPLEX ARITHMETIC

$\Longrightarrow$ Is real-valued processing possible ?

Real-valued Realization

When $h[n]$ are real-valued for all n, it is possible and desirable to obtain a realization with real arithmetic

$\Rightarrow$ exploit the symmetry of the DFT and the $e^{j2\pi k/N}$ factor.

Example: continue with the previous example, we observe $H[2] = H[1]^*$, so that we can rewrite (8) as follows,

$$H(z) = \frac{1}{3}(1-z^{-3})\left[\frac{2}{1-z^{-1}} + \frac{\frac{e^{-j2\pi/3}}{2}}{1-e^{j2\pi/3}z^{-1}} + \frac{\frac{e^{j2\pi/3}}{2}}{1-e^{-j2\pi/3}z^{-1}}\right]$$

$$= \frac{1}{3}(1-z^{-3})\left[\frac{2}{1-z^{-1}} + \frac{\cos(2\pi/3) - \cos(4\pi/3)z^{-1}}{1-2\cos(2\pi/3)z^{-1}+z^{-2}}\right]$$

$$= \frac{1}{3}(1-z^{-3})\left[\frac{2}{1-z^{-1}} + \underbrace{\frac{-\frac{1}{2}(1-z^{-1})}{1+z^{-1}+z^{-2}}}_{\text{IIR} \rightarrow 2^{nd} \text{ order resonator}}\right]$$