MIMO Broadcast Channel with an Unknown Eavesdropper: Secrecy Degrees of Freedom

Xiang He[*], Ashish Khisti[†], Aylin Yener[*]

[*] Electrical Engineering Department, The Pennsylvania State University, University Park, PA 16802
[†]Dept. of Electrical and Computer Engineering, University of Toronto, Toronto, ON, M5S 3G4, Canada
hexiang@ieee.org, akhisti@comm.utoronto.ca, yener@ee.psu.edu

Abstract—We study a multi-antenna broadcast channel with two legitimate receivers and an external eavesdropper. We assume that the channel matrix of the eavesdropper is unknown to the legitimate terminals but satisfies a maximum rank constraint. As our main result we characterize the associated secrecy degrees of freedom for the broadcast channel with common and private messages. We show that a direct extension of the single-user wiretap codebook does not achieve the secrecy degrees of freedom. Our proposed optimal scheme involves decomposing the signal space into a common subspace, which can be observed by both receivers, and private subspaces which can be observed by only one of the receivers, and carefully transmitting a subset of messages in each subspace. We also consider the case when each user's private message must additionally remain confidential from the other legitimate receiver and characterize the s.d.o.f. region in this case.

I. Introduction

Claude Shannon [1] pioneered the information theoretic approach for secure communication. Shannon's notion of *perfect secrecy* requires that the information message and the eavesdropper's observation be statistically independent. This framework was later extended to different network models, see e.g., [2]–[7], where various relaxations of perfect secrecy were considered and the associated secrecy capacity was studied. In recent years there has been a growing interest in using multiple antennas for securing wireless networks, see e.g., [8]–[14]. In these works generally some sort of side information of the eavesdropper's channel — either complete, partial or statistical — is made available to the legitimate terminals. In contrast reference [15] considers a single-user Gaussian MIMO wiretap channel when the eavesdropper's channel is unknown and time-varying, but satisfies a maximum-rank constraint. The existence of a coding scheme that simultaneously attains strong secrecy against all feasible eavesdropper channels is established. Furthermore, two receiver broadcast and multiple-access channels (MAC) are also treated in [15] when each of the legitimate terminals has an equal number of antennas and the optimality of a time-sharing based scheme is established in either case. Recently a complete characterization of the secure

degrees of freedom for the two-user MIMO MAC channel with an arbitrarily varying eavesdropper has been obtained in [16].

In this paper, we consider the two-receiver MIMO broadcast channel when there is a private message for each receiver as well as a common message for both receivers. The messages must remain confidential from an eavesdropper. We assume that the channel matrices of the legitimate terminals are known to all the terminals whereas the channel matrix of the eavesdropper is only known to the eavesdropper. However an upper bound on the rank of the eavesdropper channel matrix, or equivalently the maximum number of antennas at the eavesdropper is known. We characterize the secrecy degrees of freedom (s.d.o.f.) region for such a model, as well as a variation when the private messages must also remain mutually confidential from the other receiver. Interestingly the optimal scheme does not follow from a direct extension of the techniques used in the single-user channel [15]. Such an approach introduces independent randomization in each user's codebook and creates higher than necessary interference between users. Instead our proposed approach involves decomposing the signal space into a common subspace seen by both receivers and private subspaces seen by only one of the receivers; and transmitting a fictitious message of just enough rate such that it can simultaneously provide secrecy for *both* users. We show that the s.d.o.f. achieved by the proposed scheme are in-fact optimal and meet the natural cut-set upper bound for the broadcast network. In contrast the scheme based on the single-user codebooks is sub-optimal in general. We limit our work to the case when the eavesdropper's channel is fixed throughout the duration of communication, but unknown to the legitimate terminals.

We note that the literature on secure network coding [18], [19] is also related to our setup. The most closely related paper to our present work is reference [20], which considers an extension of secure network coding for broadcasting to two receivers. The combined message of both users maps to a syndrome vector of a maximum rank distance (MRD) code (c.f. [19]). The parity-check matrix of the MRD code is designed to be in a certain systematic form, so that each receiver is able to recover the desired message from the observed sequence. While the results in the present paper

are structurally similar to [20], our underlying approach is very different. Instead of attempting a direct extension of the MRD codes to Gaussian channels we propose a random coding technique where an explicit fictitious message, shared by both the receivers is also transmitted. The key insight in our proposed scheme is to minimize the rate associated with this fictitious messages by using a carefully constructed signal space decomposition.

In the remainder of the paper, we present the system model in Section II and a summary of the main results in Section III. In section IV we present a reduction of the MIMO broadcast channel into independent parallel channels, which is based on the Generalized Singular Value Decomposition. Thereafter sections V and VI provide proofs of the main results and section VII concludes the paper.

Throughout this paper we only focus on the case of two legitimate receivers. Unfortunately an extension of our results to more than two receivers may not be straightforward. Indeed to the best of our knowledge, the degrees of freedom of the MIMO broadcast channel even without secrecy constraints remains an open problem when both common and private (individual) messages are considered. The well known compound MIMO broadcast channel is a special case of this setup [21]–[23]. Furthermore our lower bound involves the GSVD transform, whose direct extension to more than two channels does not appear straightforward and therefore we only limit to the case of two legitimate receivers. Nevertheless we believe that the setup considered in this paper is of practical significance. Further note that in this paper we only consider the secrecy degrees of freedom (s.d.o.f.), which measures the *pre-log* of the achievable rates. While a considerably coarse measure of the capacity region, the s.d.o.f. analysis is tractable and provides important insights into the optimal scheme in the high signal-to-noise-ratio regime. For some prior works on s.d.o.f., see e.g., [4], [13]–[16], [24].

II. SYSTEM MODEL

We consider a MIMO Broadcast (BC) wiretap channel with two receivers, as shown in Figure 1. We assume that the number of antennas at the transmitter, receiver 1, receiver 2 and the eavesdropper are given by N_T, N_{R_1}, N_{R_2} and N_E respectively:

$$\mathbf{Y}_t(i) = \mathbf{H}_t \cdot \mathbf{X}(i) + \mathbf{Z}_t(i), \quad t = 1, 2 \tag{1}$$

$$\tilde{\mathbf{Y}}(i) = \tilde{\mathbf{H}} \cdot \mathbf{X}(i) \tag{2}$$

where $\mathbf{Y}_t(i)$ denotes the symbols received at the legitimate receivers at time i whereas $\tilde{\mathbf{Y}}(i)$ denotes the received symbols at the eavesdropper, $\mathbf{H}_t$ and $\tilde{\mathbf{H}}$ are the channel matrices and $\mathbf{Z}_t$ is the additive Gaussian noise observed by the intended receiver t, which is composed of independent rotationally invariant complex Gaussian random variables with unit variance.

Remark 1: Note that in (2) we do not assume any noise on the eavesdropper's channel. In practice the eavesdropper's channel will have some additive noise and its observation

will be a degraded version of (2). Thus our achievability results immediately apply to such degraded channels. As such the model we study in (2) is the worst case model among all eavesdropper channels. While the converse for the above model does not directly apply, it can be easily extended to show that the s.d.o.f. region does not increase when the eavesdropper's channel has additive noise.

Note that the rank of $\tilde{\mathbf{H}}$ in (2) is upper bounded by N_E, which is known to all the terminals. The realization $\tilde{\mathbf{H}} = \tilde{\mathbf{h}}$ is revealed only to the eavesdropper and not to the legitimate terminals. Throughout this paper we will assume that $N_E < N_T$, since otherwise the secrecy degrees of freedom is zero in the single-user setup [15]. Similarly if either $N_E \geq N_{R_1}$ or $N_E \geq N_{R_2}$ the s.d.o.f. for at least one of the receivers is zero and the problem degenerates to the single user case. Thus we also assume that $N_E < \min(N_{R_1}, N_{R_2})$. Our proposed setup guarantees confidentiality regardless of the particular channel realization of the eavesdropper. In contrast, the channel matrices $\mathbf{H}_t$ are known to both the legitimate parties and the eavesdropper(s).

