Student name: Student number:

MIDTERM 2 EXAMINATION
ECE431H1F, Digital Signal Processing

November 17, 2023
Examiner: D. Hatzinakos

Time: 4:10-5:00 pm, Room BA2195

This is a Type C exam. You may use non programmable calculators.

Exam questions

1. Explain the importance of zero padding for FFT calculations. (1 mark)

In many cases FFT implementations do not exist (ex. N is prime number) Also, radix-2 FFT algorithm is the most efficient among all other FFT algorithms. In such cases we may want to append zeros to the closest power of 2 and then implement an FFT algorithm

2. Explain the importance of knowing the region of convergence (ROC) for Z-transforms. (1 mark)

$x(n) \leftrightarrow X(z) + ROC$ ..

without ROC the $X(z)$ cannot uniquely specify $x(n)$

3. What is the purpose of an oversampling filter in a CD player and why is it used? If there was no oversampling filter, what alternative action should a designer take to achieve the same quality at the output? (2 marks)

Oversampling + filtering is used in CD player to increase the SQNR by reducing the power of the noise.
Alternative to reduce noise or increase SQNR more bits/sample must be allocated in the AD converter

4. A sinusoid at 400 Hz is sampled at 5,000 Hz. If the sinusoid is down-sampled by a factor of 10 by removing 9 samples out of each set of 10, the frequency of the down-sampled sinusoid will be 100 Hz. Explain. (2 marks)

By downsampling the underlying sampling frequency is reduced by 10 times to 500 Hz. Then, there is aliasing in the data which appears to the frequency 100 Hz. $(\underbrace{500\text{ Hz} - 400\text{ Hz}}_{100\text{ Hz}})$

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5. A linear phase FIR filter has 6 zeros. Two of the zeros are at locations, z1=0.5-j0.5 and z2= 1/3. What are the locations of the remaining 4 zeros? (2 marks)

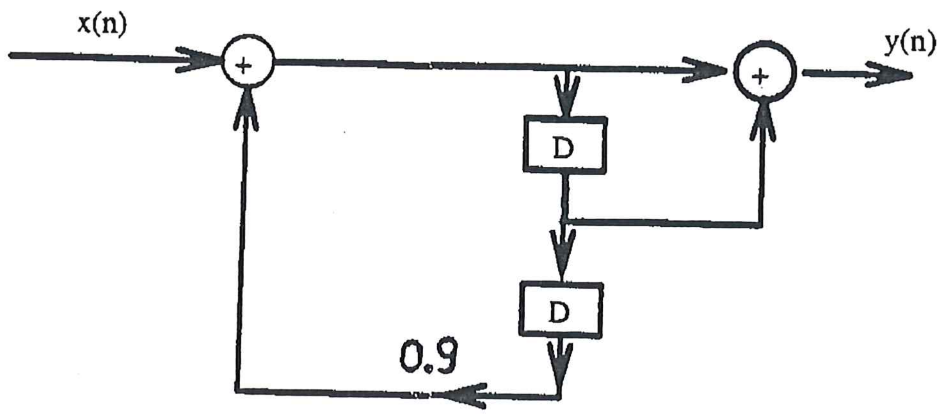
A linear phase filter satisfies the symmetry/antisymmetry conditions

for complex roots come in sets of 4: $z_1, z_1^*, \frac{1}{z_1}, \frac{1}{z_1^*}$ then

$z_0 = 0.5 - j0.5$ $z_1 = 0.5 + j0.5$, $z_2 = \frac{1}{0.5 - j0.5} = 1+j$, $z_3 = 1-j$

for real roots come in pairs so if $z_5 = \frac{1}{3}$ then $z_6 = 3$

6. Write the Linear Constant Coefficient Difference Equation (LCCDE) that describes the system below (D denotes unit delay). Calculate the Discrete-time Fourier Transform (DTFT) of the system impulse response. (2 marks)



$y(n) = x(n) + x(n-1) + 0.9\,y(n-2) \Rightarrow$

$\Rightarrow y(n) - 0.9\,y(n-2) = x(n) + x(n-1)$

$\Rightarrow Y(z)(1 - 0.9z^{-2}) = X(z)(1 + z^{-1})$

$\Rightarrow \frac{Y(z)}{X(z)} = \frac{1 + z^{-1}}{1 - 0.9z^{-2}} \Rightarrow H(\omega) = H(z = e^{j\omega}) = \frac{1 + e^{-j\omega}}{1 - 0.9e^{-2j\omega}}$

