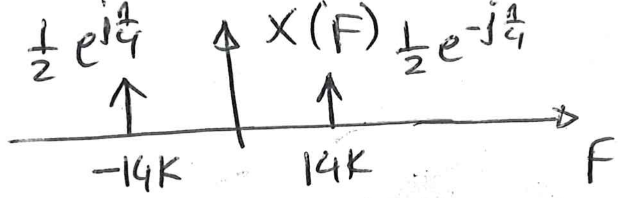
EC431H1 Digital Signal Processing
FINAL EXAM
April 26, 2004, 2:00 p.m.
Instructor: D. Hatzinakos

Instructions:

1. Type A exam
2. Non-programmable calculators are allowed
3. Please solve all five problems. All problems are equally weighted.
4. All answers must be written in the examination booklet. Do not write any answers in this problem handout.



PROBLEM 1 (10 points)

An A/D system converts by ideal sampling at 16kHz.

(a) If a continuous time signal of the form $x(t) = cos(2.8\pi 10^4 t + \pi/4)$ is sampled, sketch, with careful labeling of axes and impulses, the DTFT of the sequence $x[n] = x(nT)$.

(b) A reconstruction is made by interpolating the samples according to

$$y(t) = \sum_{-\infty}^{\infty} x[n] sinc\left(\frac{2t - nT}{T}\right) = \sum x(n) \operatorname{sinc}\left(\frac{t - n\frac{T}{2}}{\frac{T}{2}}\right)$$

with T as in part (a) and $sinc(t) = sin(\pi t)/\pi t$. Find $y(t)$ in its simplest form.

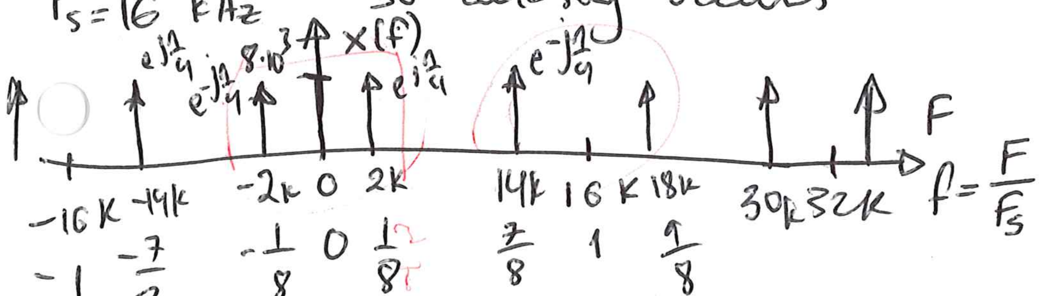
(c) The signal $y(t)$ of (b) is not the same as the $x(t)$ we sampled in (a). You are restricted to using the interpolator as in part (b), but now you may use a linear discrete time filter to process $x[n]$ before reconstructing. The goal is to make the reconstructed signal $\hat{x}(t)$ identical to $x(t)$. In other words, you will create $\tilde{x}[n] = \sum_{-\infty}^{\infty} x(m)h(n-m)$, then reconstruct

$$\hat{x}(t) = \sum_{-\infty}^{\infty} \tilde{x}[n] sinc\left(\frac{2t - nT}{T}\right)$$

with T fixed as above. Design an FIR filter to achieve this, or show why you cannot do it.

a) 4

$F_s = 16$ kHz   So aliasing occurs



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b) 3 This is equivalent to reconstructing with sampling period $\frac{T}{2}$ instead of T or filter ideally with cutoff frequency F_s instead of $\frac{F_s}{2}$

So the output will be $y(t) = A\cos(2 \cdot 10^3 2\pi t + \theta) + B\cos(14 \cdot 10^3 2\pi t + \varphi)$

c) 3 Not possible since if we try to remove the frequency at 2K we will remove ~~of~~ ~~freq~~ freq at 14K as well (discrete FIR is periodic in freq)

PROBLEM 2 (10 points)
Consider the finite length sequence

$$x[n] = 2\delta[n] + \delta[n-1] + \delta[n-3]$$

length 7

We perform the following operation on this sequence:

(i) We compute the five-point DFT $X[k]$

(ii) We compute a five-point inverse DFT of $Y[k] = X^2[k]$ to obtain a sequence $y[n]$.

5

a) Determine the sequence $y[n]$ for $n = 0, 1, 2, 3, 4$.

