Section 9.4 LECTURE 18 19

RADIX-2 Decimation in Frequency FFT. algorithm ($N=2^9$)

Idea: Divide $X(k)$ into smaller and smaller sequences: to perform smaller DFT's

* $N=2^9 \Rightarrow$ N even, Then,

step 1

$$X(k) = \sum_{n=0}^{N-1} x(n) W_N^{nk} = \sum_{n=0}^{\frac{N}{2}-1} x(n) W_N^{nk} + \sum_{n=\frac{N}{2}}^{N-1} x(n) W_N^{nk} =$$

$$= \sum_{n=0}^{\frac{N}{2}-1} x(n) W_N^{nk} + W_N^{\frac{N}{2}k} \sum_{m=0}^{\frac{N}{2}-1} x(m+\tfrac{N}{2}) W_N^{mk} \qquad (n = m+\tfrac{N}{2})$$

Now: $\boxed{W_N^{\frac{N}{2}k} = (-1)^k}$.

$$X(k) = \sum_{n=0}^{\frac{N}{2}-1} \left[x(n) + (-1)^k x(n+\tfrac{N}{2}) \right] W_N^{nk}$$

k: even $= 2r$ $\quad X(k) = X(2r) = \sum_{n=0}^{\frac{N}{2}-1} \left[x(n) + x(n+\tfrac{N}{2}) \right] W_N^{2rn}$

k: odd $= 2r+1$ $\quad X(k) = X(2r+1) = \sum_{n=0}^{\frac{N}{2}-1} \left[x(n) - x(n+\tfrac{N}{2}) \right] W_N^{2rn} W_N^{n}$

Also: $\boxed{W_N^{2rn} = W_{\frac{N}{2}}^{rn}}$ Thus:

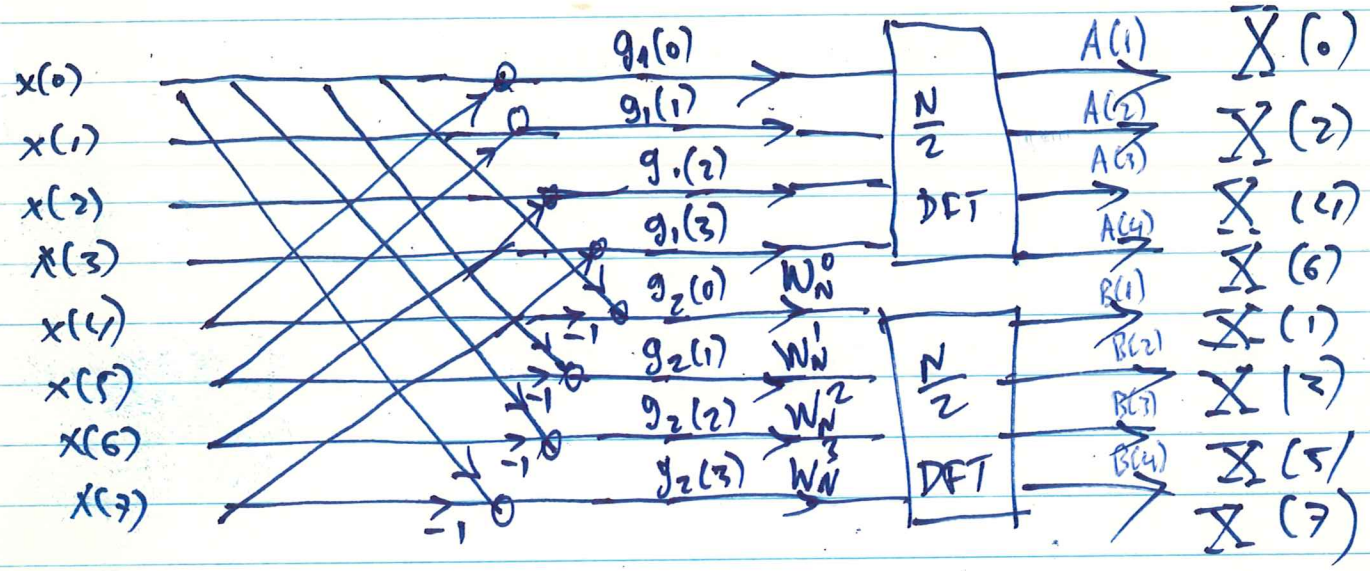
$$A(r) = X(2r) = \sum_{n=0}^{\frac{N}{2}-1} \underbrace{\left[x(n) + x(n+\tfrac{N}{2}) \right]}_{g_1(n)} W_{\frac{N}{2}}^{rn}$$

$\frac{N}{2}$-DFT of $g_1(n)$, $r = 0, 1, \ldots, \frac{N}{2}-1$

$$B(r) = X(2r+1) = \sum_{n=0}^{\frac{N}{2}-1} \underbrace{\left[x(n) - x(n+\tfrac{N}{2}) \right] W_N^{n}}_{g_2(n)} W_{\frac{N}{2}}^{rn}$$