In practice note that the complete lack of the eavesdropper's CSI at the legitimate terminals is far more realistic than the previous models studied with partial or full knowledge, given that the eavesdropper is a passive observer who does not transmit signals. The limit on N_E can be justified since the passive eavesdropping device must be stealth, and hence be limited in number of antennas due to size limitations.

The input symbols in (2), denoted by $\mathbf{X}(i)$, must satisfy the average power constraint:

$$E \left[\frac{1}{n} \sum_{i=1}^{n} \text{trace} \left(\mathbf{X}(i) \mathbf{X}^H(i) \right) \right] \leq \bar{P}. \tag{3}$$

We next define the associated secure broadcast code. Receiver t must decode a confidential message W_t, and a common confidential message W_0 over n channel uses. The messages (W_0, W_1, W_2) must be kept jointly confidential from the eavesdropper. Let $\tilde{\mathbf{Y}}_{\tilde{\mathbf{h}}}^n$ denote the signals received by the eavesdropper when its channel matrix $\tilde{\mathbf{H}}$ equals $\tilde{\mathbf{h}}$. We impose the following secrecy constraint:

$$w \left(\lim_{n \to \infty} \frac{1}{n} \sup_{\tilde{\mathbf{h}}} I(W_0, W_1, W_2; \tilde{\mathbf{Y}}_{\tilde{\mathbf{h}}}^n) \right) = 0, \tag{4}$$

where $w(x) = \lim_{\bar{P} \to \infty} \frac{x}{\log_2 \bar{P}}$. To interpret the secrecy constraint in (4), note that $\frac{1}{n} I(W_0, W_1, W_2; \tilde{\mathbf{Y}}_{\tilde{\mathbf{h}}}^n)$ is the information leakage-rate [30, sec 22.1, pp. 550] at the eavesdropper. The constraint in (4) only requires that the pre-log of the asymptotic leakage-rate at the eavesdropper be zero. Note that this condition is weaker than the usual notion of *weak secrecy* which requires that the information leakage-rate approach must zero asymptotically in n. Strictly speaking, we should refer to (4) as the secrecy-DOF constraint, but we drop the "DOF" for simplicity in this paper. We primarily consider this notion as it suffices to highlight the key ideas in the coding scheme proposed in the paper. We point the reader to [15]
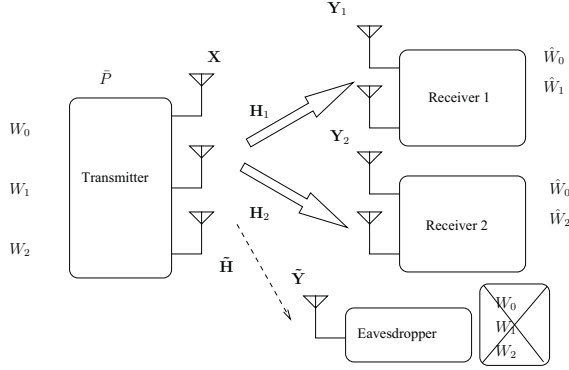
Fig. 1. The MIMO Broadcast Wiretap Channel where $N_T = 3, N_{R_1} = N_{R_2} = 2, N_E = 1$.
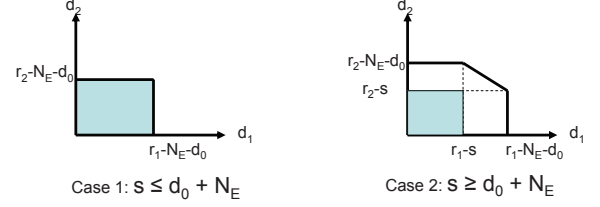

Fig. 2. Achievable Secrecy Degrees of Freedom for the two-user MIMO broadcast channel with an external eavesdropper. We fix the s.d.o.f. of the common message to d_0 and plot (d_1, d_2). The shaded area corresponds the s.d.o.f. achievable with mutual privacy constraint. The figure on the left corresponds to the case when $s \leq d_0 + N_E$ while the figure on the right corresponds to the case when $s \geq d_0 + N_E$.

be the dimension of the common row-space of $\mathbf{H}_1$ and $\mathbf{H}_2$.

Theorem 1: The secrecy degrees of freedom region for the MIMO broadcast wiretap channel in absence of the mutual privacy constraint is given by all non-negative triples (d_0, d_1, d_2) that satisfy the following constraints:

$$0 \leq d_0 + d_i \leq \{r_i - N_E\}^+, \quad i = 1, 2 \tag{9}$$
$$0 \leq d_0 + d_1 + d_2 \leq \{r_0 - N_E\}^+ \tag{10}$$

where we use the notation that $\{v\}^+ \triangleq \max(0, v)$. □

The inequalities in Theorem 1 can be interpreted as the cut-set bounds in the broadcast network. The two inequalities in (9) are single user bounds, whereas the inequality in (10) corresponds to the case when both the receivers are allowed to cooperate. Our proof of the coding theorem shows that these bounds are also achievable, whereas the converse involves selecting the specific eavesdropper channel gains that lead to these upper bounds.

Theorem 2: The secrecy degrees of freedom region for the MIMO broadcast wiretap channel in presence of the mutual privacy constraint (5) and (6) consists of all non-negative triples (d_0, d_1, d_2) that satisfy the following constraints:

$$0 \leq d_0 + d_i \leq \{r_i - N_E\}^+, \quad i = 1, 2, \tag{11}$$
$$0 \leq d_i \leq \{r_i - s\}^+, \quad i = 1, 2. \tag{12}$$

□

The inequalities in (11) corresponds to single-user bounds associated with each receiver, whereas the inequalities in (12) correspond to transmission of only a private message to each receiver, with the other receiver as the only eavesdropper in the network. Note that the sum-constraint is not active in Theorem 2.

Fig. 2 compares the results in Theorem 1 and 2. We observe that the structure of the capacity region takes one of two forms. In case 1, we assume that

$$N_E' \triangleq d_0 + N_E \geq s. \tag{13}$$

for the for the analysis of strong secrecy and time-varying eavesdropper channels in the single user case.

When an additional constraint of mutual privacy is imposed on the messages W_1 and W_2 we further require that:

$$w\left(\lim_{n \to \infty} \frac{1}{n} I(W_1; \mathbf{Y}_2^n) \right) = 0 \tag{5}$$

$$w\left(\lim_{n \to \infty} \frac{1}{n} I(W_2; \mathbf{Y}_1^n) \right) = 0 \tag{6}$$

The secrecy rate tuple $(R_{s,0}, R_{s,1}, R_{s,2})$ is achievable if $R_{s,i} = \lim_{n \to \infty} \frac{1}{n} H(W_i), i = 0, 1, 2$ and a sequence of encoding and decoding functions exists (indexed by n) such that the error probability in decoding of $\{W_0, W_t\}$ by receiver t approaches zero as $n \to \infty$ and furthermore the secrecy constraints (4) is satisfied. In addition when a mutual privacy constraint is imposed we also require that (5) and (6) be satisfied.

In this paper, we use the secrecy degrees of freedom (s.d.o.f.) region as a characterization of the high SNR behaviour of the secrecy capacity for this channel. The s.d.o.f. pair (d_0, d_1, d_2) is achievable if there exists a sequence of achievable rates $(R_{s,0}(\bar{P}), R_{s,1}(\bar{P}), R_{s,2}(\bar{P}))$, indexed by $\bar{P}$, such that $d_i = \limsup_{\bar{P} \to \infty} \frac{R_{s,i}(\bar{P})}{\log_2 P}$ for $i = 0, 1, 2$. The set of all achievable (d_0, d_1, d_2) is called the s.d.o.f. region.

