# LECTURE 10

## • PROPERTIES OF THE N-DFT

Remember that in this case a finite length sequence is represented as one period of a periodic sequence (N)

Let us define the function $R_N(n) = \begin{cases} 1 & , n=0,\dots,N-1 \\ 0 & , \text{otherwise} \end{cases}$

Then, $\tilde{x}(n)R_N(n) = \left[\sum_\ell x(n+\ell N)\right]\cdot R_N(n) \equiv \tilde{x}(n)$, $n=0,1,\dots,N-1$

1) LINEARITY: Let $a_1, a_2$ any two constants. Then, if $x_1(n)$ and $x_2(n)$ are two discrete finite length signals, so that

$$\tilde{x}_1(n)R_N(n) \overset{N\text{-DFT}}{\longleftrightarrow} X_1(k) = \tilde{X}_1[k]R_N(k)$$
$$\tilde{x}_2(n)R_N(n) \overset{N\text{-DFT}}{\longleftrightarrow} X_2(k)$$
$$\Rightarrow \left[a_1\tilde{x}_1(n) + a_2\tilde{x}_2(n)\right]R_N(n) \overset{N\text{-DFT}}{\longleftrightarrow} a_1X_1(k) + a_2X_2(k)$$

2) SYMMETRY: $x(n)$ real $\Rightarrow$ $X(k) = X^*(-k) = X^*(N-k)$, $k=0,\dots,\frac{N}{2}$

$x(n)$ real and even $\Rightarrow$ $X(k)$ real and even

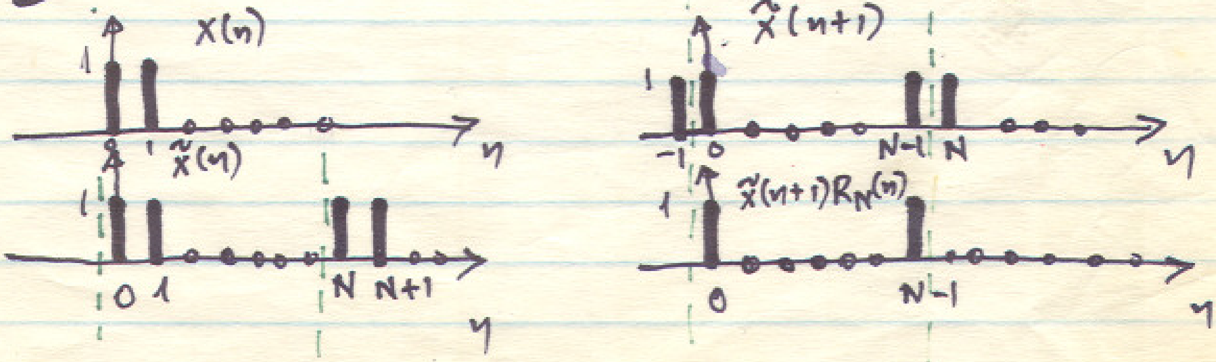
$x(n)$ real and odd $\Rightarrow$ $X(k)$ imaginary.

3) TIME SHIFT or CIRCULAR SHIFT: Given $x(n)$ of finite length

Let $\tilde{x}(n)\cdot R_N(n) \overset{N\text{-DFT}}{\longleftrightarrow} X(k)$ $\quad k=0,\dots,N-1$

Then, $$\left[\tilde{x}(n-n_0)R_N(n) \overset{N\text{-DFT}}{\longleftrightarrow} e^{-j\frac{2\pi k n_0}{N}} X(k)\right]$$

