# DECIMATION/INTERPOLATION

$x[n] \rightarrow [\downarrow M] \rightarrow y[n]$

**DECIMATION**: $y[n] = x[Mn]$

If $x[n] = x(nT_s)$, then $y[n] = x(nMT_s)$. So, can think of decimation as reducing sampling frequency by factor of $M$.

**Interpolation**: $y[n] = x[n/L]$

Problem: How to define $y[n]$ when $n/L$ not integer?

Examples: (a) Zero Interpolation (Upsampling)

$y[n] = 0$ $n/L$ not integer

$= x[n/L]$ $n/L$ integer

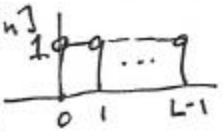
$x[n] \rightarrow [\uparrow L] \rightarrow y[n]$ (basis for considering other interpolation methods)

(b) Step Interpolation:

$y[n] = x[\lfloor n/L \rfloor]$ $n/L$ not integer

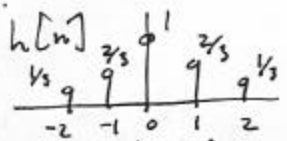
$= x[n/L]$ $n/L$ integer

$x[n] \rightarrow [\uparrow L] \rightarrow [h[n]] \rightarrow y[n]$



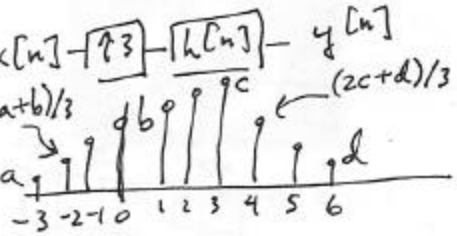

(c) Linear Interpolation: $y[n]$ is a weighted combination of past, present and future (non-causal) samples

Ex:



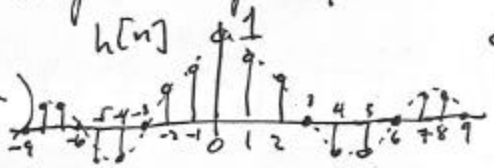


$x[n] \rightarrow [\uparrow 3] \rightarrow [h[n]] \rightarrow y[n]$




(because of "1" at time zero, will preserve original signal points)

Ex: ("Truncated sinc")




← Longer impulse response, closer approximation of $y[n]$ to $x(nT_s/L)$ (Assuming original signal was sampled at Nyquist rate or higher).

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Q: Is Interpolation inverse operation of Decimation?

Not necessarily.

$x[n] \rightarrow [\uparrow X] \rightarrow [h[n]] \rightarrow [\downarrow X] \rightarrow y[n]$ ← $y[n] = x[n]$ always (assuming $h[0] = 1$, $h[Xk] = 0, k = \pm 1, \pm 2, \ldots$)

$x[n] \rightarrow [\downarrow X] \rightarrow [\uparrow X] \rightarrow [h[n]] \rightarrow z[n]$ ← $z[n] = x[n]$ in only special cases.

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**Fractional Delay**: How to implement $y[n] = x[n - 1/2]$? ($y[n] = x[n - \frac{M}{L}]$ in general)

Use interpolation, shift, then decimation

$x[n] \rightarrow [\uparrow 2] \rightarrow [h[n]] \rightarrow [\text{DELAY BY 1}] \rightarrow [\downarrow 2] \rightarrow x[n - \tfrac{1}{2}] = y[n]$

$x[n/2]$ $x[\frac{n-1}{2}]$ $x[2n-1]$

In general:

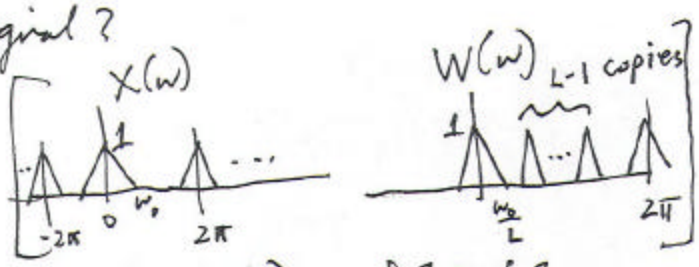
$x[n] \rightarrow [\uparrow L] \rightarrow [h[n]] \rightarrow [\text{Delay by } M] \rightarrow [\downarrow L] \rightarrow y[n]$

$x[n/L]$ $x[\frac{n-M}{L}]$ $x[nL-M]$

What is effect on freq. content of signal?

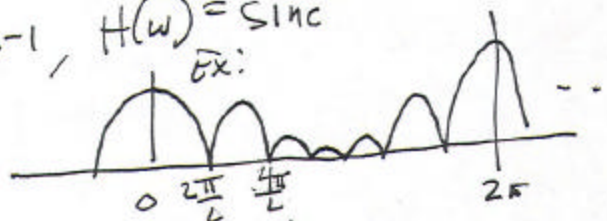
$x[n] \rightarrow \boxed{\uparrow L} \rightarrow w[n]$

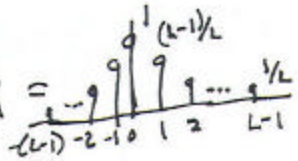
Can show that $W(\omega) = X(\omega L)$




So, $h[n]$ will shape $w[n]$ to give $y[n]$. Ex: $h[n] = \delta[n]$, $H(\omega) = 1$, $Y(\omega) = W(\omega)$, $y[n] = w[n]$ (Upsampling)

Ex: $h[n] = 1$ $n = 0, \ldots, L-1$, $H(\omega) =$ "sinc" (Step Interpolation)




Ex: $h[n] =$  $(H(\omega) =$ "$\text{sinc}^2$")



Ex: $h[n] =$  $H(\omega) =$ rectangle, then $y[n] = x\left(\frac{nT_s}{L}\right)$.

(sinc)