$\frac{N}{2}$-DFT of $g_2(n) \cdot W_N^n$

Ex $N = 8 = 2^3$

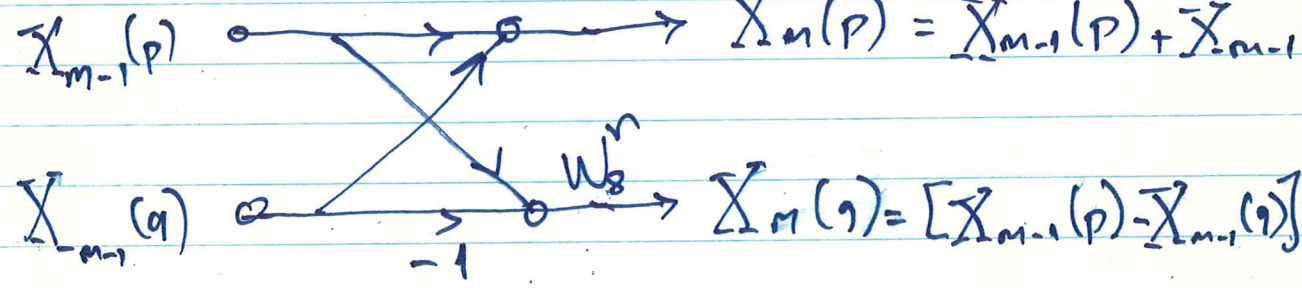


step 2 $\frac{N}{2}$ is even $\Rightarrow$ repeat procedure for each of the sequences $A(r)$, $r = 0, \ldots, \frac{N}{2}-1$ and $B(r)$, $r = 0, \ldots, \frac{N}{2}-1$

$\vdots$

Stop at 2-DFT after q steps

BASIC BUTTERFLY ELEMENT (at m-th stage)



ELE 431 S'

In bit reversed order

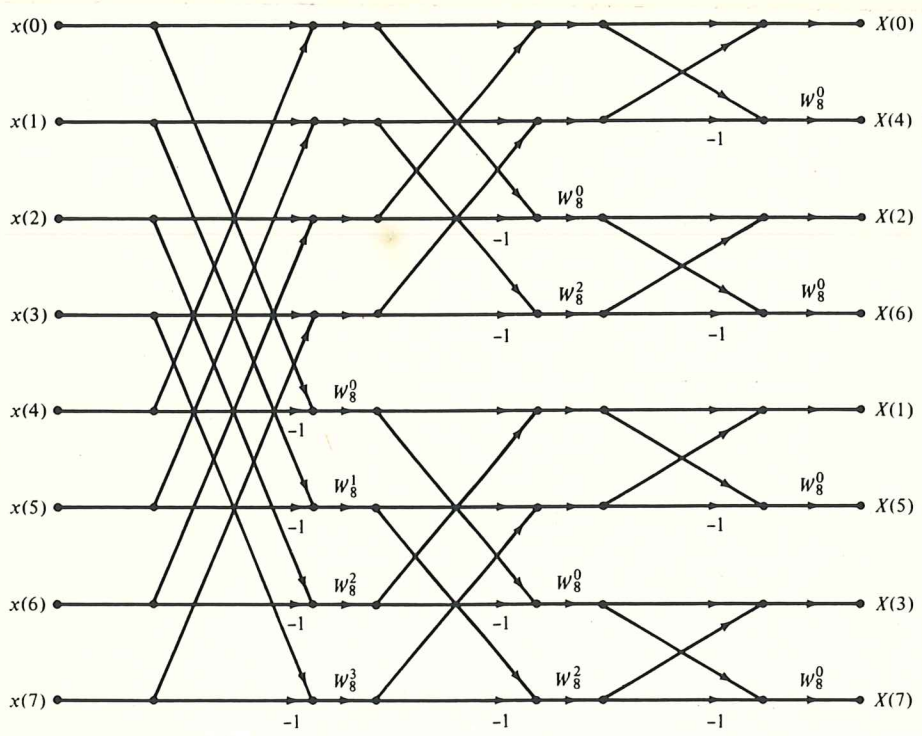


FIGURE $N = 8$-point decimation-in-frequency FFT algorithm.

stage 1 stage 2 stage 3.

In general for $N = 2^q$-DFT (Deimation in Frequency)

1) $q = \log_2 N$ stages, $N/2$ butterflies / stage.

2) m-th stage, 2^{q-m} different butterflies { multiply lower branch of each butterfly with $W_N^{\ell N/2^{q-m+1}}$, $\ell = 0, 1, .., 2^{m-1} - 1$ }

3) Input in normal order, output in bit reversed order.

4) # of $\otimes_c$: 1/butterfly $\times \frac{N}{2}$ but./stage $\times \log_2 N$ stages $= \frac{N}{2}\log_2 N$

of $\oplus_c$: 2/ " $\times \frac{N}{2}$ " $\times$ " $= N \log_2 N$

of storage locations: 2 / butt. $\times \frac{N}{2}$ butterflies (in place computation) $= N$.

5) $2^{\log_2 N - 1}$ exponentials stored in look up table.