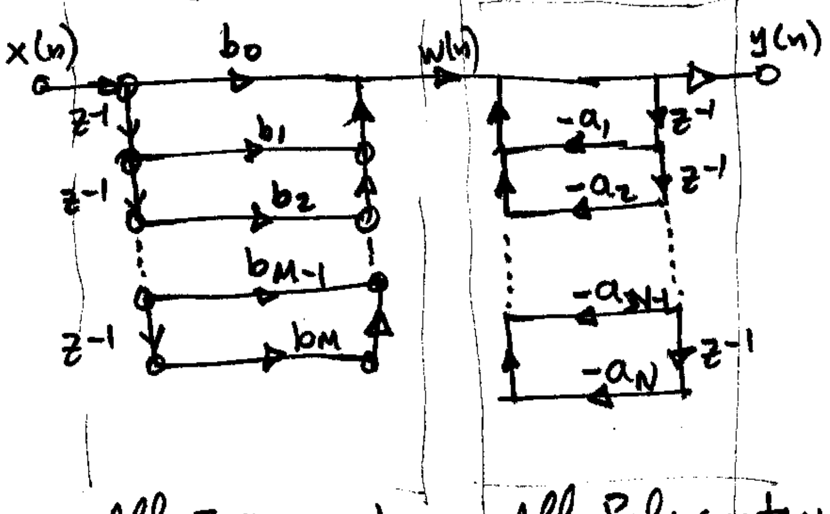
Section 6.3 Realization for IIR Systems

②

(.1) $$* \quad y(n) = -\sum_{k=1}^{N} a_k y(n-k) + \sum_{k=0}^{M} b_k x(n-k), \qquad ** \quad H(z) = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1+\sum_{k=1}^{N} a_k z^{-k}}.$$

DIRECT FORM I (because * can be written by inspection from ** without any rearrangement)

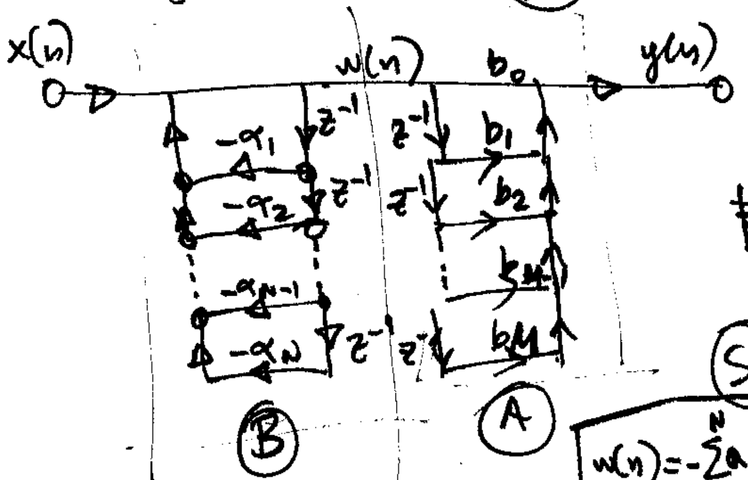


All-zero system (A)　　All-Pole system (B)

- M+N+1 multiplications
- M+N additions
- M+N memory locations for the signal
- more nodes than necessary however consistent with the fact that digital hardware or software typically involves an addition of two numbers at a time
- sensitive to quantization effects

DIRECT FORM II

Interchange (B) and (A) $(A*B = B*A)$



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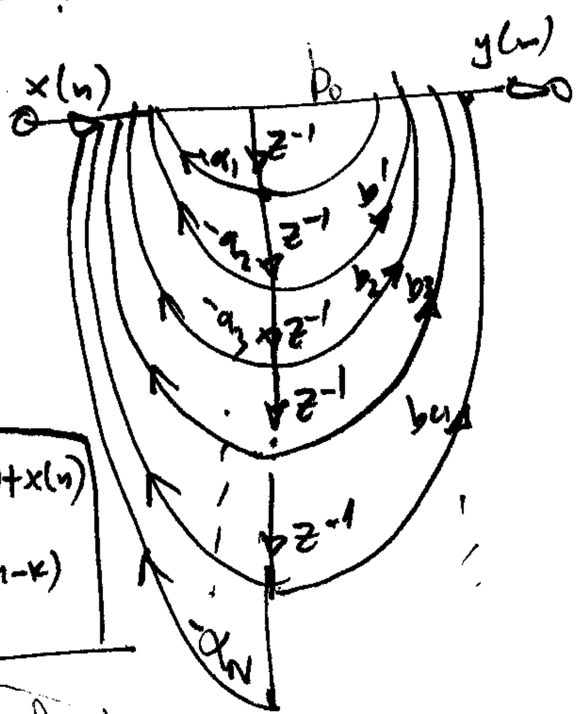