III. MAIN RESULTS

The secrecy degrees of freedom region is characterized using rank of the associated channel matrices. Let r_1, r_2 be the rank of $\mathbf{H}_1$ and $\mathbf{H}_2$ respectively. Let

$$r_0 = \text{rank} \begin{bmatrix} \mathbf{H}_1 \\ \mathbf{H}_2 \end{bmatrix} \tag{7}$$

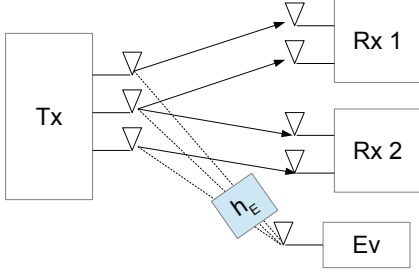and let

$$s = r_1 + r_2 - r_0 \tag{8}$$

Fig. 3. The decomposition of the $3 \times 2 \times 2 \times 1$ MIMO Broadcast Wiretap channel (cf. Fig. 1) using the GSVD transform. The channel matrices of the legitimate receivers are scaled versions of $[\mathbf{I}_2, \mathbf{0}]$ and $[\mathbf{0}, \mathbf{I}_2]$ respectively, while the eavesdropper channel matrix is of rank at most 1.

It can be verified that the two constraints in (9) imply (10) (by adding the constraints (9) and using (8) and (13)). Therefore the projection of the s.d.o.f. region in the (d_1, d_2) plane reduces to a rectangle

$$d_i \leq \{r_i - N'_E\}^+, \quad i = 1, 2 \qquad (14)$$

and the sum-rate constraint is not active. Furthermore upon examining (11) and (12), one can conclude that the same region is also achieved in Theorem 2, where an additional mutual privacy constraint is imposed.

In case 2, which corresponds to $N'_E \leq s$, the sum-rate constraint (10) is active. It can be easily verified that the constraints (9) and (10), using (8), reduce to $d_i \leq r_i - N'_E$, and $d_1 + d_2 \leq r_1 + r_2 - s - N'_E$. Thus as shown in Fig. 2, $(d_1, d_2) = (r_1 - s, r_2 - N'_E)$ and $(d_1, d_2) = (r_1 - N'_E, r_2 - s)$ are the two corner points in the (d_1, d_2) plane. Furthermore examining (11), (12) in Theorem 2, the active constraints in the case when $N'_E \leq s$ are $d_i \leq \{r_i - s\}^+$ for $i = 1, 2$. This region is in general smaller than the region achieved in Theorem 1.

As a final remark we note that when $N_E = 0$, i.e., the eavesdropper is absent, the result here is equivalent to the degrees of freedom for the two-user MIMO broadcast channel with common and private messages [25].

IV. GENERALIZED SINGULAR VALUE DECOMPOSITION

A common element in code construction in both Theorem 1 and 2 is the Generalized Singular Value Decomposition (GSVD) [26] previously used in [9] in the MIMO wiretap channel literature. The GSVD transform can be used to decompose the channel in (1) into parallel and independent channels, which are more amenable to analysis.

Theorem 3: [26] There exist unitary matrices $\mathbf{U}, \mathbf{V}, \mathbf{W}, \mathbf{Q}$ and a nonsingular upper triangular matrix $\mathbf{R}$ such that

$$\mathbf{U}^H \mathbf{H}_1 \mathbf{Q} = \mathbf{\Sigma}_{1(N_{R_1} \times r_0)} \left[\mathbf{W}^H \mathbf{R}_{(r_0 \times r_0)}, \mathbf{0} \right]_{(r_0 \times N_T)} \qquad (15)$$

$$\mathbf{V}^H \mathbf{H}_2 \mathbf{Q} = \mathbf{\Sigma}_{2(N_{R_2} \times r_0)} \left[\mathbf{W}^H \mathbf{R}_{(r_0 \times r_0)}, \mathbf{0} \right]_{(r_0 \times N_T)} \qquad (16)$$

$$\mathbf{\Sigma}_1 = \begin{bmatrix} \mathbf{I}_{1(\tilde{r}_1 \times \tilde{r}_1)} & & \\ & \mathbf{S}_{1(s \times s)} & \\ & & \mathbf{O}_{1((N_{R_1} - \tilde{r}_1 - s) \times \tilde{r}_2)} \end{bmatrix} \qquad (17)$$

$$\mathbf{\Sigma}_2 = \begin{bmatrix} \mathbf{O}_{2((N_{R_2} - \tilde{r}_2 - s) \times \tilde{r}_1)} & & \\ & \mathbf{S}_{2(s \times s)} & \\ & & \mathbf{I}_{2(\tilde{r}_2 \times \tilde{r}_2)} \end{bmatrix} \qquad (18)$$

where $\mathbf{S}_i, i = 1, 2$ are $s \times s$ diagonal matrices with positive real elements on the diagonal line, $\mathbf{I}_i, i = 1, 2$ are $\tilde{r}_i \times \tilde{r}_i$ identity matrices and the matrices $\mathbf{O}_i, i = 1, 2$ are zero matrices. For clarity, the dimension of each matrix is shown in the parenthesis in the subscript. Recall from (7) and (8) that r_0 equals the rank of $\begin{bmatrix} \mathbf{H}_1 \\ \mathbf{H}_2 \end{bmatrix}$, r_1 and r_2 equal the rank of $\mathbf{H}_1$ and $\mathbf{H}_2$, and we let $s = r_1 + r_2 - r_0$. The constants $\tilde{r}_i$, for $i = 1, 2$, are given by $\tilde{r}_i = r_i - s$. $\square$

We next demonstrate the simultaneous reduction of the channel matrices $\mathbf{H}_i, i = 1, 2$ into parallel and independent channels using the GSVD transform. Let the decomposition of $\mathbf{H}_i$ be as in (15) and (16). We left-multiply the transmitted signals with $\mathbf{Q}$, left-multiply the received signals with $\mathbf{U}^H$ at receiver 1, and left-multiply the received signals with $\mathbf{V}^H$ at receiver 2. Since $\mathbf{Q}$, $\mathbf{U}^H$ and $\mathbf{V}^H$ are all unitary matrices the setup is equivalent to the following:

$$\mathbf{Y}_t(i) = \mathbf{\Sigma}_t \left[\mathbf{P}_{(r_0 \times r_0)}, \mathbf{0} \right] \mathbf{X}(i) + \mathbf{Z}_t(i), \quad t = 1, 2, \quad (19)$$

$$\tilde{\mathbf{Y}}(i) = \tilde{\mathbf{H}} \mathbf{X}(i), \qquad (20)$$

where we have introduced the matrix $\mathbf{P} \triangleq \mathbf{W}^H \mathbf{R}$. We also set the last $N_T - r_0$ component of $\mathbf{X}$ to zero and design the achievable scheme for the following channel model:

$$\mathbf{Y}_t(i) = \mathbf{\Sigma}_t \mathbf{P}_{(r_0 \times r_0)} \mathbf{X}_{(r_0 \times 1)}(i) + \mathbf{Z}_t(i), t = 1, 2, \quad (21)$$

$$\tilde{\mathbf{Y}}(i) = \{\tilde{\mathbf{H}} \mathbf{Q}\}_{(N_E \times r_0)} \mathbf{X}_{(r_0 \times 1)}(i), \qquad (22)$$

where $\{\tilde{\mathbf{H}} \mathbf{Q}\}_{(N_E \times r_0)}$ denotes the first r_0 columns of the matrix $\tilde{\mathbf{H}} \mathbf{Q}$. Since $\mathbf{P}$ is nonsingular, without loss of generality, we can view $\mathbf{P} \mathbf{X}_{(r_0 \times 1)}(i)$ as the input to the channel. The main channel can then be expressed as $\mathbf{Y}_t(i) = \mathbf{\Sigma}_t \mathbf{X}_{(r_0 \times 1)}(i) + \mathbf{Z}_t(i), t = 1, 2$ and the eavesdropper channel reduces to $\tilde{\mathbf{Y}}(i) = \{\tilde{\mathbf{H}} \mathbf{Q}\}_{(N_E \times r_0)} \mathbf{P}^{-1} \mathbf{X}_{(r_0 \times 1)}(i)$. Note that the eavesdropper channel state matrix is arbitrary, and $\mathbf{Q}$ is a unitary matrix, and thus it can be easily seen that the rank of $\{\tilde{\mathbf{H}} \mathbf{Q}\}_{(N_E \times r_0)} \mathbf{P}^{-1}$ is the same as rank of $\tilde{\mathbf{H}}_{(N_E \times r_0)}$. Therefore we can simply replace $\{\tilde{\mathbf{H}} \mathbf{Q}\}_{(N_E \times r_0)} \mathbf{P}^{-1}$ with $\tilde{\mathbf{H}}_{(N_E \times r_0)}$. Thus it suffices to consider the following channel model instead:

