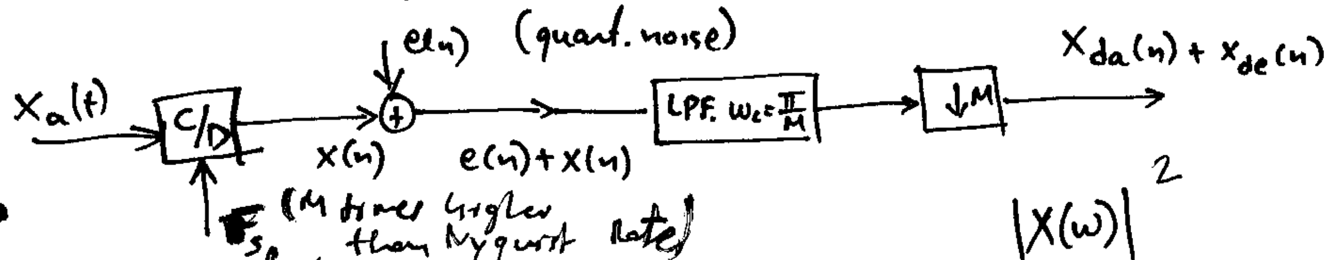
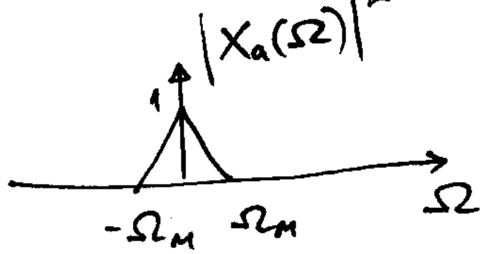
Oversampling in A/D conversion — Lecture 14



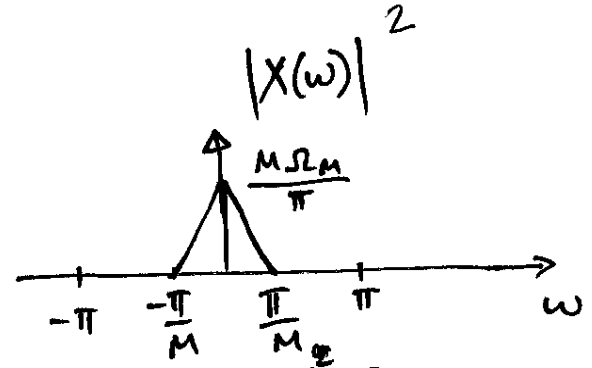
Ω_M



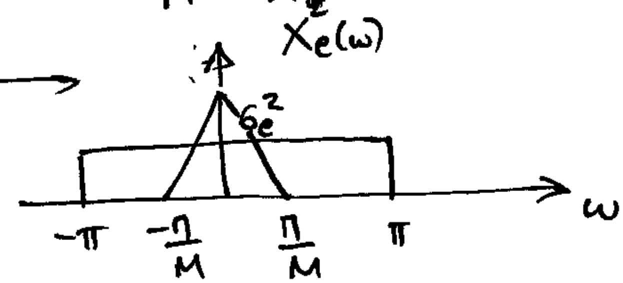
Oversampling by a factor M $\longrightarrow$

$2\pi F_s = 2M\Omega_M$

$\Rightarrow T = \frac{\pi}{M\Omega_M}$



In terms of power $\longrightarrow$



Quantization noise (random, variance $\sigma_e^2 = \frac{\Delta^2}{12}$

Explain that the total power of $e(n)$, and the total power of $x(n)$ are the same irrespectively of M $(M \geq 1)$

However, the power of the noise can be significantly reduced via filtering

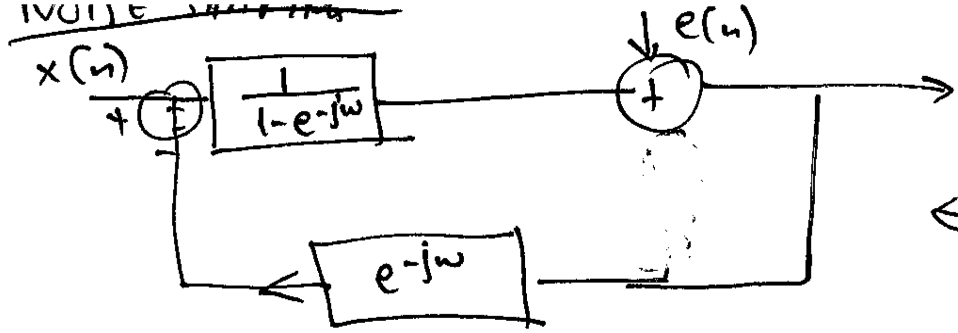
So $SQNR = 10\log_{10} \frac{\text{Power Signal}}{\text{Power of noise}} = 10\log_{10}\frac{P_{da}}{P_{de}} = 10\log_{10}\frac{P_{da}}{\frac{\sigma_e^2}{M}} =$