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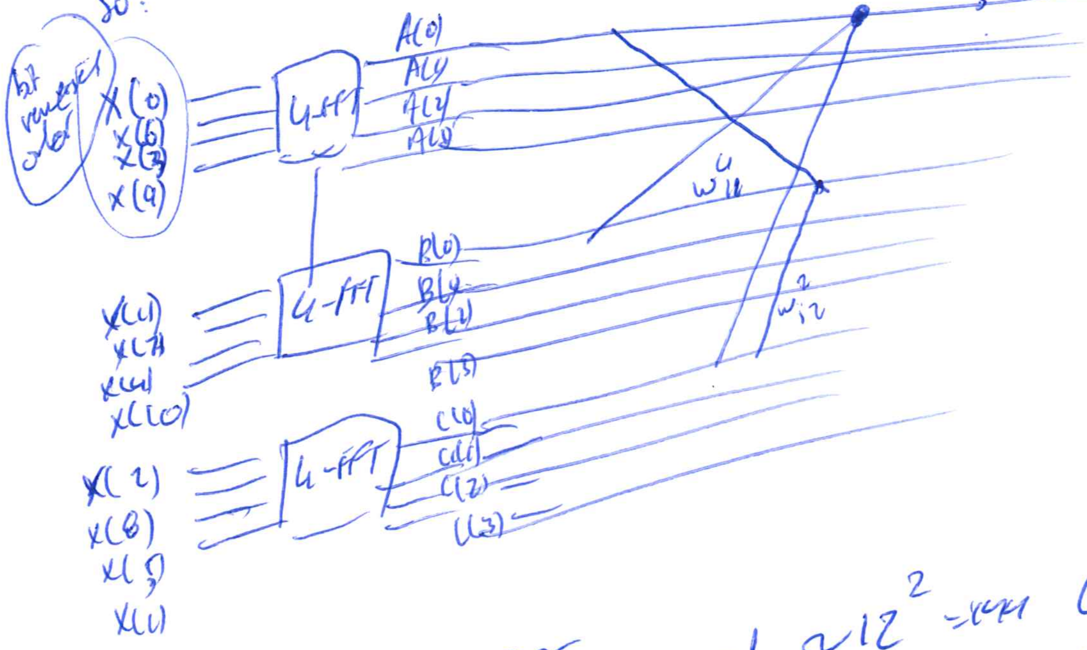
7. A designer has available a number of four-point FFT chips. Show explicitly how he/she should interconnect three such chips to compute a 12-point DFT (including diagram and expressions). What is the number of multiplications and additions required? What are the savings in computations compared to direct computation of 12 point DFT? (5 marks)

$$X(k) = \sum_{m=0}^{3} x(3m) W_{12}^{3mk} + W_{12}^{k} \sum_{m=0}^{2} x(3m+1) W_{12}^{(3m+1)k} + W_{12}^{2k} \sum_{m=0}^{3} x(3m+2) W_{12}^{(3m+2)k}$$

$$\Rightarrow X(k) = \underbrace{\sum_{m=0}^{2} x(3m) W_4^{mk}}_{A(k)-4DFT} + W_{12}^{k} \underbrace{\sum_{m=0}^{2} x(3m+1) W_4^{mk}}_{B(k)-4DFT} + W_{12}^{2k} \underbrace{\sum_{m=0}^{2} x(3m+2) W_4^{mk}}_{C(k)-4DFT}$$

$k = 0, 1, \ldots, 11$

So:



$X(0) = A(0) + B(0) + C(0)$

$X(1) = A(1) + W_{12} B(1) + W_{12}^2 C(1)$

$\vdots$

$X(11) = A(0) + W_{12}^{11} B(4) + W_{12}^{22} C(4)$

For regular 12-DFT we need $\sim 12^2 = 144$ complex multiplication and addition

For this implementation we need $\sim 3 \cdot (\frac{4}{2} \log_2 4) + 10 \cdot 2 =$

$= 3 \cdot 2 \cdot 2 + 20 = 12 + 20 = 32$ complex multiplications and addition

So we have $\frac{32}{144}$ efficiency.