$$\mathbf{Y}_t(i) = \mathbf{\Sigma}_t \mathbf{X}_{(r_0 \times 1)}(i) + \mathbf{Z}_t(i), t = 1, 2 \qquad (23)$$

$$\tilde{\mathbf{Y}}(i) = \tilde{\mathbf{H}}_{(N_E \times r_0)} \mathbf{X}_{(r_0 \times 1)}(i) \qquad (24)$$

subject to the following constraint:

$$E\left[\frac{1}{n}\text{trace}\left\{(\mathbf{PX}^n)(\mathbf{PX}^n)^H\right\}\right] \leq \bar{P} \quad (25)$$

where $\mathbf{PX}^n$ denotes the vector formed by concatenating $\{\mathbf{PX}(i)\}_{1 \leq i \leq n}$. Recall that if $s_{\mathbf{P}}^2$ denotes the largest eigenvalue of $\mathbf{P}^H\mathbf{P}$ then we have that:

$$\text{trace}\{(\mathbf{PX}(i))(\mathbf{PX}(i))^H\} \leq s_{\mathbf{P}}^2\text{trace}\{\mathbf{X}(i)\mathbf{X}(i)^H\}. \quad (26)$$

Hence when designing achievable scheme, we use the following power constraint, which is a sufficient condition for (25) to hold:

$$E\left[\frac{1}{n}\text{trace}(\mathbf{X}^n(\mathbf{X}^n)^H)\right] \leq \frac{\bar{P}}{s_{\mathbf{P}}^2} \quad (27)$$

We further reduce the channel (23) to obtain equivalent parallel channels. Let $s_{\min}$ be the minimal nonzero element among all diagonal elements in $\mathbf{S}_1$ and $\mathbf{S}_2$. Replace all diagonal nonzero elements of $\mathbf{\Sigma}_t$ with $s_{\min}$ and let the resulting matrix be $\bar{\mathbf{\Sigma}}_t$. We present our coding scheme for the following channel:

$$\mathbf{Y}_t(i) = \bar{\mathbf{\Sigma}}_t\mathbf{X}_{(r_0 \times 1)}(i) + \mathbf{Z}_t(i), \quad t = 1, 2 \quad (28)$$

$$\tilde{\mathbf{Y}}(i) = \tilde{\mathbf{H}}_{(N_E \times r_0)}(i)\mathbf{X}_{(r_0 \times 1)}(i) \quad (29)$$

If we let:

$$\mathbf{X}_{(r_0 \times 1)}(i) = \begin{bmatrix} \mathbf{X}_{\tilde{r}_1}(i) \\ \mathbf{X}_s(i) \\ \mathbf{X}_{\tilde{r}_2}(i) \end{bmatrix} \begin{matrix} \}\tilde{r}_1 \text{ rows} \\ \}s \text{ rows} \\ \}\tilde{r}_2 \text{ rows} \end{matrix} \quad (30)$$

then the two equations given in (28) reduce to the following

$$\mathbf{Y}_1(i) = s_{\min}\begin{bmatrix} \mathbf{X}_{\tilde{r}_1}(i) \\ \mathbf{X}_s(i) \end{bmatrix} + \mathbf{Z}_1(i), \quad (31)$$

$$\mathbf{Y}_2(i) = s_{\min}\begin{bmatrix} \mathbf{X}_s(i) \\ \mathbf{X}_{\tilde{r}_2}(i) \end{bmatrix} + \mathbf{Z}_2(i). \quad (32)$$

Thus (31) and (32) denote a collection of parallel, independent and identically distributed channels between the transmitter and the legitimate receivers. The channels with input $\mathbf{X}_s(i)$ denote the common channels observed by both receivers, whereas the channels with input $\mathbf{X}_{\tilde{r}_1}(i)$ and $\mathbf{X}_{\tilde{r}_2}(i)$ are only observed by receivers 1 and 2 respectively.

V. Proof of Theorem 1

We first present the key ideas in the coding scheme for a special example, and then present the coding scheme for the general case.

A. A Motivating Example: $3 \times 2 \times 2 \times 1$ Channel

Consider the special case when $N_T = 3$, $N_E = 1$ and $N_{R_1} = N_{R_2} = 2$. For simplicity, we assume that the common message W_0 is not present. Assume that $r_1 = r_2 = 2$ and $r_0 = 3$ and all the channel matrices are full rank. Following

the reduction in (31) and (32), the channel matrices of the two legitimate receivers reduce to:

$$\mathbf{H}_1 = [\mathbf{I}_{(2 \times 2)}, \mathbf{0}_{(2 \times 1)}], \quad \mathbf{H}_2 = [\mathbf{0}_{(2 \times 1)}, \mathbf{I}_{(2 \times 2)}] \quad (33)$$

while the effective channel matrix of the eavesdropper is an arbitrary rank one matrix. Assume that we do not impose a mutual secrecy constraint and let $d_0 = 0$. Thus according to Theorem 1 we seek to achieve $d_1 = d_2 = 1$.

Recall that in (1), the vector $\mathbf{X}$ denotes the transmitter input. Since the transmitter has three antennas, i.e., $N_T = 3$, $\mathbf{X}$ has three components. To achieve $d_1 = 1$, a single-user wiretap codebook $\mathcal{C}_1$ for user 1 requires transmission over the first and the second component of $\mathbf{X}$. Likewise to achieve $d_2 = 1$, a single-user wiretap codebook $\mathcal{C}_2$ requires transmission over the second and third component of $\mathbf{X}$. Thus the two codebooks must share the second component of $\mathbf{X}$. However, since W_1 and W_2 are independent, the signals that $\mathcal{C}_1$ uses to represent W_1 over the second component of $\mathbf{X}$ in general do not agree with the signals that $\mathcal{C}_2$ uses to represent W_2 over this component, causing a conflict. Thus we cannot simultaneously achieve $d_1 = 1$ and $d_2 = 1$ using this approach.

Our proposed scheme in Theorem 1 resolves this conflict by constructing three codebooks, one for each component of $\mathbf{X}$. A codebook for the second component of $\mathbf{X}$, $\mathcal{C}_E$, is used to transmit a fictitious message W_E via a codeword $X_E^n(W_E)$. An independent codebook $\mathcal{C}_1$ is used to jointly encode (W_E, W_1) into a codeword $X_1^n(W_E, W_1)$ which is transmitted over the first component of $\mathbf{X}$. Another codebook $\mathcal{C}_2$ for the third component of $\mathbf{X}$ is used to transmit a codeword $X_2^n(W_E, W_2)$. Through random coding analysis, as will be discussed later, one can show that users 1 and 2 can decode (W_1, W_E) and (W_2, W_E) upon observing (X_1^n, X_E^n) and (X_2^n, X_E^n) respectively, with high probability. Furthermore as will be shown in the sequel, the messages (W_1, W_2) remain simultaneously confidential from any eavesdropper with a single receive antenna.

To summarize the above example, note that the naive extension of the single-user codebook involves *independent* randomization in the codebooks of the two users. This effectively injects fictitious messages of a higher rate and in turn reduces the message rate. In contrast the proposed scheme introduces a fictitious message of minimum possible rate needed to guarantee secrecy.

In generalizing the above example to arbitrary number of antennas, we use three i.i.d. Gaussian codebooks, and assign a subset of parallel channels for each codeword. The rate of the codebooks is selected such that the average error probability at the legitimate receivers under maximum likelihood decoding is arbitrarily small. We also show in section V-C that if the information messages are revealed to the eavesdropper, the error probability in decoding the fictitious message given the eavesdropper's observation, also vanishes to zero. For a codebook satisfying these properties, we provide the secrecy analysis in section V-D and complete the proof of the coding theorem. We note that this approach of secrecy analysis, where

the eavesdropper is able to decode the fictitious message given side information, is routinely used when establishing the achievability of weak-secrecy.