5

b) if N-point DFTs are used in the two step procedure, how should we choose N so that $y[n] = x[n] * x[n]$ for $n = 0, 1, 2, 3, 4$?

$N \geq 7$

a) $y[n] = x[n] \circledast_5 x[n] =$

~~period of~~

So $y[n]$ is the 5 point periodic extension of $x[n] * x[n]$

$$x[n] * x[n] = \left[2\delta[n] + \delta[n-1] + \delta[n-3]\right] * \left[2\delta[n] + \delta[n-1] + \delta[n-3]\right]$$

$$= 4\delta[n] + 2\delta[n-1] + 2\delta[n-3] + 2\delta[n-1] + \delta[n-2] + \delta[n-4] + 2\delta[n-3] + \delta[n-4] + \delta[n-6]$$

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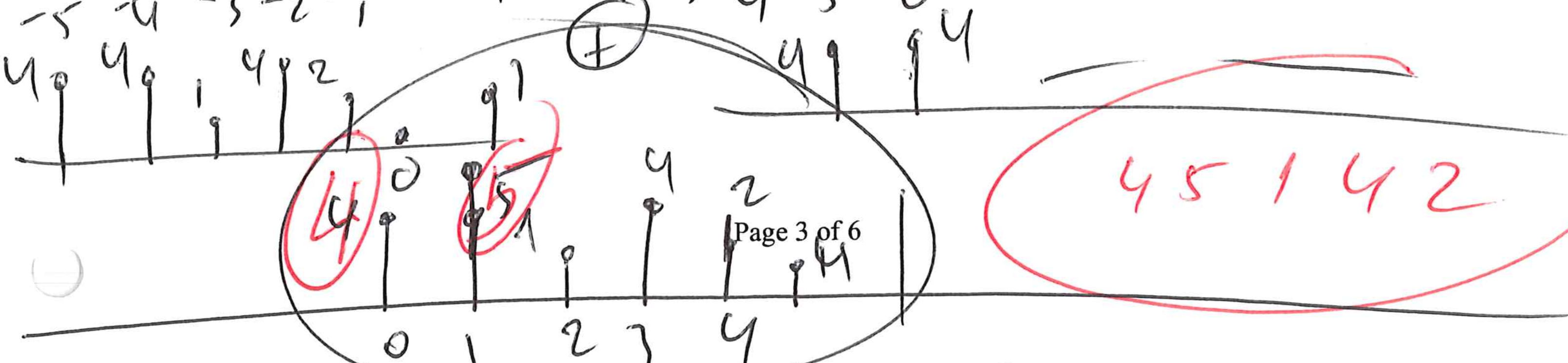
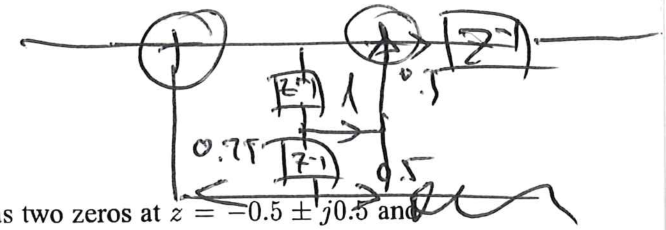
$$= 4\delta[n] + 4\delta[n-1] + \delta[n-2] + 4\delta[n-3] + 2\delta[n-4] + \delta[n-6]$$



4 5 1 4 2

So $y[n] = 4\delta[n] + 5\delta[n-1] + \delta[n-2] + 4\delta[n-3] + 2\delta[n-4]$

(3) $H(z) = 0.3\, z^{-1} \frac{1+z^{-1}+0.5z^{-2}}{(1-0.25z^{-2})}$



PROBLEM 3 (10 points)

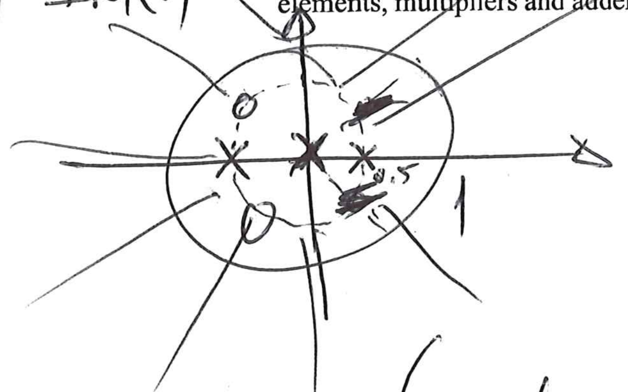
The transfer function $H(z)$ of a stable discrete time filter has two zeros at $z = -0.5 \pm j0.5$ and three poles at $z = -0.5, 0, 0.5$.

