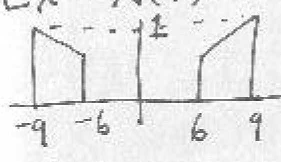
BANDPASS SAMPLING

RYAN

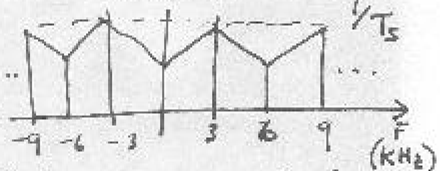
It is not always necessary to sample a bandpass signal at the Nyquist rate (or higher) to avoid aliasing distortion

Ex: $X(F)$

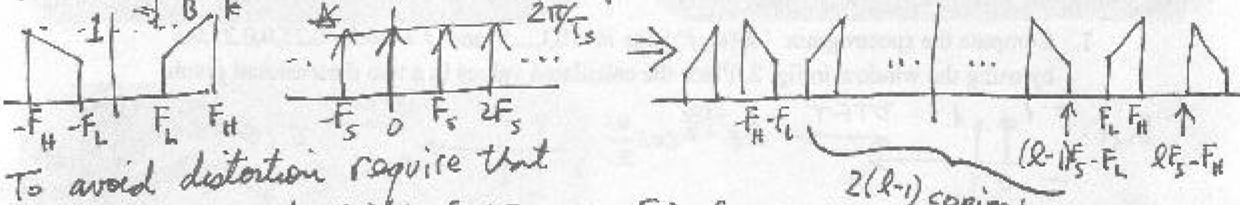


The Nyquist rate is 18 kHz here. But, consider using 6 kHz for samp. freq. Then the spectrum of sampled signal:

$X_s(F)$



So, there is no aliasing distortion, but need a BPF to recover original sig. What are the allowable rates for a bandpass signal to avoid aliasing distortion? Consider bandpass signal with bandwidth B, and use F_s to samp:



To avoid distortion require that

$lF_s - F_H > F_H$ and $(l-1)F_s - F_L < F_L$ so $F_s > \frac{2F_H}{l}$ and $F_s < \frac{2F_L}{l-1}$

thus $\boxed{\frac{2F_H}{l} < F_s < \frac{2F_L}{l-1}}$ and $\frac{\cancel{2}F_H}{l} < \frac{\cancel{2}F_L}{l-1}$ so $\boxed{l < \frac{F_H}{F_H-F_L} = F_H/B}$

Since l must be an integer, the allowable values are $l = 1, 2, \ldots \lfloor F_H/B \rfloor$.

For each l get an allowable range for F_s. For example, if we choose $l=1$, then $2F_H < F_s < \infty$, this is the Nyquist rate condition.

What is the minimum value F_s can take and under what conditions?

When $F_H/B = \lfloor F_H/B \rfloor$, then $F_{s,min} = 2B$. This special case is called integer band positioning. It means that both F_H and F_L are integer multiples of B. In this case we can sample at 2B and avoid aliasing distortion.

This is the case shown in the example above.

To have the lowest possible F_s we should choose the highest l. Are there any constraints for doing this? As l increases the range of allowable F_s decreases. Since oscillators have finite precision this is an important issue. For a given l, the range of allowable F_s is $\Delta F_s = \frac{2F_L}{l-1} - \frac{2F_H}{l}$, thus

$\Delta F_s = \frac{2[F_H - lB]}{l(l-1)}$, or $\frac{\Delta F_s}{2B} = \frac{F_H/B - l}{l(l-1)}$. The precision of an oscillator will put a bound on how low $\Delta F_s/2B$ can be. This puts an upper bound on how large l can be.