$= 10\log_{10}\frac{P_{da}}{\sigma_e^2} + 10\log_{10} M = 10\log_{10} P_{da} + 10\log_{10}\frac{M}{\sigma_e^2}$

$= 10\log_{10} P_{da} + 10\log_{10}\frac{M}{\frac{X_m^2}{\frac{2^{2B}}{12}}} = 10\log_{10} P_{da} + 10\log_{10}\frac{M\cdot 2^{2B}}{12 X_m^2}$

So for $M \cdot 2^{2B} = A$ ~~for every~~ You save one bit for every increase of M by $\underline{4}$

~~Noise shaping~~



$$y(n) = y_x(n) + y_e(n)$$

← noise shaping or sigma delta modulator

(1) $$\frac{Y_x(\omega)}{X(\omega)} = \frac{\frac{1}{1-e^{-j\omega}}}{1+\frac{e^{-j\omega}}{1-e^{-j\omega}}} = \frac{\frac{1}{1-e^{-j\omega}}}{\frac{1-\cancel{e^{-j\omega}}+\cancel{e^{-j\omega}}}{1-e^{-j\omega}}} = 1$$

(2) $$\frac{Y_e(\omega)}{E(\omega)} = \frac{1}{1-\frac{-e^{-j\omega}}{1-e^{-j\omega}}} = \frac{1-e^{-j\omega}}{1-e^{-j\omega}+e^{-j\omega}} = 1-e^{-j\omega} =$$

$$= e^{-j\frac{\omega}{2}}\left(e^{j\frac{\omega}{2}} - e^{-j\frac{\omega}{2}}\right) = 2j e^{-j\frac{\omega}{2}} \sin\left(\frac{\omega}{2}\right)$$

$$= 2\sin\left(\frac{\omega}{2}\right) e^{-j\left(\frac{\omega}{2}-\frac{\pi}{2}\right)} = H_e(\omega)$$

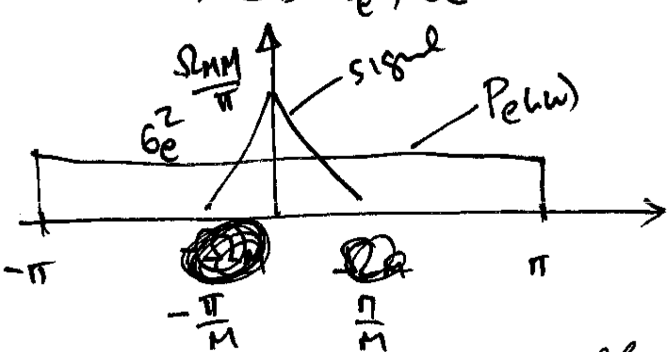
Power spectral density:

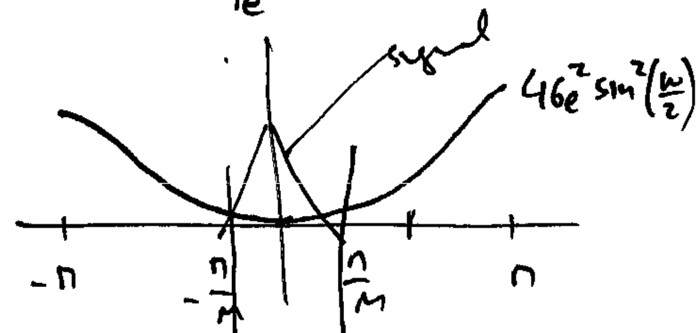
$$y_e(n) = e(n) * h_e(n)$$

~~$Y_e(\omega) = E(\omega)\cdot H_e(\omega) = H_e(\omega)$~~

random $E(\omega)$

$PSD = P_e(\omega) = \sigma_e^2$ $\quad \longrightarrow P_{Y_e}(\omega) = \sigma_e^2 \cdot |H_e(\omega)|^2$





in this case we find that $2^{2B} M^3 = ct$ →
→ even higher bit savings

filter here