2+2 (a) Draw the zero pole diagram and mark the ROC of $H(z)$. Write down the complete transfer function $H(z)$, assuming that the magnitude of the filter response to zero-frequency signals is 1.

(b) Find the impulse response, $h[n]$, of the system.

(c) Carefully draw a diagram of a realization of this system using the minimum number of delay elements, multipliers and adders.

a)



to satisfy Stabilit $z=1 \in$ ROC

4

b) $$H(z) = A\frac{\left(z-(-0.5+j0.5)\right)\left(z-(-0.5-j0.5)\right)}{z(z+0.5)(z-0.5)}$$

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$$= A\frac{z^2+z+0.5}{z(z+0.5)(z-0.5)} = A\frac{z^2+z+0.5}{z\cdot(z^2-0.25)}$$

$$H(e^{j\omega})\Big|_{\omega=0} = A\frac{e^{j2\omega}+e^{j\omega}+0.5}{e^{j\omega}(e^{j2\omega}-0.25)}\Bigg|_{\omega=0} = 1 \Rightarrow A\frac{2.5}{0.75}=1$$

$$\Rightarrow A = \frac{0.75}{2.5}$$

$$\Rightarrow A = 0.3$$ (2)

(3)

c) $$H(z) = 0.3\left(\frac{A}{z} + \frac{B}{z+0.5} + \frac{C}{z-0.5}\right) \Rightarrow$$

$$\Rightarrow h(n) = 0.3A\,\delta[n-1] + 0.3B(-0.5)^{n-1}u[n-1] + 0.3C(0.5)^{n-1}u[n-1]$$

PROBLEM 4 (10 points)
A designer has available a number of six-point FFT chips. Show explicitly how he/she should interconnect two such chips to compute a twelve-point DFT. What is the number of multiplications and additions required? What are the savings in computations compared to direct computation of twelve-point DFT?

$$X[k] = \sum_{n=0}^{12} x(n) W_{12}^{kn} = \sum_{n=2m} x[2m] W_{12}^{2km} + \sum_{n=2m+1} x[2m+1] W_{12}^{k(2m+1)} =$$

$$= \underbrace{\sum_{m=0}^{5} x[2m] W_6^{km}}_{A(k)} + W_{12}^{k} \underbrace{\sum_{m=0}^{5} x[2m+1] W_6^{km}}_{B[k]}$$

6

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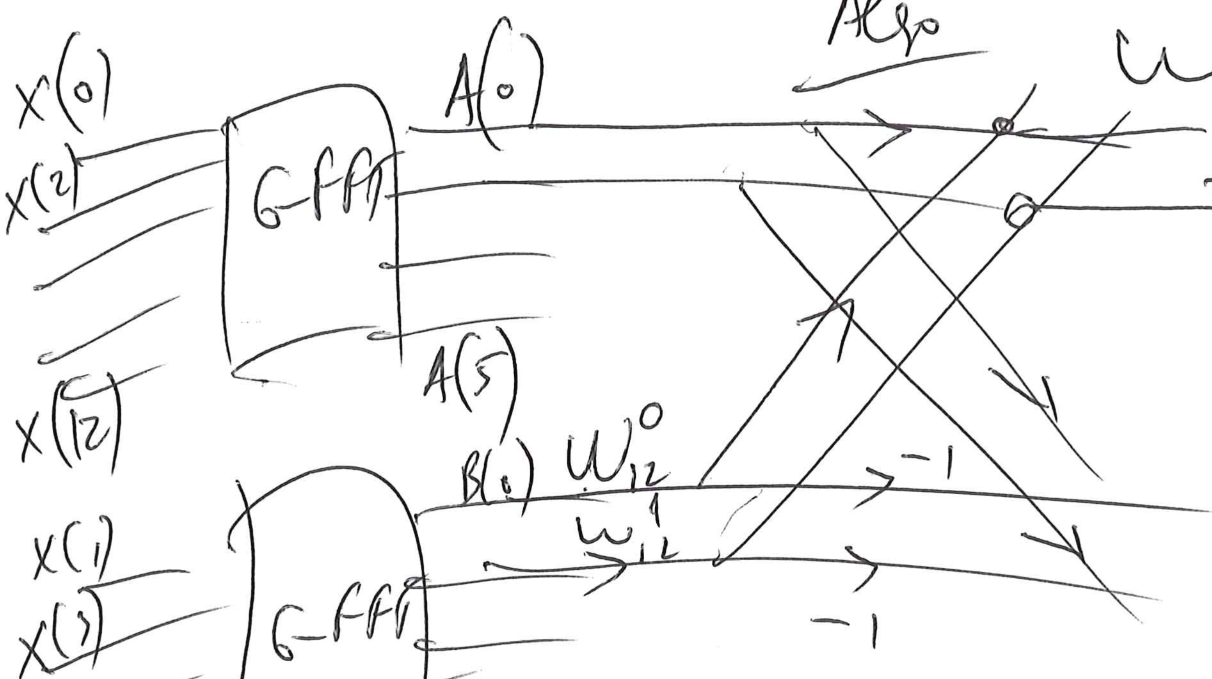
Algo

