LECTURE 11

PROPERTIES OF DFT (Continued)

LINEAR AND CIRCULAR CONVOLUTION USING THE DFT

Given, $x_1(n) = \begin{cases} \neq 0 & , n = 0, \dots, N_1 - 1 \\ 0 & , \text{otherwise} \end{cases}$, $x_2(n) = \begin{cases} \neq 0 & , n = 0, \dots, N_2 - 1 \\ 0 & , \text{otherwise} \end{cases}$

→ LINEAR CONVOLUTION

$$x_L(n) = x_1(n) * x_2(n) = \sum_{k=0}^{N_1-1} x_1(k) x_2(n-k) = \begin{cases} \neq 0, & 0 \leq n \leq N_1 + N_2 - 2 \\ 0, & \text{otherwise} \end{cases}$$

or in DTFT terms:

$$\underline{X_L(\omega) = X_1(\omega) X_2(\omega)}$$

By sampling $X_L(\omega)$: $X(k) = X_L\left(\omega = \frac{2\pi k}{N}\right) = X_1(k) X_2(k)$

Then we know that: $X(k) \overset{N\text{-IDFT}}{\longleftrightarrow} \sum_{\ell} x_L(n + \ell N)$

→ CIRCULAR CONVOLUTION

$$x_c(n) = x_1(n) \underset{N}{\circledast} x_2(n) = \left[\sum_{\ell} x_L(n + \ell N)\right] R_N(n) \quad , n = 0, 1, \dots, N-1$$

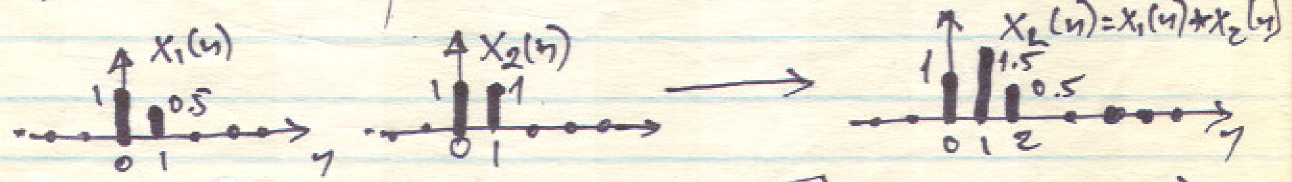
in N-DFT terms

$$\underline{X_c(k) = X_1(k) \cdot X_2(k)} \quad , k = 0, 1, \dots, N-1$$

Thus: * In discrete time domain circular convolution is one period $^{(N)}$ of the periodic extension of linear convolution with period N

* If $N \geq N_1 + N_2 - 1$ then $x_c(n) \equiv x_L(n)$

Ex:



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EXERSISE: Given the $x_1(n)$, $x_2(n)$ of previous example.

Perform the steps

① $x_1(n) \xrightarrow{\text{2-DFT}} X_1(k)$

② $x_2(n) \xrightarrow{\text{2-DFT}} X_2(k)$

③ $X_c(k) = X_1(k) \cdot X_2(k)$

$k=0,1$

④ $X_c(k) \xrightarrow{\text{2-IDFT}} x_c(n)$, $n=0,1$ (Verify result in A)

(Repeat with 4-DFT and 4-IDFT $\longrightarrow$ verify result in B)

Convolution in frequency domain.

Based on duality: $x_1(n) \cdot x_2(n) \xleftrightarrow{\text{N-DFT}} \frac{1}{N} X_1(k) \circledast_N X_2(k)$

Ex. WINDOWING

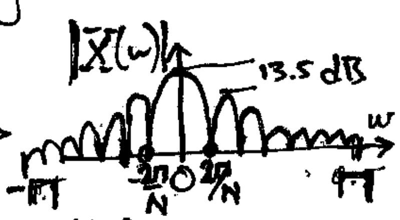
Consider the infinite length sequence $x(n)$, $n=0, \dots, \infty$ $(x(n) \leftrightarrow X(\omega))$

They define the windowed

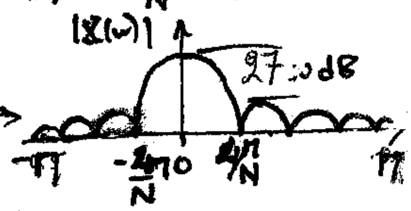
- DTFT $X_w(\omega) = \sum_{n=0}^{N-1} x(n) \cdot w(n) e^{-j\omega n} = \frac{1}{2\pi} X(\omega) * W(\omega)$
- N-DFT $X_w(k) = \sum_{n=0}^{N-1} x(n) w(n) e^{-j\frac{2\pi k n}{N}} = \frac{1}{N} X(k) \circledast_N X_2(k)$