B. Achievability

Since the secrecy rate is zero whenever $r_0 \leq N_E$ (c.f. [15]), without loss of generality, we assume $r_0 > N_E$ and consider $\tilde{\mathbf{H}}$ that has the following form:

$$\tilde{\mathbf{H}} = [\mathbf{I}_{N_E \times N_E}, \mathbf{0}_{N_E \times (r_0 - N_E)}]\mathbf{U}_E \triangleq \tilde{\mathbf{U}}_E \qquad (34)$$

where $\mathbf{U}_E$ is a unitary matrix, which is only known by the eavesdropper, and $\tilde{\mathbf{U}}_E$ represents the first N_E rows of $\mathbf{U}_E$. Furthermore recall that the legitimate receiver's channel are parallel independent broadcast channels:

$$\mathbf{Y}_1(i) = s_{\min}\begin{bmatrix} \mathbf{X}_{\tilde{r}_1}(i) \\ \mathbf{X}_s(i) \end{bmatrix} + \mathbf{Z}_1(i) \qquad (35)$$

$$\mathbf{Y}_2(i) = s_{\min}\begin{bmatrix} \mathbf{X}_s(i) \\ \mathbf{X}_{\tilde{r}_2}(i) \end{bmatrix} + \mathbf{Z}_2(i) \qquad (36)$$

As in [15], [27], we then introduce artificial noise into $\mathbf{X}$ as:

$$\mathbf{X}(i) = \bar{\mathbf{X}}_{(r_0 \times 1)}(i) + \mathbf{N}(i) \qquad (37)$$

where $\mathbf{N}$ is the $r_0 \times 1$ artificial noise vector consisting of independent rotationally invariant complex Gaussian random variables with zero mean and unit variance. In contrast $\bar{\mathbf{X}}$ is the information bearing signal which will be used in the codebook transmission.

Let P be such that $P = \frac{\bar{P}}{s_\mathbf{P}^2} - r_0$ (c.f. (27)). We shall allocate a total power of r_0 units on artificial noise $\mathbf{N}$ in (37) and P units on $\bar{\mathbf{X}}$. Let the rate R be defined by:

$$R = C(s_{\min}^2(P/r_0)/(s_{\min}^2 + 1)) \qquad (38)$$

where $C(x) \triangleq \log_2(1+x)$. Note that R is the rate supported over each parallel channel in (35) and (36).

We sample our codebooks from an i.i.d. Gaussian random ensemble as discussed next. Let $\bar{\epsilon} > 0$ be a fixed constant. Let $Q_k(\mathbf{x})$ denote the k-dimensional rotationally invariant complex Gaussian distribution with covariance matrix $(P(1-\bar{\epsilon})/r_0)\mathbf{I}_{(k \times k)}$ where $\mathbf{I}_{k \times k}$ denotes an identity matrix of dimension k. Define the n-letter Gaussian input distribution $Q_k(\mathbf{x}^n)$ as $Q_k(\mathbf{x}^n) = \prod_{i=1}^{n} Q_k(\mathbf{x}_i)$. In the following we consider transmission of four messages $W_i \in [1, 2^{nd_iR}]$, $W_0 \in [1, 2^{nd_0R}]$ and $W_E \in [1, 2^{nN_ER}]$, where W_0 is the common message and W_1 and W_2 are the private messages that need to be decoded by receivers 1 and 2 respectively. The message W_E is a fictitious message. As in (13) we define N_E' using:

$$N_E' = N_E + d_0. \qquad (39)$$

Such a notation is again convenient, since in our coding scheme we will jointly code the message pair (W_E, W_0) of a total rate $2^{nR(N_E+d_0)}$. We separately consider the case when $s \leq N_E'$ and when $s > N_E'$.
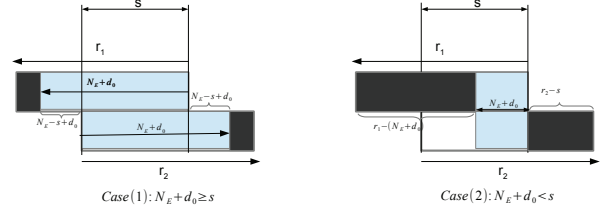

Fig. 4. Codebook generation: (1) $s \leq N_E + d_0 \leq \min\{r_1, r_2\}$ (2) $0 \leq N_E + d_0 < s$. Here $s = r_1 + r_2 - r_0$ denotes the dimension of the common subspace. The shaded blue region indicates the dimensions where the common and fictitious messages i.e., $\bar{\mathbf{X}}_B^n(W_0, W_E)$ are transmitted. The shaded black portion indicate the dimensions where the private messages are transmitted. In case (1), in addition to the common subspace, we further need to use $(N_E + d_0 - s)$ dimensions of each private subspace for transmitting the common message. In case (2), we have a surplus $(s - N_E - d_0)$ dimensions in the common subspace. These can be used for transmitting private messages. Therefore the sum-rate constraint in Theorem 1 is active in this case.

1) Case 1 ($s \leq N_E' \leq \min(r_1, r_2)$): In this case recall from Theorem 1 and Fig. 2 that the (d_1, d_2) region is a rectangle. It suffices to show that any triple $(\tilde{d}_0, \tilde{d}_1, \tilde{d}_2)$ such that $\tilde{d}_0 \leq N_E' - N_E$ and $\tilde{d}_i \leq r_i - N_E'$ for $i = 1, 2$, is achievable. Following the decomposition illustrated in Figure 4 let:

$$\bar{\mathbf{X}}_{(r_0 \times 1)}(i) = \begin{bmatrix} \bar{\mathbf{X}}_A(i) \\ \bar{\mathbf{X}}_B(i) \\ \bar{\mathbf{X}}_C(i) \end{bmatrix} \begin{matrix} \}r_1 - N_E' \text{ rows} \\ \}2N_E' - s \text{ rows} \\ \}r_2 - N_E' \text{ rows} \end{matrix} \qquad (40)$$

We let the three components above correspond to three different codebooks $\mathcal{C}_A$, $\mathcal{C}_B$ and $\mathcal{C}_C$ indicated below.

- The codebook $\mathcal{C}_B$ maps the message-pair (W_0, W_E) to a codeword $\bar{\mathbf{X}}_B^n(W_0, W_E)$. It consists of $2^{n(N_E'R)}$ codewords. Each codeword is sampled in an i.i.d. fashion from the distribution $Q_{(2N_E'-s)}(\mathbf{x})$. The codeword is transmitted through the component $\bar{\mathbf{X}}_B$ in (40) as discussed below.

$$\bar{\mathbf{X}}_B(i) = \begin{bmatrix} \bar{\mathbf{X}}_{B1}(i) \\ \bar{\mathbf{X}}_{B0}(i) \\ \bar{\mathbf{X}}_{B2}(i) \end{bmatrix} \begin{matrix} \}N_E' - s \text{ rows} \\ \}s \text{ rows} \\ \}N_E' - s \text{ rows} \end{matrix} \qquad (41)$$

and let $\bar{\mathbf{X}}_s(i) = \bar{\mathbf{X}}_{B0}(i)$, and furthermore

$$\bar{\mathbf{X}}_{\tilde{r}_1}(i) = \begin{bmatrix} \bar{\mathbf{X}}_A(i) \\ \bar{\mathbf{X}}_{B1}(i) \end{bmatrix} \quad \bar{\mathbf{X}}_{\tilde{r}_2}(i) = \begin{bmatrix} \bar{\mathbf{X}}_{B2}(i) \\ \bar{\mathbf{X}}_C(i) \end{bmatrix}, \quad (42)$$

where the vectors $\bar{\mathbf{X}}_s(i)$, $\bar{\mathbf{X}}_{\tilde{r}_1}(i)$ and $\bar{\mathbf{X}}_{\tilde{r}_2}(i)$ are inputs into the parallel channels in (30).

