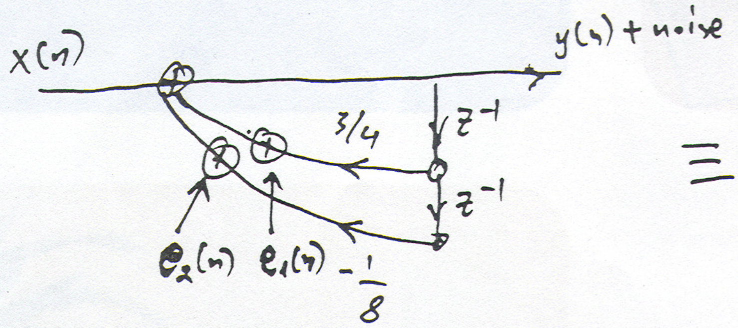
[redacted]

a) $H(z) = \dfrac{z^2}{z^2 - \frac{3}{4}z + \frac{1}{8}} = \dfrac{1}{1 - \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}} \overset{\text{PFE}}{\Longleftarrow} \dfrac{2}{1 - \frac{1}{2}z^{-1}} + \dfrac{-1}{1 - \frac{1}{4}z}$

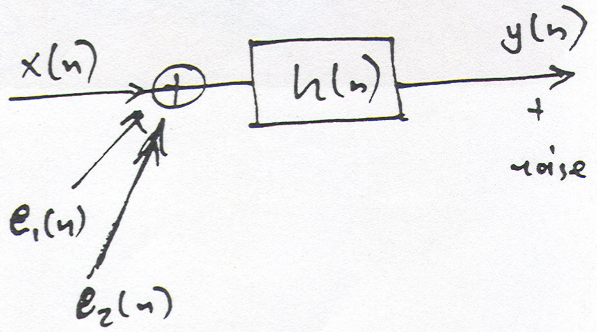
(assuming causal filter)

$$\boxed{h(n) = \left[2\left(\tfrac{1}{2}\right)^n - \left(\tfrac{1}{4}\right)^n\right] u(n)}$$

DIRECT I Form



$\equiv$



Assuming that the noise sources are white - zero mean with $\sigma_{e_1}^2 = \sigma_{e_2}^2 = 1$ independent from each other and the signal and noise sources, then the variance of the noise at the output of the filter is

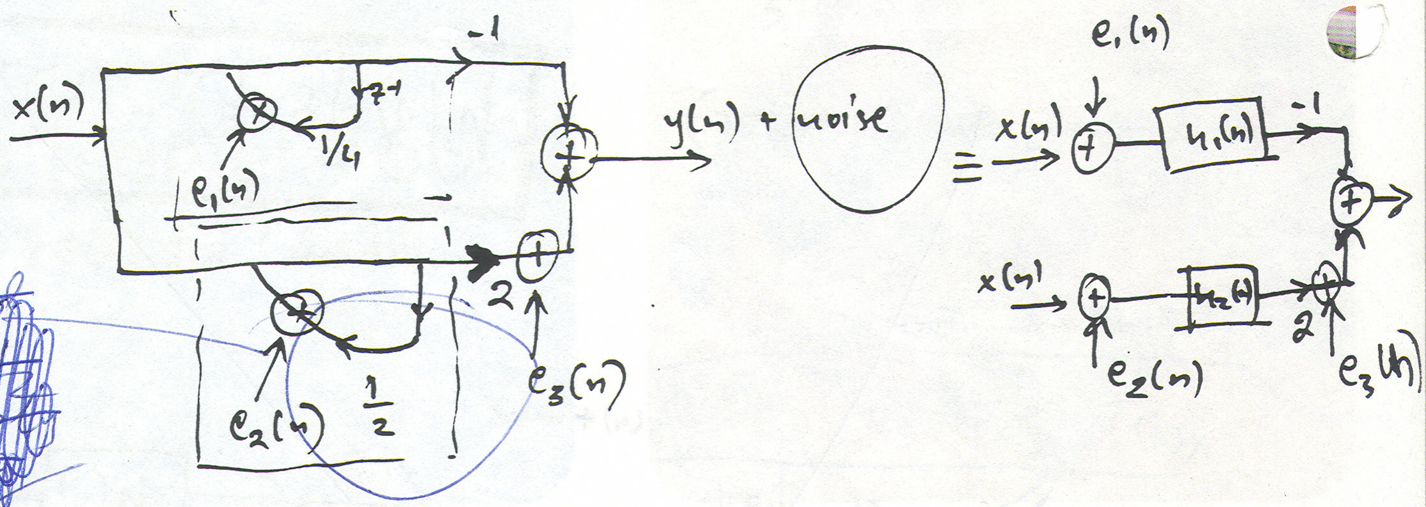
$$\sigma_f^2 = \left[\sigma_{e_1}^2 + \sigma_{e_2}^2\right]\left[\sum_{n=\infty}^{\infty} h^2(n)\right] = 2\left[\sum_{n=0}^{\infty}\left[2\left(\tfrac{1}{2}\right)^n - \left(\tfrac{1}{4}\right)^n\right]^2\right]$$