Where $w(n)$: a discrete time window of length N $(n=0,\dots,N-1)$

— Rectangular window (default): $w(n) = \begin{cases} 1, & n=0,\dots,N-1 \\ 0, & \text{other} \end{cases}$ $\longrightarrow$
(brutally truncates signal)



— Bartlett window: $w(n) = \begin{cases} 1 - \frac{|n - N/2|}{N/2}, & n=0,\dots,N-1 \\ 0, & \text{other} \end{cases}$ $\longrightarrow$
(less brutal truncation of signal)

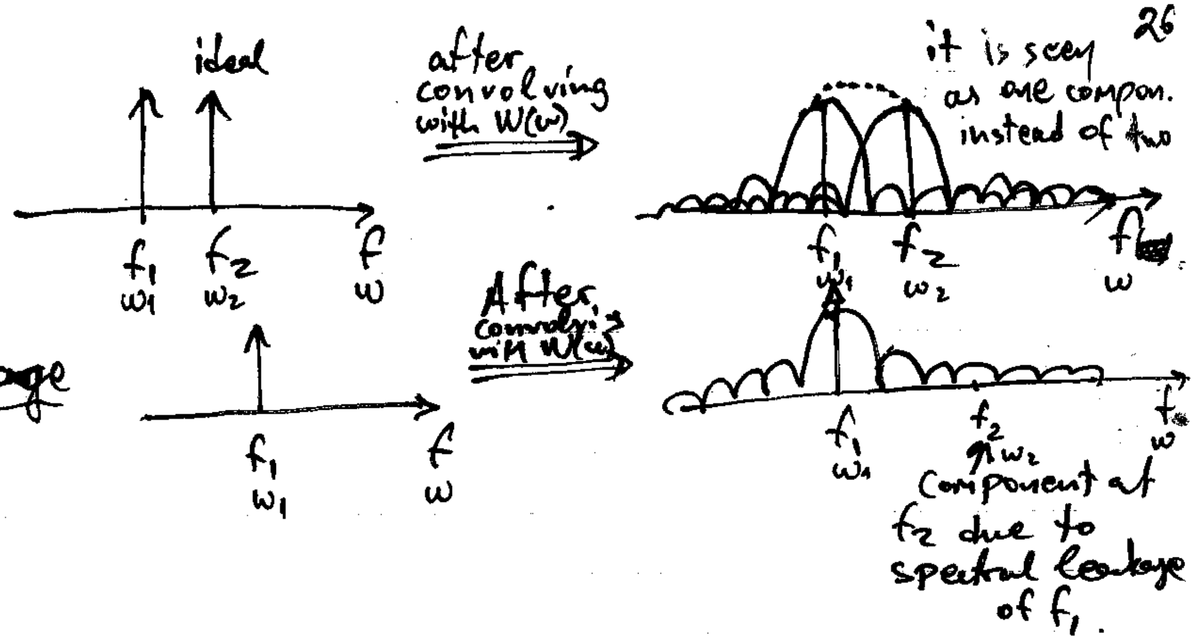


WIDTH OF MAINLOBE Vs SIZE OF SIDELOBES

(less Resolution due to convolution in frequency domain) Vs (Spectral leakage)

Resolution:

