LECTURE

Section 9.3

Radix-2 Decimation in time FFT algorithms.

<u>1st step:</u> From last lecture

$$X(k) = \underbrace{\sum_{m=0}^{\frac{N}{2}-1} x(2m) W_{N/2}^{mk}}_{A(k),\ \frac{N}{2}\text{-DFT},\ k=0,1,2,\ldots,\frac{N}{2}-1} + W_N^k \underbrace{\sum_{m=0}^{N/2-1} x(2m+1) W_{N/2}^{mk}}_{B(k),\ \frac{N}{2}\text{-DFT},\ k=0,\ldots,\frac{N}{2}-1} \equiv \begin{cases} X(k) = A(k) + W_N^k B(k) \\ X(k+\frac{N}{2}) = A(k) - W_N^k B(k) \end{cases} \quad k=0,1,\ldots,\frac{N}{2}-1$$

<u>2nd step</u> ($\frac{N}{2}$ even)

$a(m) = x(2m)$

$$A(k) = \sum_{m=0}^{\frac{N}{2}-1} a(m) W_{N/2}^{mk} = \sum_{\substack{l=0 \\ (m=2l)}}^{\frac{N}{4}-1} a(2l) W_{\frac{N}{2}}^{2lk} + \sum_{\substack{l=0 \\ (m=2l+1)}}^{\frac{N}{4}-1} a(2l+1) W_{N/2}^{(2l+1)k} \Rightarrow$$

$$\to A(k) = \underbrace{\sum_{l=0}^{\frac{N}{4}-1} \underset{x(4l)}{a(2l)} W_{N/4}^{lk}}_{C(k),\ N/4\text{-DFT},\ k=0,1,\ldots,\frac{N}{4}-1} + \underset{\underset{W_N^{2k}}{\|}}{W_{N/2}^{k}} \underbrace{\sum_{l=0}^{\frac{N}{4}-1} \underset{x(4l+2)}{a(2l+1)} W_{N/4}^{lk}}_{D(k),\ N/4\text{-DFT},\ k=0,1,\ldots,\frac{N}{4}-1}$$

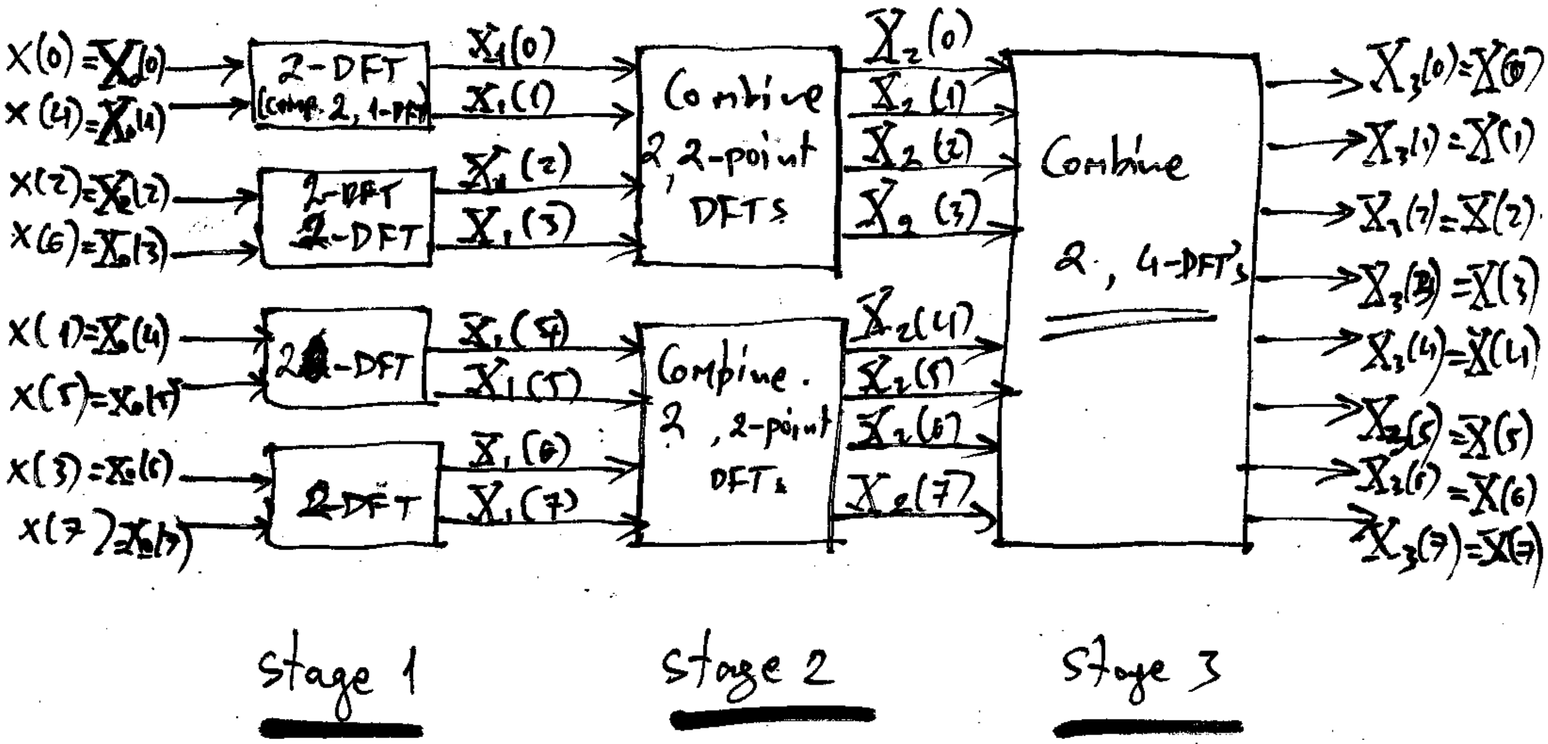
Similarly for, $B(k) = \sum_{l=0}^{\frac{N}{4}-1} x(4l+1) W_{N/4}^{lk} + W_{N/2}^{k} \sum_{l=0}^{\frac{N}{4}-1} x(4l+3) \ldots$

<u>3rd step</u> $\Rightarrow \frac{N}{4}$ is even $\longrightarrow$ Break each of the $\frac{N}{4}$-DFTs to two $\frac{N}{8}$-DFTs and so on

⋮

~~[illegible]~~ . . .