- The codebook $\mathcal{C}_A$ maps a message pair (W_0, W_1, W_E) to a codeword $\bar{\mathbf{X}}_A^n(W_0, W_1, W_E)$. It consists of a total of $2^{n((\tilde{d}_0 + \tilde{d}_1 + N_E)R)}$ codewords each sampled in an i.i.d. fashion from the distribution $Q_{(r_1 - N_E')}(\mathbf{x})$. The codeword will be transmitted through $\bar{\mathbf{X}}_A$.

- The codebook $\mathcal{C}_C$ maps the message pair (W_0, W_2, W_E) to a codeword $\bar{\mathbf{X}}_C^n(W_0, W_2, W_E)$. It consists of a total of $2^{n((\tilde{d}_0 + \tilde{d}_2 + N_E)R)}$ codewords. Each codeword is sampled in an i.i.d. fashion from the distribution $Q_{(r_2 - N_E')}(\mathbf{x})$.

Given a message pair (W_0, W_1, W_2, W_E) the encoder generates the associated sequence $\bar{\mathbf{X}}^n$ (c.f. (40)) and transmits $\mathbf{X}^n$ (c.f. (37)) over n channel uses. We declare an error if $\mathbf{X}^n$ does not satisfy the average power constraint (cf. (3)). By selecting n to be sufficiently large, this error can be made arbitrarily small.

The received signal $\mathbf{Y}_1^n$ (c.f. (31)) at receiver 1 can be expressed as:

$$\mathbf{Y}_1(i) = \begin{bmatrix} \mathbf{Y}_A(i) \\ \mathbf{Y}_{B1}(i) \\ \mathbf{Y}_s(i) \end{bmatrix} \begin{matrix} \}r_1 - N_E' \text{ rows} \\ \}N_E' - s \text{ rows} \\ \}s \text{ rows} \end{matrix} \quad . \qquad (43)$$

Receiver 1 decodes (W_0, W_1, W_E) in the following order:

1) Decode (W_0, W_E) from $(\mathbf{Y}_{B1}^n, \mathbf{Y}_s^n)$ using a maximal likelihood decoder:

$$(\hat{W}_0, \hat{W}_E) = \arg \max_{w_0, w_E} \Pr\left(\bar{\mathbf{X}}_B^n(w_0, w_E)|\mathbf{Y}_{B1}^n, \mathbf{Y}_s^n\right)$$
$$(44)$$

2) Decode W_1 from $\mathbf{Y}_A^n$ using a maximal likelihood decoder:

$$\hat{W}_1 = \arg \max_{w_1} \Pr\left(\bar{\mathbf{X}}_A^n(\hat{W}_0, w_1, \hat{W}_E)|\mathbf{Y}_A^n\right) \qquad (45)$$

It can be shown through standard analysis[1] that the error probability in (44) approaches zero provided (R_0, R_E) satisfy the following:

$$R_0 + R_E < I(\bar{\mathbf{X}}_B; \mathbf{Y}_{B1}, \mathbf{Y}_s) = N_E' R \qquad (46)$$

where the rate R is the rate associated with each parallel channel (38). This shows that any $\tilde{d}_0 \leq N_E' - N_E = d_0$ (c.f. (39)) is achievable at user 1. Furthermore the error probability in (45) vanishes to zero provided that

$$R_1 < I(\bar{\mathbf{X}}_A; \mathbf{Y}_A) = (r_1 - N_E')R \qquad (47)$$

is satisfied i.e., $\tilde{d}_1 \leq r_1 - N_E'$ is achievable for user 1. In an analogous manner we can show that $\tilde{d}_0 = N_E' - N_E$ and $\tilde{d}_2 = r_2 - N_E'$ are achievable for user 2.

2) Case 2 ($N_E' < s$): In this case the sum-rate constraint in Theorem 1 is active. We show that $\tilde{d}_0 \leq N_E' - N_E$ as well as the corner point $(\tilde{d}_1, \tilde{d}_2) = (r_1 - N_E', r_2 - s)$ is achievable. By a symmetric argument it follows that the corner point $(r_1 - s, r_2 - N_E')$ is also achievable. The achievability of the entire region then follows using a time-sharing argument.

To define our code construction, we begin by splitting the input symbols $\bar{\mathbf{X}}_s$ in (30) into two groups:

$$\bar{\mathbf{X}}_s = \begin{bmatrix} \bar{\mathbf{X}}_{s1} \\ \bar{\mathbf{X}}_{s2} \end{bmatrix} \begin{matrix} \}s - N_E' \text{ rows} \\ \}N_E' \text{ rows} \end{matrix} \qquad (48)$$

---

[1]If a joint-typicality based decoder is used we still obtain the same rate. However the maximum likelihood decoder also guarantees that the error probability approaches zero exponentially with the block-length [28, (7.3.22)]. This particular scaling is useful in showing the existence of a single universal codebook that remains confidential against all eavesdropper channels simultaneously as done in the single user case [15].

Define $\bar{\mathbf{X}}_A$ as

$$\bar{\mathbf{X}}_A = \begin{bmatrix} \bar{\mathbf{X}}_{\tilde{r}_1} \\ \bar{\mathbf{X}}_{s1} \end{bmatrix} \begin{matrix} \}\tilde{r}_1 \text{ rows} \\ \}s - N_E' \text{ rows} \end{matrix} , \qquad (49)$$

where $\bar{\mathbf{X}}_{\tilde{r}_1}$ constitutes the input to parallel channels of receiver 1 in (30). Let $\bar{\mathbf{X}}_B = \bar{\mathbf{X}}_{s2}$ and $\bar{\mathbf{X}}_C = \bar{\mathbf{X}}_{\tilde{r}_2}$. The overall input vector is expressed via $\bar{\mathbf{X}} = \begin{bmatrix} \bar{\mathbf{X}}_A \\ \bar{\mathbf{X}}_B \\ \bar{\mathbf{X}}_C \end{bmatrix}$.

As before we use $\bar{\mathbf{X}}_A$ to send W_1 to user 1, use $\bar{\mathbf{X}}_C$ to send W_2 to user 2, and use the second group $\bar{\mathbf{X}}_B$ for sending W_0 and W_E. The associated codebook construction is discussed next.

- The codebook $\mathcal{C}_B$ that maps the message pair (W_0, W_E) to a codeword $\bar{\mathbf{X}}_B^n(W_0, W_E)$. It consists of $2^{n(N_E' R)}$ codewords. Each codeword is sampled in an i.i.d. fashion from the distribution $Q_{N_E}(\mathbf{x})$.
- The codebook $\mathcal{C}_A$ that maps each message pair (W_0, W_1, W_E) to a codeword $\bar{\mathbf{X}}_A^n(W_0, W_1, W_E)$. It consists of a total of $2^{n((r_1 - N_E')R)}$ codewords. Each codeword is sampled in an i.i.d. fashion from the distribution $Q_{(r_1 - N_E')}(\mathbf{x})$.
- The codebook $\mathcal{C}_C$ maps the message pair (W_0, W_2, W_E) to a codeword $\bar{\mathbf{X}}_C^n(W_0, W_2, W_E)$. It consists of a total of $2^{n((r_2 - s)R)}$ codewords each sampled in an i.i.d. fashion from the distribution $Q_{(r_2 - s)}(\mathbf{x})$.

Upon receiving $\mathbf{Y}_1^n$ (c.f. (31)), receiver 1 decomposes each of its component $\mathbf{Y}_1(i)$ as:

$$\mathbf{Y}_1(i) = \begin{bmatrix} \mathbf{Y}_A(i) \\ \mathbf{Y}_s(i) \end{bmatrix} \begin{matrix} \}r_1 - N_E' \text{ rows} \\ \}N_E' \text{ rows} \end{matrix} . \qquad (50)$$

Receiver 1 decodes (W_0, W_1, W_E) in the following order:

1) Decode (W_0, W_E) from $\mathbf{Y}_s^n$ using the maximal likelihood decoder:

$$(\hat{W}_0, \hat{W}_E) = \arg \max_{w_0, w_E} \Pr\left(\bar{\mathbf{X}}_B^n(w_0, w_E)|\mathbf{Y}_s^n\right) \quad (51)$$

2) Decode W_1 from $\mathbf{Y}_A^n$ using the maximal likelihood decoder in (45).

