3.22. A. The system is linear, so we can find the response to each term in the input expression and add the responses together.

For input $2\cos\left(\frac{\pi}{2}n\right)$, we can evaluate $H(z)$ at $z = e^{j\frac{\pi}{2}}$. The steady-state response is then $\left|H\left(e^{j\frac{\pi}{2}}\right)\right| 2\cos\left(\frac{\pi}{2}n + \measuredangle H\left(e^{j\frac{\pi}{2}}\right)\right)$.

For input $u[n]$, the steady-state response is equal to the DC gain; that is, $H\left(e^{j0}\right)$.

B. Given $H(z) = \dfrac{1-4z^2}{1+0.5z^{-1}}$, we have

$$H\left(e^{j\frac{\pi}{2}}\right) = \frac{1-4e^{-j\pi}}{1+0.5e^{-j\frac{\pi}{2}}} = \frac{5}{1-j0.5} = 4.47e^{j0.464}.$$

Then $y_1[n] = 8.94\cos\left(\frac{\pi}{2}n + 0.464\right)$.

Next, $H\left(e^{j0}\right) = \dfrac{1-4}{1+0.5} = -2.00$, so that $y_2[n] = -2.00 \times 1 = -2.00$.

As n gets large the response becomes

$$y[n] = y_1[n] + y_2[n] = -2.00 + 8.94\cos\left(\tfrac{\pi}{2}n + 0.464\right).$$

3.32. (a)

$$X(z) = \frac{1}{(1+\frac{1}{2}z^{-1})^2(1-2z^{-1})(1-3z^{-1})} \qquad \frac{1}{2} < |z| < 2$$

$$= \frac{\frac{1}{35}}{(1+\frac{1}{2}z^{-2})^2} + \frac{\frac{88}{1225}}{(1+\frac{1}{2}z^{-1})} - \frac{\frac{1568}{1225}}{(1-2z^{-1})} + \frac{\frac{2700}{1225}}{(1-3z^{-1})}$$

Therefore,

$$x[n] = \frac{1}{35}(n+1)\left(\frac{-1}{2}\right)^{n+1}u[n+1] + \frac{58}{(35)^2}\left(\frac{-1}{2}\right)^n u[n] + \frac{1568}{(35)^2}(2)^n u[-n-1] - \frac{2700}{(35)^2}(3)^n u[-n-1]$$

(b)

$$X(z) = e^{z^{-1}} = 1 + z^{-1} + \frac{z^{-2}}{2!} + \frac{z^{-3}}{3!} + \frac{z^{-4}}{4!} + \ldots$$

Therefore, $x[n] = \frac{1}{n!}u[n]$.

(c)

$$X(z) = \frac{z^3 - 2z}{z-2} = z^2 + 2z + \frac{2}{1-2z^{-1}} \qquad |z| < 2$$

Therefore,

$$x[n] = \delta[n+2] + 2\delta[n+1] - 2(2)^n u[-n-1]$$

3.46. 1. Given $x[n] = -\frac{1}{3}\left(\frac{1}{2}\right)^n u[n] - \frac{4}{3}(2)^n u[-n-1]$, we have

$$X(z) = \frac{-\frac{1}{3}}{1-\frac{1}{2}z^{-1}} + \frac{\frac{4}{3}}{1-2z^{-1}}$$

$$= \frac{1}{\left(1-\frac{1}{2}z^{-1}\right)\left(1-2z^{-1}\right)}, \quad \tfrac{1}{2} < |z| < 2.$$

2. The expression

$$Y(z) = \frac{1-z^{-2}}{\left(1-\frac{1}{2}z^{-1}\right)\left(1-2z^{-1}\right)}$$

can converge for a) $|z| < \frac{1}{2}$, b) $\frac{1}{2} < |z| < 2$, or c) $2 < |z|$. However, $Y(z) = H(z)X(z)$, where $H(z)$ is the system function of the given LTI system, and the ROC of $Y(z)$ must contain the intersection of the ROC of $H(z)$ with the ROC of $X(z)$. Since $X(z)$ has ROC $\frac{1}{2} < |z| < 2$, only choice b) is possible.

3. The system function $H(z)$ is given by

$$H(z) = \frac{Y(z)}{X(z)} = 1 - z^{-2}, \quad 0 < |z|.$$

The only choice for difference equation is

$$y[n] = x[n] - x[n-2].$$

4. The only choice for impulse response is

$$h[n] = \delta[n] - \delta[n-2].$$

3.48. (a) Since $y[n]$ is stable, its ROC contains the unit-circle. Hence, $Y(z)$ converges for $\frac{1}{2} < |z| < 2$.

(b) Since the ROC is a ring on the z-plane, $y[n]$ is a two-sided sequence.

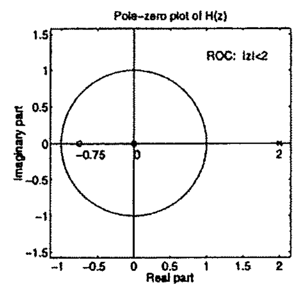
(c) $x[n]$ is stable, so its ROC contains the unit-circle. Also, it has a zero at ∞ so the ROC includes ∞. ROC: $|z| > \frac{3}{4}$.

(d) Since the ROC of $x[n]$ includes ∞, $X(z)$ contains no positive powers of z, and so $x[n] = 0$ for $n < 0$. Therefore $x[n]$ is causal.

(e)

$$\begin{aligned} x[0] &= X(z)|_{z=\infty} \\ &= \frac{A(1-\frac{1}{4}z^{-1})}{(1+\frac{3}{4}z^{-1})(1-\frac{1}{2}z^{-1})}|_{z=\infty} \\ &= 0 \end{aligned}$$

(f) $H(z)$ has zeros at -.75 and 0, and poles at 2 and ∞. Its ROC is $|z| < 2$.



(g) Since the ROC of $h[n]$ includes 0, $H(z)$ contains no negative powers of z, which implies that $h[n] = 0$ for $n > 0$. Therefore $h[n]$ is anti-causal.