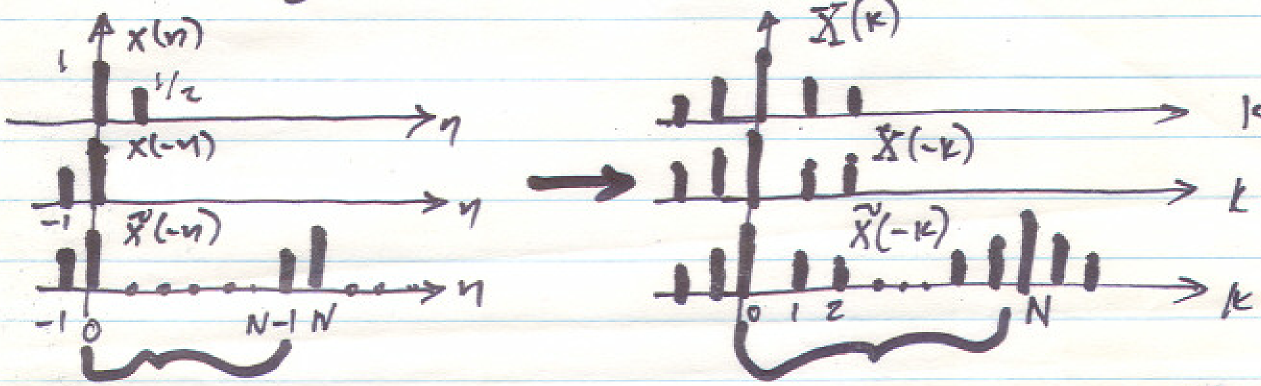
**Ex**



4) <u>FREQUENCY SHIFT</u> $\left[ \left[x(n)e^{\frac{j2\pi k_0 n}{N}}\right]^{\sim} R_N(n) \xleftrightarrow{N\text{-DFT}} \overset{N}{X}(k-k_0)\, R_N(k) \right]$

5) <u>TIME REVERSAL</u> $\left[ \tilde{x}(-n) R_N(n) \xleftrightarrow{N\text{-DFT}} \overset{N}{X}(-k) R_N(k) \right]$

**Ex**



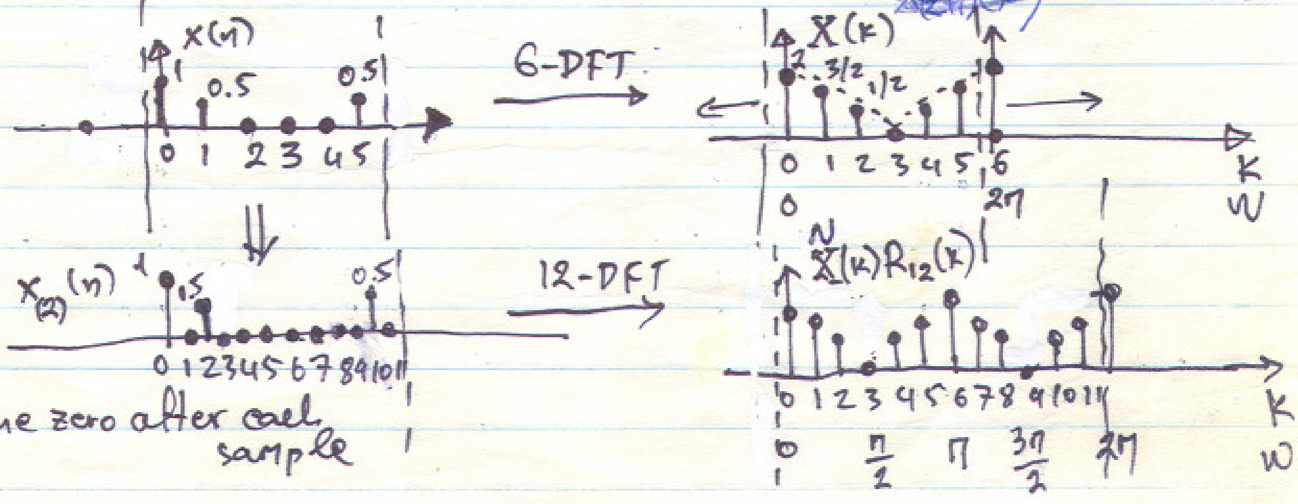

6) <u>PARSEVAL'S RELATION</u> $\left[ \sum_{n=0}^{N-1} |\tilde{x}(n)|^2 = \frac{1}{N}\sum_{k=0}^{N-1} |X(k)|^2 . \right]$

7) Let $x_{(Z+1)}(n) = \begin{cases} x[n/(Z+1)] & \text{, if } n/(Z+1) \text{ integer} \\ 0 & \text{, otherwise.} \end{cases}$

<u>TIME EXPANSION</u> (i.e. you put Z zeros after each sample of x(n))

Then, if $\left[ \begin{array}{l} \tilde{x}(n)\cdot R_N(n) \xleftrightarrow{N\text{-DFT}} \overset{N}{X}(k)\, R_N(k) \\ \tilde{x}_{(Z+1)}(n)\, R_{N(Z+1)}(n) \xleftrightarrow{N(Z+1)\text{-DFT}} \overset{N}{X}[k]\cdot R_{N(Z+1)}(k) \end{array} \right]$