It can be shown that the error probability associated with (51) vanishes to zero if $R_0 + R_E < I(\bar{\mathbf{X}}_B; \mathbf{Y}_s) = N_E' R$. This shows that $\tilde{d}_0 \leq N_E' - N_E = d_0$ is achievable. Likewise the error probability associated with message W_1 vanishes to zero if $R_1 < I(\bar{\mathbf{X}}_A; \mathbf{Y}_A) = (r_1 - N_E')R$. This shows that $\tilde{d}_1 \leq r_1 - N_E'$ is achievable for user 1.

In an analogous matter we can show that the error probability at user 2 vanishes to zero if $\tilde{d}_0 \leq N_E' - N_E$ and $\tilde{d}_2 \leq r_2 - s$.

C. Side Information Assisted Decoding at Eavesdropper

We next argue that in the codebook ensemble $(\mathcal{C}_A \times \mathcal{C}_B \times \mathcal{C}_C)$ there exists at-least one codebook such that the eavesdropper can also reliably decode W_E given (W_0, W_1, W_2). Such a codebook will be used in the analysis of equivocation in the

next sub-section. Our proposed decoder is also a maximum likelihood decoder as stated below:

$$\hat{W}_E = \arg\max_{w_E} \Pr\left(\bar{\mathbf{X}}_A^n(\hat{W}_0, \hat{W}_1, w_E), \bar{\mathbf{X}}_B^n(\hat{W}_0, w_E),\right.$$
$$\left.\bar{\mathbf{X}}_C^n(\hat{W}_0, \hat{W}_2, w_E)|\tilde{\mathbf{Y}}_{\tilde{\mathbf{h}}}^n\right) \quad (52)$$

where $(\hat{W}_0, \hat{W}_1, \hat{W}_2)$ denote the messages revealed to the eavesdropper. The error probability decays to zero if the rate R_E satisfies:

$$R_E < I(\bar{\mathbf{X}}_A, \bar{\mathbf{X}}_B, \bar{\mathbf{X}}_C; \tilde{\mathbf{Y}}_{\tilde{\mathbf{h}}}) \quad (53)$$
$$= I(\bar{\mathbf{X}}_A, \bar{\mathbf{X}}_B, \bar{\mathbf{X}}_C; \tilde{\mathbf{U}}_E\bar{\mathbf{X}} + \tilde{\mathbf{U}}_E\mathbf{N}) \quad (54)$$
$$= \log\det\left(\mathbf{I} + \frac{P}{r_0}\tilde{\mathbf{U}}_E\tilde{\mathbf{U}}_E^H\right) \quad (55)$$
$$= N_E \log\left(1 + \frac{P}{r_0}\right) \quad (56)$$

where $\tilde{\mathbf{U}}_E$ denotes the first N_E rows of the unitary matrix $\mathbf{U}_E$ (c.f. (34)). We further substitute (37) in (54), (55) follows from the fact that the entries of $\tilde{\mathbf{X}}$ are sampled i.i.d. from $\mathcal{CN}(0, P/r_0)$ and the last relation uses $\tilde{\mathbf{U}}_E\tilde{\mathbf{U}}_E^H = \mathbf{I}_{N_E}$. Since we select $R_E = N_E R$, where R (c.f. (38)) satisfies $R \le \log\left(1 + \frac{P}{r_0}\right)$, it follows (56) is indeed satisfied. This shows that in the ensemble of codebooks there exists at-least one codebook such that W_E is reliably decoded by the eavesdropper with a fixed channel matrix $\tilde{\mathbf{h}}$. Thus using Fano's inequality:

$$H\left(W_E|\tilde{\mathbf{Y}}_{\tilde{\mathbf{h}}}^n, W_0, W_1, W_2\right) \le n\epsilon_n \quad (57)$$

for some sequence ϵ_n that converges to zero as $n \to \infty$. Using a standard union bound argument it follows that there is at-least one codebook which can be reliably decoded by the legitimate receivers and that satisfies (57).

In order to establish the secrecy constraint (4), we need to demonstrate that there exists a single codebook that satisfies (57) for every possible realization of the eavesdropper channel matrix $\tilde{\mathbf{H}}$, whose rank equals N_E. The existence of such codebooks generally follows from the compound channel coding theorem [30, Theorem 7.1 (pp. 170), Remark 7.3 (pp. 172)], [31, Eq. (11)]. We remark that when the set of possible states $\tilde{\mathbf{H}}$ is finite, the proof of the compound channel coding theorem exploits a union bound argument over the states. In the present case $\tilde{\mathbf{H}}$ belongs to a continuous set. Therefore a simple union bound argument cannot be applied. Suitable quantization of the channel matrices is needed to show that the error probability simultaneously goes to zero for each state. We refer the reader to [15] where such an argument was carefully outlined for the single-user case, but leave out a detailed argument in the present paper as it is analogous[2].

---

[2]One key difference required in the extension to continuous set of channels is that we cannot sample the codewords i.i.d. and then use expurgation to satisfy the power constraint as is commonly done (see e.g., [29, pp. 243-245]). Instead we need to sample codewords from a normalized distribution, so that they lie within a ball. We refer the reader to [15] for further details.

We note that such a codebook guarantees that (57) is satisfied for any $\tilde{\mathbf{H}}$ whose rank equals N_E i.e.,

$$\sup_{\tilde{\mathbf{h}}:\ \mathrm{rank}(\tilde{\mathbf{h}})=N_E} H\left(W_E|\tilde{\mathbf{Y}}_{\tilde{\mathbf{h}}}^n, W_0, W_1, W_2\right) \le n\epsilon_n. \quad (58)$$

D. Equivocation analysis

It suffices to demonstrate the secrecy constraint when the rank of $\tilde{\mathbf{H}}$ equals N_E. If the rank is smaller than N_E the secrecy constraint clearly holds for this weaker eavesdropper. We shall only present the secrecy analysis for $N_E' \ge s$. The analysis for $N_E' < s$ is completely analogous. As discussed in section V-C, we consider a codebook that satisfies (58).

$$H\left(W_0, W_1, W_2|\tilde{\mathbf{Y}}_{\tilde{\mathbf{h}}}^n\right) \ge I\left(W_0, W_1, W_2; \bar{\mathbf{X}}^n|\tilde{\mathbf{Y}}_{\tilde{\mathbf{h}}}^n\right) \quad (59)$$
$$= H\left(\bar{\mathbf{X}}^n|\tilde{\mathbf{Y}}_{\tilde{\mathbf{h}}}^n\right) - H\left(\bar{\mathbf{X}}^n|\tilde{\mathbf{Y}}_{\tilde{\mathbf{h}}}^n, W_0, W_1, W_2\right) \quad (60)$$
$$= H\left(\bar{\mathbf{X}}^n|\tilde{\mathbf{Y}}_{\tilde{\mathbf{h}}}^n\right) - H\left(W_E|\tilde{\mathbf{Y}}_{\tilde{\mathbf{h}}}^n, W_0, W_1, W_2\right) \quad (61)$$
$$= H\left(\bar{\mathbf{X}}^n\right) - I\left(\bar{\mathbf{X}}^n; \tilde{\mathbf{Y}}_{\tilde{\mathbf{h}}}^n\right) - H\left(W_E|\tilde{\mathbf{Y}}_{\tilde{\mathbf{h}}}^n, W_0, W_1, W_2\right) \quad (62)$$

where (61) follows from the fact that $\bar{\mathbf{X}}^n$ is a deterministic function of (W_0, W_E, W_1, W_2).