$$W_{12}^{k+6} = W_{12}^{6} W_{12}^{k} = (-1) W_{12}^{k}$$



x(11)

Direct # of ⊗: $12^2 = 144$

4 #X¢ $< 2 \cdot 6^2 + 5 = 72 + 5 = 77$

$h(n) = x[n-1] * x[-n]$

3

$H(w) = X(w) e^{-jw} \cdot X(-w)$

$= |X(w)|^2 e^{-jw}$

PROBLEM 5 (10 points)

(a) An audio signal $x(t)$ is bandlimited to $15kHz$, and its values may be assumed uniformly distributed on $[-1, 1]$. This signal must be sampled and transmitted aliasing-free with signal to quantization noise ratio of at least 40 dB. What is the minimum number of bits per second necessary for this?

(b) Consider the following two signals: $\{x[n]\}_{n=0}^{4} = \{1, 0, 0, 1, -1\}$ and $\{y[n]\}_{n=0}^{2} = \{1, -1, 1\}$. Assuming the N-point DFTs of these sequences are $X[k]$ and $Y[k]$ respectively, choose N so that

$$X[k] = Y[k]e^{j(2\pi k2/N)}$$

Is your choice for N unique?

$H[k]$ $\tilde{y}[n]$

(c) Show that the filter with impulse response $h[n] = x[n-1] * x[-n]$, where $x[n]$ is a real finite length sequence of length M, is a linear phase FIR filter.

a) ①

$\rightarrow F_s = 30$ kHz

4

$\rightarrow$ ② $SNR = 6.02B + 10.8 \geq 40$

$\Rightarrow B > \frac{30}{6} \Rightarrow B > 5$

Thus Minimum # bits/sec = 150 K

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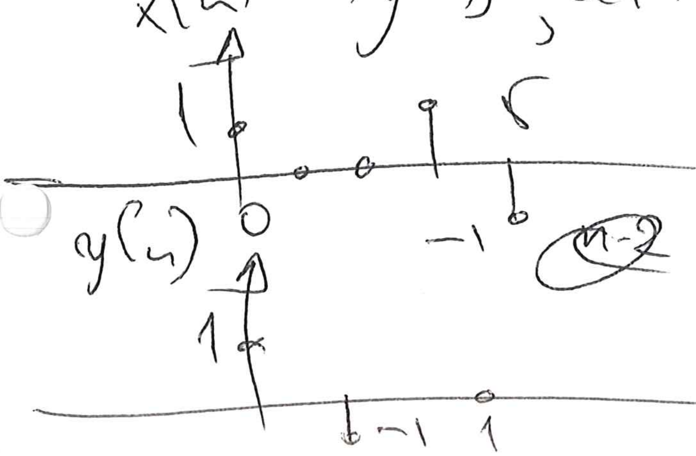
(b) $\tilde{y}[n]$ = We want to recall, $\tilde{y}[n] = \tilde{x}[n-2]$

where $\tilde{y}[n]$, $\tilde{x}[n]$ are the periodic extensions of $y[n]$, $x[n]$ with period N

$x(n)$



$N = 5$

3

$x((n-2) \bmod N)$

$= y(n \bmod N)$

$\Rightarrow$