PARALLEL FORM REALIZATION

Express $H(z)$ as a partial-fraction expansion in the form of first- and second-order system (or may group the real poles and conjugate-complex poles together)

Assuming that $N \geq M$ and that all poles are distinct (no loss of generality)

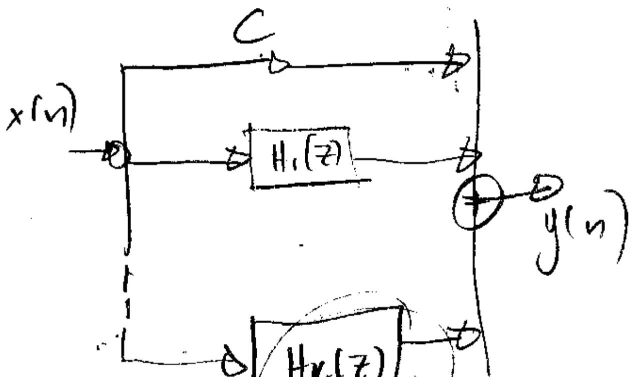
$$H(z) = C + \sum_{k=1}^{K} H_k(z) = C + \sum_{k=1}^{K_1} \frac{b_{k0} + b_{k1} z^{-1}}{1 + a_{k1} z^{-1} + a_{k2} z^{-2}} =$$

second order terms

$C = 0$ if $M < N$

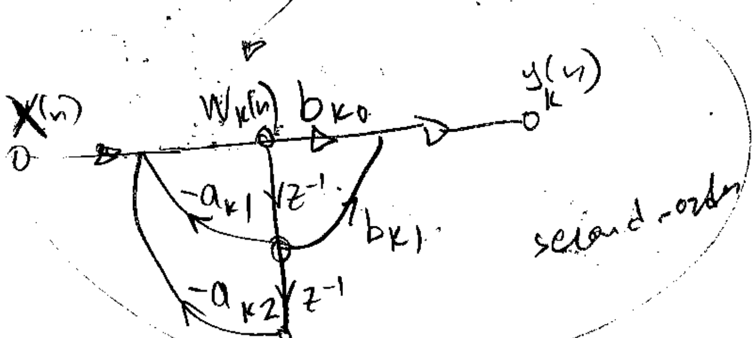
$$= C + \sum_{k=1}^{N} \frac{A_k}{1 + p_k z^{-1}}$$

– First order terms



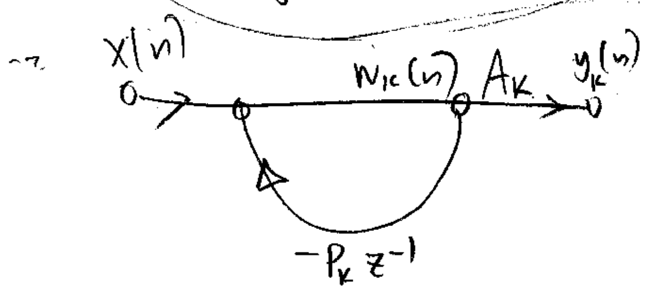
→ Depending on realization sensitivities to quantiz. errors

→ High throughput rate



D.E $k = 1, \ldots, K$

$$w_k(n) = -a_{k1} w_k(n-1) - a_{k2} w_k(n-2) + x(n)$$
$$y_k(n) = b_{k0} w_k(n) + b_{k1} w_k(n-1)$$
$$y(n) = C x(n) + \sum_{k=1}^{K} y_k(n)$$

