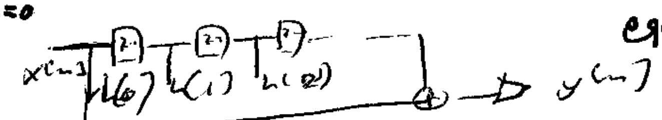
ECE 431 F / Fall 1999

Lecture 20 23 ~~XXXX~~

① Realizations for FIR systems/filters

$$y(n) = \sum_{k=0}^{M} b_k x(n-k) = \sum_{k=0}^{M} h(k)\, x(n-k)$$ (i.e. with $N=0$ in LCCDE equation)



- Direct form realization also known as "tapped delay line" or "transversal" filter.

- Cascade realization.
- There is no parallel realization.

[M+1: multiplications
M: additions
M: delay elements]

LINEAR PHASE FIR filters

FIR Filters can be designed to have exactly linear phase, (actually generalized linear phase) a very desirable operation in all filtering operations i.e in this case

$$H(\omega) = |H(\omega)|\, e^{-(j a\omega + j\beta)} \quad , a, \beta \text{ constants.}$$

so: $\left[\arg[H(\omega)] = -a\omega - \beta\right]$ and group delay of the system is constant, i.e,

$$\left[\mathrm{grd}[H(\omega)] = -\frac{d}{d\omega}\arg[H(\omega)] = a\right]$$

- For filters with linear phase ($\beta=0$) $x(n) \rightarrow \boxed{|H(\omega)|} \rightarrow \boxed{e^{-ja\omega}} \rightarrow y(n)$ (pure delay by a. in time.) so linear phase introduces constant delay in the input signal
- Similarly for "narrowband signals", generalized linear phase (i.e. constant group delay) introduced pure delay in the envelope of the input signal. ......

● Causal FIR systems have generalized linear phase if they have an impulse response of length $M+1$ if they satisfy either of two. symmetry conditions $h(n) = \pm\, h(M-n)$, $n=0,1,\dots,M$

(sufficient condition, $+$: symmetry , $-$: antisymmetry)

Ex. Let M = even no loss of generality and $h(n) = h(M-n)$

Then,
$$H(z) = \sum_{n=0}^{M} h(n) z^{-n} = \sum_{n=0}^{\frac{M}{2}-1} h(n) z^{-n} + h\left(\tfrac{M}{2}\right) z^{-\frac{M}{2}} + \sum_{n=\frac{M}{2}+1}^{M} h(n) z^{-n} =$$

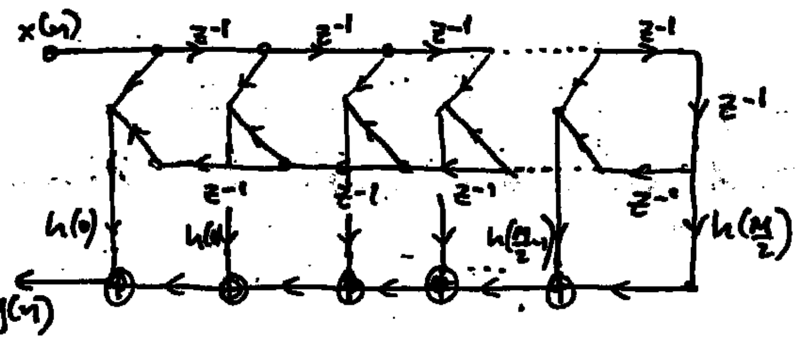
$$(*) \quad = \sum_{n=0}^{\frac{M}{2}-1} h(n) z^{-n} + h\left(\tfrac{M}{2}\right) z^{-\frac{M}{2}} + \sum_{n=0}^{\frac{M}{2}-1} h(M-n) z^{-(M-n)} =$$

$$= z^{-\frac{M}{2}} \left[h\left(\tfrac{M}{2}\right) + \sum_{n=0}^{\frac{M}{2}-1} h(n) \left[z^{-(n-\frac{M}{2})} + z^{-(\frac{M}{2}-n)} \right] \right]$$

So:
$$H(\omega) = H(z)\Big|_{z=e^{j\omega}} = e^{-j\omega\frac{M}{2}} \left[h\left(\tfrac{M}{2}\right) + \sum_{n=0}^{\frac{M}{2}-1} 2h(n)\cos\left[\omega\left(n-\tfrac{M}{2}\right)\right] \right]$$

In other words $\boxed{\arg\left(H(\omega)\right) = -\omega\frac{M}{2} + \begin{cases} 0 \\ \pi \end{cases}}$: generalized linear phase for $h(n)$: real.

Also:



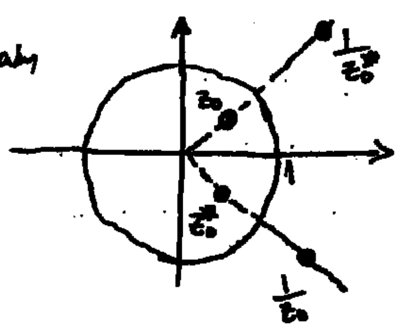
needs only $\left(\frac{M}{2}+1\right)$ multiplications

Also: (From $(*)$ above)

$$H(z) = z^{-M} \left[\sum_{n=0}^{\frac{M}{2}-1} h(n) z^{-(n-M)} + h\left(\tfrac{M}{2}\right) z^{\frac{M}{2}} + \sum_{n=0}^{\frac{M}{2}-1} h(M-n) z^{n} \right] = z^{-M} H(z^{-1})$$

Thus If $z = z_0$ is a zero of $H(z)$ then $z = z_0^{-1}$ is also a zero of $H(z)$

z-domain



- If a real LP FIR filter has zeros at $z = z_0$ it also has zeros at $z = z_0^*$, $z = \frac{1}{z_0}$, $z = \frac{1}{z_0^*}$ i.e. zeros appear in groups of 4.
- If a complex LP FIR system has zero at $z = z_0$ it also has zero at $z = 1/z_0$