Spectral leakage



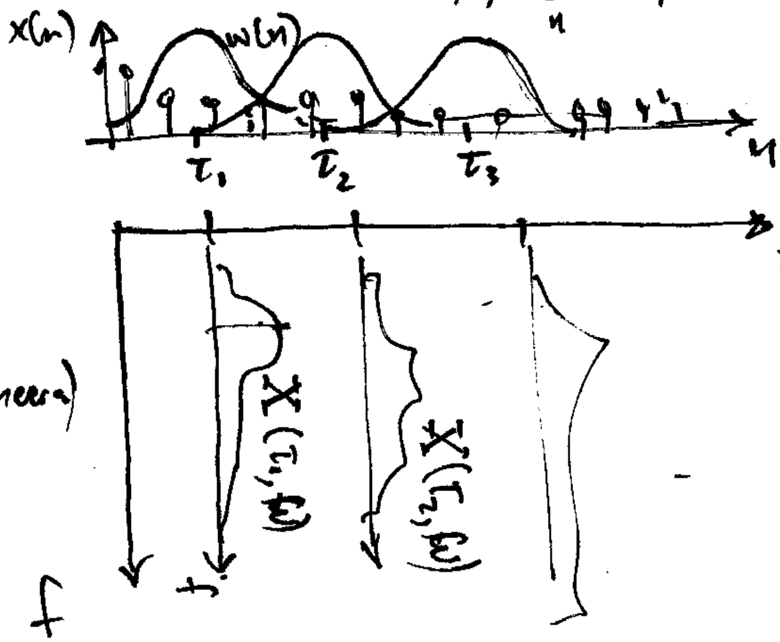
Ex. Short-time Fourier transform
SDTFT or N-SDFT

Consider the transform

- S-DTFT $X(\tau,\omega)=\sum_n x(n)w(n-\tau)e^{-j\omega n}$
- N-SDFT $X(\tau,k)=\sum_n x(n)w(n-\tau)e^{-j\frac{2\pi kn}{N}}$

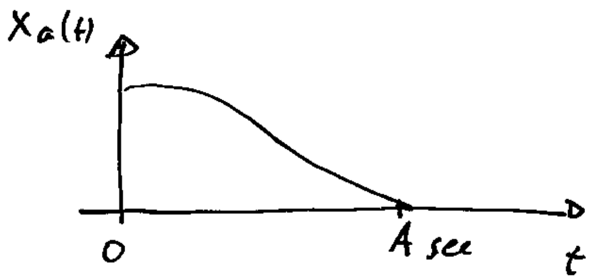
Two dimensional sequence (τ, ω)

w(n) window of length N shifted in time by integer (non-integer in general) # of samples)



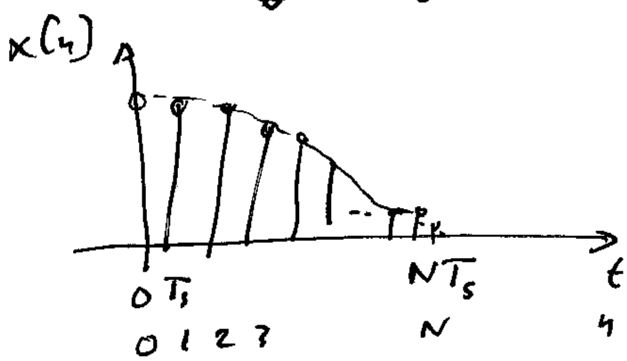
* The SDTFT or N-SDFT represent better the local characteristics of a signal (e.g. speech, ... (locally stationary))

MORE ON SPECTRAL RESOLUTION (DISCUSSION)



Resolution in frequency $(\Delta\Omega) = \frac{1}{A\,\text{sec}}$.

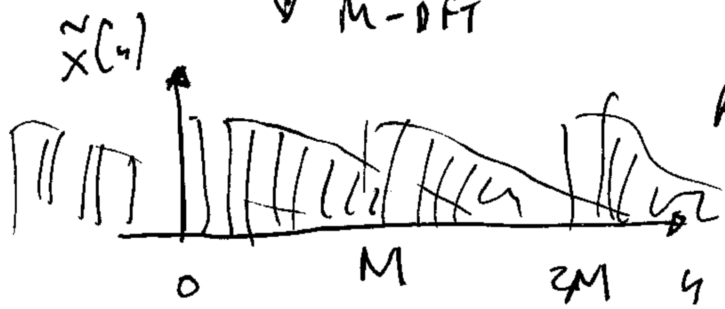
$\downarrow$ sampling (T_s)



Resolution in frequency $= \frac{1}{NT_s}$ $(\Delta\Omega)$

$= \frac{1}{N}$ $(\Delta\omega)$

$\downarrow$ M-DFT



Actual Resolution in frequency — same
Visual resolution decreases and is variable depending on M
however
visual MDFT spectral sample distance $= \frac{2\pi}{M}$
can go to $\rightarrow 0$ as $M \rightarrow \infty$
however this is different from frequency resolution which remains unchanged

$\boxed{\text{VISUAL RESOLUTION} \neq \text{FREQUENCY RESOLUTION}}$