$$= 2\sum_{n=0}^{\infty} 4\left(\tfrac{1}{2}\right)^{2n} + 2\sum_{n=0}^{\infty}\left(\tfrac{1}{4}\right)^{2n} - 4\sum_{n=0}^{\infty}\left(\tfrac{1}{8}\right)^n =$$

$$= 2\frac{4}{1-\left(\frac{1}{2}\right)^2} + 2\frac{1}{1-\left(\frac{1}{4}\right)^2} - 2\frac{4}{1-\frac{1}{8}} = \frac{8}{3/4} + \frac{2}{\frac{15}{16}} - \frac{8}{\frac{7}{8}} =$$

$$= 3.657.$$

(2) PARALLEL FORM let

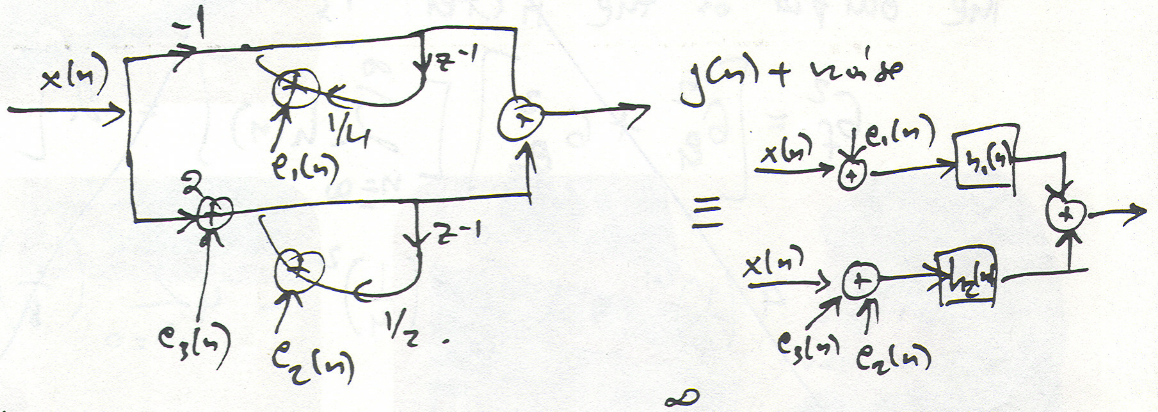


$$\sigma_f^2 = \sigma_{e_1}^2 \sum_{n=-\infty}^{\infty} h_1^2(n) + 2^2\left[\sigma_{e_2}^2 \sum_{n=-\infty}^{\infty} h_2^2(n)\right] + \sigma_{e_3}^2 =$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{4}\right)^{2n} + 4\sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{2n} + 1 = \frac{1}{1-\left(\frac{1}{4}\right)^2} + 4\frac{1}{1-\left(\frac{1}{2}\right)^2} + 1 =$$

$$= \frac{1}{15/16} + \frac{4}{3/4} + 1 = \frac{16}{15} + \frac{16}{3} + 1 = \frac{16}{15} + \frac{90}{15} + 1 = \frac{106}{15} + 1 = \frac{131}{15} \approx 8.733$$

Alternatively:



So

$$S_f^2 = \sigma_{e_1}^2 \sum_{n=-\infty} h_1(n) + \left[\sigma_{e_2}^2(n) + \sigma_{e_3}^2(n)\right] \sum_{n=-\infty}^{\infty} h_2(n) =$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{4}\right)^{2n} + 2\sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{2n} = \frac{1}{15/16} + \frac{2}{3/4} = \frac{16}{15} + \frac{8}{3} = \frac{56}{15} \cong 3.8$$