Ex Complete Radix-2, Decim. in time FFT flowchart for $N = 8 = 2^3$

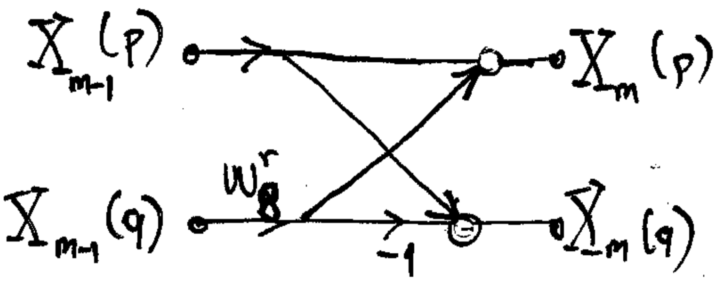


* total of $3 = \log_2 8$ stages
* All stages have 8 inputs, 8 outputs
* Input data in bit reversed order, *i.e.* for $n = 0, 1, \ldots, 7$

$$X_0(n_0, n_1, n_2) = x(n_2, n_1, n_0) \quad \text{in } \underline{3 \text{ bit number repres.}} \ (8 = 2^3)$$

for ex. $X_0(4) = X_0(100) = x(001) = x(1)$

* Basic computation element for the m-th stage.

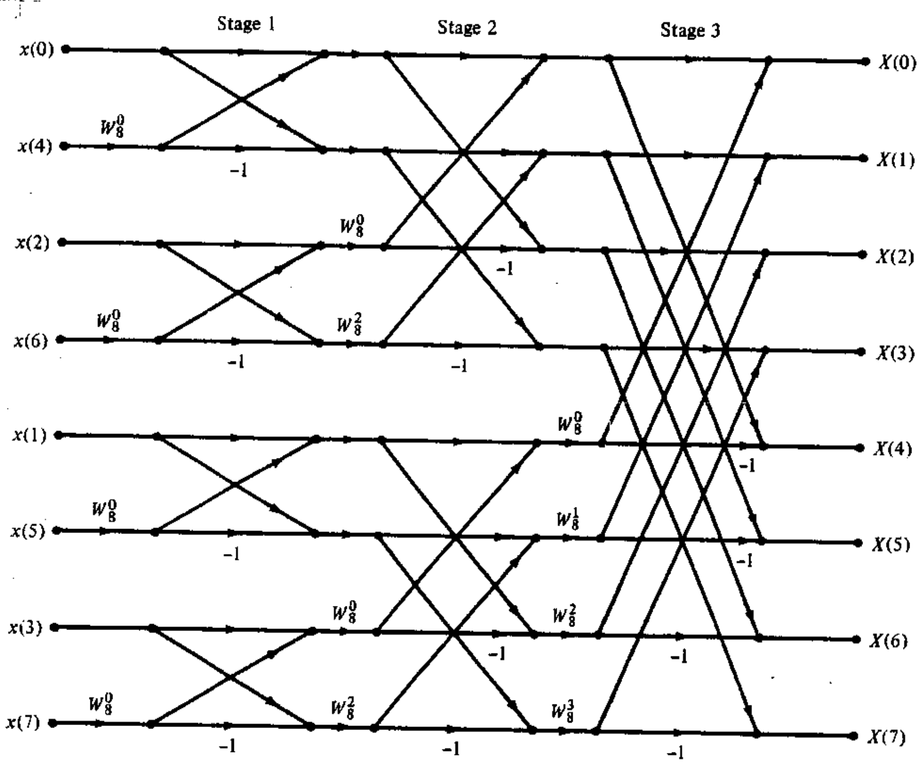


BUTTERFLY COMPUTATION

- 1 multiplication
- 2 additions
- 2 storage locations (in place computations)
- r changes from stage to stage and butterfly to butterfly.

ELE 431S

Input in bit reversed order



Output in normal order

FIGURE Eight-point decimation-in-time FFT algorithm.

In general for $N=2^q$ - FFT (Decimation in time).

1) $q=\log_2 N$ stages, $N/2$ butterflies / stage

2) m-th stage, 2^{m-1} different butterflies $\left\{ \text{Multiply lower branches of each butterfly } W_N^{\ell N/2^m},\ \ell=0,\dots,2^{m-1}-1 \right\}$

3) # of $\otimes_c$: 1/butterfly $\times \frac{N}{2}$ butt/stage $\times \log_2 N$ stages $= \frac{N}{2}\log_2 N$

of $\oplus_c$: 2/ " $\times \frac{N}{2}$ " $\times$ " " $= N\log_2 N$

of storage locations: 2 / butterfly $\times \frac{N}{2}$ butt. $=$ (in place computation) $= N$

4) $2^{\log_2 N-1}$ exponentials stored in look up table

5) $G(N) = \dfrac{\#\text{ of FFT } \otimes}{\#\text{ of DFT } \otimes} = \dfrac{1}{2N}\log_2 N \ll 1$. For ex. $G(8) = \frac{3}{16}$

$G(4096) \simeq 0.0015$