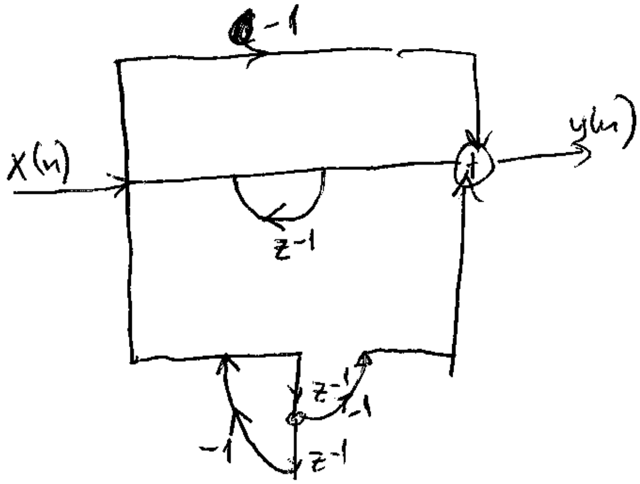


DE

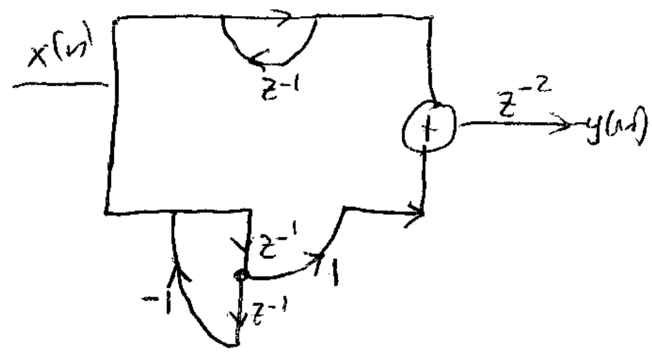
$$w_k(n) = -P_k w_k(n-1) + x(n)$$
$$y_k(n) = A_k \, w_k(n)$$
$$y(n) = C x(n) + \sum_{k=1} y_k(n)$$

$$H(z) = \frac{z+1}{(z-1)(z+j)(z-j)} = \frac{z+1}{(z-1)(z^2+1)} = \frac{z^{-3}}{z^{-3}}\left(\quad \right) = \frac{z^{-2}(1+z^{-1})}{(1-z^{-1})(1+z^{-2})}$$

$$= \begin{cases} \dfrac{z^{-2}+z^{-3}}{(1-z^{-1})(1+z^{-2})} = -1 + \dfrac{A}{(1-z^{-1})} + \dfrac{B+Cz^{-1}}{1+z^{2}} = -1 + \dfrac{1}{1-z^{-1}} + \dfrac{z^{-1}}{1+z^{2}} \\[2ex] z^{-2}\left[\dfrac{1+z^{-1}}{(1+z^{-1})(1+z^{-2})} \right] = z^{2}\left[\dfrac{D}{1-z^{-1}} + \dfrac{E+Fz^{-1}}{1+z^{2}} \right] = z^{2}\left[\dfrac{1}{1-z^{-1}} + \dfrac{z^{-1}}{1+z^{-2}} \right] \end{cases}$$



Higher throughput rate



Lower roundoff noise power

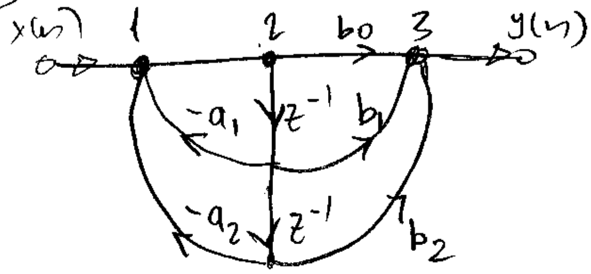
TRANSPOSED FORMS REALIZATIONS

(5)

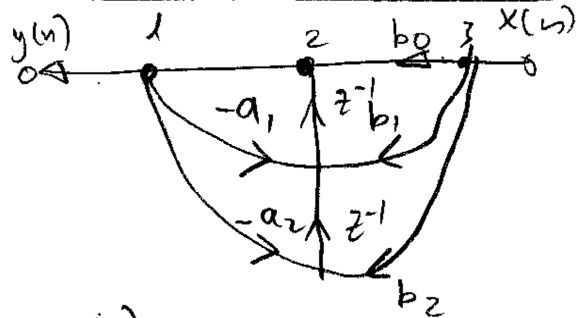
Transposition or flow-graph reversal Theorem:

if we reverse the directions of all branch transmittances and interchange the input and output in the flow graph, the system function remains unchanged. The resulting structure is called "transposed structure" or "transposed form".

Ex. Direct form II of second order system

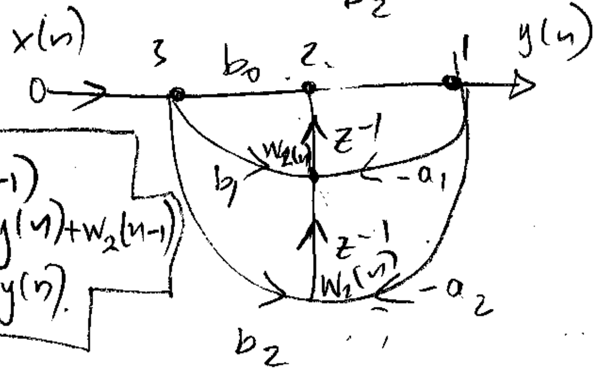


Transposed Direct II



- branching nodes → become addition nodes and vice versa.

or



D.E

$$\begin{aligned} y(n) &= b_0 x(n) + w_1(n-1) \\ w_1(n) &= b_1 x(n) - a_1 y(n) + w_2(n-1) \\ w_2(n) &= b_2 x(n) - a_2 y(n) \end{aligned}$$

- So in all previous realizations you can replace subsystem realization by transposed forms

- Same # of mult., addit., memory locations as original form
- Same system function.