**Ex.**



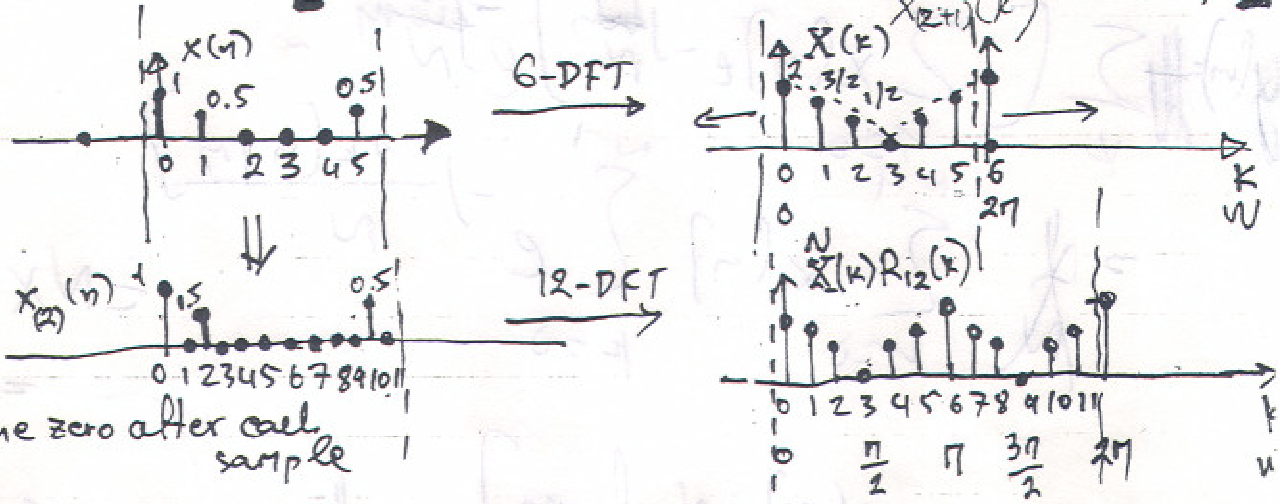
7) Let $x_{(Z+1)}(n) = \begin{cases} x\left[n/(Z+1)\right] & \text{, if } n/(Z+1) \text{ integer} \\ 0 & \text{, otherwise.} \end{cases}$

TIME EXPANSION (i.e. you put $Z$ zeros after each sample of $x(n)$)

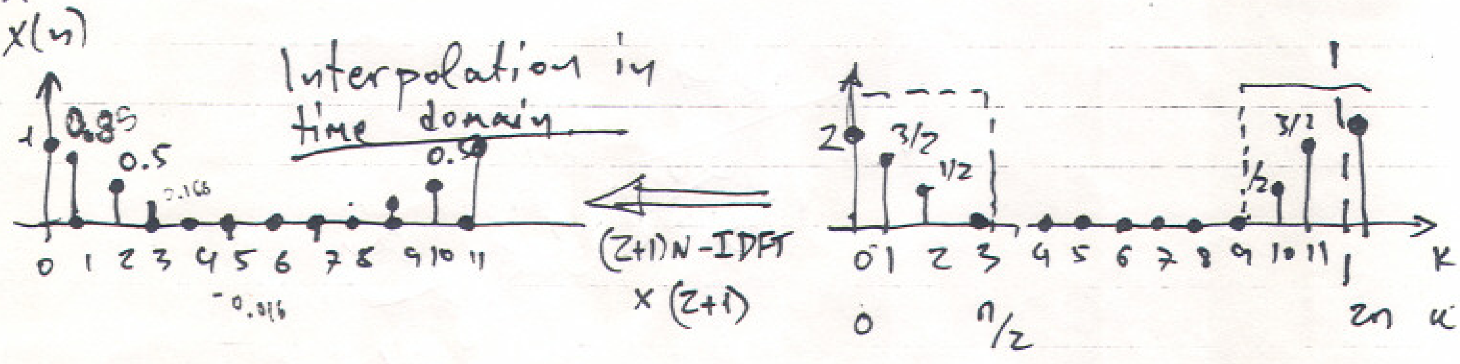
Then, if $\left[\begin{array}{l} \tilde{x}(n)\cdot R_N(n) \xleftrightarrow{N\text{-DFT}} \tilde{X}(k)\, R_N(k) \\ \tilde{x}_{(Z+1)}(n)\, R_{N(Z+1)}(n) \xleftrightarrow{N(Z+1)\text{-DFT}} \tilde{X}[k]\cdot R_{N(Z+1)}(k) \end{array}\right]$

$\tilde{X}_{(Z+1)}(k)$

**Ex.**




Now if we filter $\tilde{X}(k)\, R_{12}(k)$ with an ideal low pass filter with cuttoff frequency $w = \frac{\pi}{2}$ $(k=3)$ and gain 2 it is equivalent to zero padd 6-$X(k)$ with 6 zeros.




Thus: Alternative way for digital interpolation.