So

$$\boxed{\begin{aligned} w(n) &= -\sum_{k=1}^{N} a_k w(n-k) + x(n) \\ y(n) &= \sum_{k=0}^{M} b_k w(n-k) \end{aligned}}$$

- M+N+1 multiplications
- M+N additions
- Max{M,N} memory locations (for the signal) (minimum # of delays) "canonic form"
- sensitive to quantization effects

CASCADE FORM (7.3.3) 6.3, 6.4 (3)

$$H(z) = G \prod_{k=1}^{K} H_k(z) = G \prod_{k=1}^{K} \frac{1 + b_{k1} z^{-1} + b_{k2} z^{-2}}{1 + a_{k1} z^{-1} + a_{k2} z^{-2}}$$

G: fixed gain parameter

$\{a_{ki}\}, \{b_{ki}\}$ real.

Group ~~together~~ first (ones pairs of complex conjugate poles and zeros) (increase to have real coefficients)

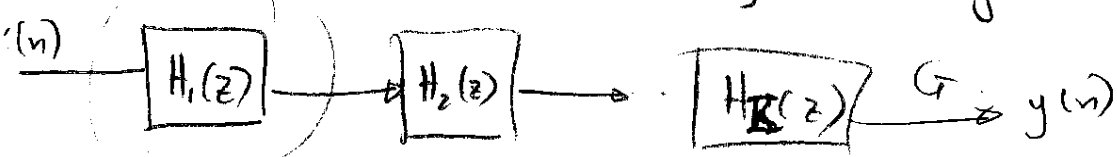
and then pairs of real poles or zeros

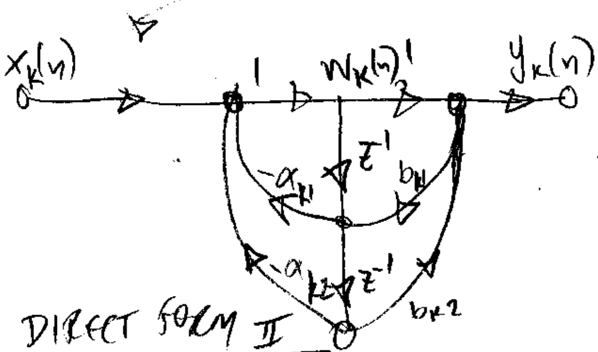
Thus the quadratic factor the numerator or denominator may consist either of a pair of complex-conjugate roots or a pair of real roots.

Assuming in general that $M \leq N$

if N is odd we may have also a first-order component

- First decompose $H(z)$ as above
- Realize each of the quadratic subsystems using direct I or II form





DIRECT FORM II

Difference equations

for K system $k = 1, \ldots, K$.

$$w_k(n) = -a_{k1} w_k(n-1) - a_{k2} w_k(n-2) + x_k(n)$$

$$y_k(n) = b_{k1} w_k(n-1) + b_{k2} w_k(n-2) + w_k(n)$$

$y_{k-1}(n)$

- Different ways of pairing poles and zeros result in ~~different ways of~~ various cascade realizations which are equivalent for infinite precision but may differ significantly when implemented with finite-precision arithmetic.

Problem 4 (12.5 points)

1. Consider the causal linear time invariant discrete time filter with system function

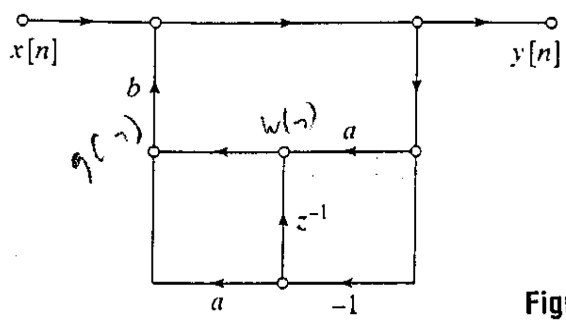
$$H(z) = \frac{1 - \frac{1}{3}z^{-1}}{(1 - \frac{1}{2}z^{-1} + \frac{1}{3}z^{-2})(1 + \frac{1}{4}z^{-1})}$$

   (a) Draw the Direct form II realization.

   (b) Draw the Cascade form realization using first- and second-order direct form II sections.

   (c) Draw the Parallel form realization using first- and second-order direct form II sections.

2. The flow graph shown in the figure is noncomputable, i.e., it is not possible to compute the output using the difference equations represented by the flowgraph because it contains a closed loop having no delay elements.



Figure

   (a) Write the difference equations for the system in the figure, and from them, find the system function $H(z) = \frac{Y(z)}{X(z)}$.

   (b) From the system function, obtain a flowgraph that is computable.