On the other hand since $\mathrm{rank}(\tilde{\mathbf{h}}) = N_E$ it can be shown that:

$$w\left(\lim_{n\to\infty} \frac{1}{n} I\left(\bar{\mathbf{X}}^n; \tilde{\mathbf{Y}}_{\tilde{\mathbf{h}}}^n\right)\right) \le N_E. \quad (63)$$

For the first term in (62), since $H(\bar{\mathbf{X}}^n) = H(W_0, W_1, W_2, W_E)$ we have:

$$w\left(\lim_{n\to\infty} \frac{1}{n} H(\bar{\mathbf{X}}^n)\right) = \tilde{d}_0 + \tilde{d}_1 + \tilde{d}_2 + N_E \quad (64)$$

Applying (58), (63) and (64) to (59)-(62), we have that

$$w\left(\lim_{n\to\infty} \frac{1}{n} \sup_{\tilde{\mathbf{h}}} H(W_0, W_1, W_2|\mathbf{Y}_{\tilde{\mathbf{h}}}^n)\right) \ge \tilde{d}_0 + \tilde{d}_1 + \tilde{d}_2 \quad (65)$$

as required. This completes the proof of our coding theorem.

E. Converse

We now establish the upper bounds stated in (9) and (10). The two upper bounds in (9) are single-user bounds whereas the upper bound in (10) involves the sum-rate. These bounds correspond to the three cuts in the broadcast network [30]. For each cut, as discussed below we find the worst case eavesdropper channel.

Recall that the rank of $\mathbf{H}_1$ is r_1. To establish (9), we express (after row permutation if necessary) $\mathbf{H}_1$ as

$$\mathbf{H}_1 = \begin{bmatrix} \mathbf{H}_{11} \\ \mathbf{H}_{12} \end{bmatrix} \begin{matrix} \}N_E \text{ rows} \\ \}N_{R_1} - N_E \text{ rows} \end{matrix}. \quad (66)$$

such that the matrix $\mathbf{H}_{12}$ has a rank of $\{r_1 - N_E\}^+$. Since the eavesdropper channel state is arbitrary, we consider an

eavesdropper channel for which $\tilde{\mathbf{H}} = \mathbf{H}_{11}$ whose rank clearly equals N_E. For any coding scheme we can upper bound the rate as follows:

$$n(R_0 + R_1) = H(W_0, W_1) \tag{67}$$

$$\leq I(W_0, W_1; \mathbf{Y}_1^n) + n\epsilon_n \tag{68}$$

$$\leq I(W_0, W_1; \mathbf{Y}_1^n) - I(W_0, W_1; \tilde{\mathbf{Y}}^n) + 2n\epsilon_n \tag{69}$$

$$\leq I(W_0, W_1; \mathbf{Y}_1^n, \tilde{\mathbf{Y}}^n) - I(W_0, W_1; \tilde{\mathbf{Y}}^n) + 2n\epsilon_n \tag{70}$$

$$= I(W_0, W_1; \mathbf{Y}_1^n | \tilde{\mathbf{Y}}^n) + 2n\epsilon_n \tag{71}$$

$$\leq h(\mathbf{Y}_1^n | \tilde{\mathbf{Y}}^n) - h(\mathbf{Y}_1^n | \tilde{\mathbf{Y}}^n, W_0, W_1) + 2n\epsilon_n \tag{72}$$

$$\leq h(\mathbf{Y}_1^n | \tilde{\mathbf{Y}}^n) - h(\mathbf{Y}_1^n | \tilde{\mathbf{Y}}^n, W_0, W_1, \mathbf{X}^n) + 2n\epsilon_n \tag{73}$$

$$= I(W_0, W_1, \mathbf{X}^n; \mathbf{Y}_1^n | \tilde{\mathbf{Y}}^n) + 2n\epsilon_n \tag{74}$$

$$= I(\mathbf{X}^n; \mathbf{H}_1 \mathbf{X}^n + \mathbf{Z}_1^n | \mathbf{H}_{11} \mathbf{X}^n) + 2n\epsilon_n \tag{75}$$

$$\leq I(\mathbf{X}^n; \mathbf{H}_{12} \mathbf{X}^n + \mathbf{Z}_{12}^n) + 2n\epsilon_n \tag{76}$$

where $\mathbf{Z}_{12}$ is the last $N_{R_1} - N_E$ rows of $\mathbf{Z}_1$. The step (68) follows from Fano's inequality since (W_0, W_1) must be decodable by receiver 1 while (69) is a consequence of the equivocation constraint. Finally (75) follows from the Markov Condition that $(W_0, W_1) \rightarrow \mathbf{X}^n \rightarrow \mathbf{Y}_1^n$. Since the right hand side of (76) is upper bounded by the capacity of a MIMO Gaussian channel [32] and by our construction, the rank of $\mathbf{H}_{12}$ equals $(r_1 - N_E)$, it follows that

$$d_0 + d_1 \leq \{r_1 - N_E\}^+. \tag{77}$$

In a similar fashion, we can show that

$$d_0 + d_2 \leq \{r_2 - N_E\}^+. \tag{78}$$

To establish sum-rate bound on (W_0, W_1, W_2) we let the two users cooperate and obtain with $\mathbf{H}_\cup = \begin{bmatrix} \mathbf{H}_1 \\ \mathbf{H}_2 \end{bmatrix}$ and $\mathbf{Z} = \begin{bmatrix} \mathbf{Z}_1 \\ \mathbf{Z}_2 \end{bmatrix}$:

$$n(R_0 + R_1 + R_2) \leq I(\mathbf{X}^n; \mathbf{H}_\cup \mathbf{X}^n + \mathbf{Z}^n | \tilde{\mathbf{Y}}^n) + 2n\epsilon_n. \tag{79}$$

Since the rank of $\mathbf{H}_\cup$ equals $r_0 = r_1 + r_2 - s$, we can express (after suitable row permutations)

$$\mathbf{H}_\cup = \begin{bmatrix} \mathbf{A} \\ \mathbf{B} \end{bmatrix} \begin{matrix} \}N_E \text{ rows} \\ \}N_{R_1} + N_{R_2} - N_E \text{ rows} \end{matrix}. \tag{80}$$

where the rank of $\mathbf{B}$ equals $(r_0 - N_E)$. We further select $\tilde{\mathbf{H}} = \mathbf{A}$ of rank N_E. Using $\mathbf{Z}_A$ and $\mathbf{Z}_B$ to denote the projection of the noise vector $\mathbf{Z}$ onto the rows corresponding to matrices $\mathbf{A}$ and $\mathbf{B}$ respectively, the sum-rate can be upper bounded as follows:

$$n(R_0 + R_1 + R_2) \leq I(\mathbf{X}^n; \mathbf{H}_\cup \mathbf{X}^n + \mathbf{Z}^n | \tilde{\mathbf{Y}}^n) + 2n\epsilon_n \tag{81}$$

$$= I(\mathbf{X}^n; \mathbf{A}\mathbf{X}^n + \mathbf{Z}_A^n, \mathbf{B}\mathbf{X}^n + \mathbf{Z}_B^n | \mathbf{A}\mathbf{X}^n) + 2n\epsilon_n \tag{82}$$

$$\leq I(\mathbf{X}^n; \mathbf{B}\mathbf{X}^n + \mathbf{Z}_B^n) + 2n\epsilon_n \tag{83}$$

Since (83) is upper bounded by the capacity of a MIMO channel with channel matrix $\mathbf{B}$ of rank $(r_0 - N_E)$ it follows that

$$d_0 + d_1 + d_2 \leq \{r_0 - N_E\}^+. \tag{84}$$

This completes the proof of the converse.

VI. PROOF OF THEOREM 2

We outline how our results change when an additional mutual privacy constraints across the two receivers are imposed.

For the case when $d_0 + N_E \geq s$, we can in fact use the same construction as in Section V-B1. Since the message W_1 is transmitted only through symbols $\bar{\mathbf{X}}_A(i)$ which are not observed by user 2, its mutual privacy constraint is clearly satisfied. Similarly message W_2 is transmitted only through symbols $\bar{\mathbf{X}}_C(i)$ which are not observed by user 1. Hence its mutual privacy constraint is also satisfied. Clearly the addition of the mutual privacy constraint can only reduce the achievable d.o.f. and hence the proposed scheme achieves optimal d.o.f.

When $d_0 + N_E \leq s$ we use construction in section V-B2 with $N_E' = s$. Thus the codebook $\mathcal{C}_B$ uses all available $\bar{\mathbf{X}}_s$ symbols whereas the codebooks $\mathcal{C}_A$ and $\mathcal{C}_C$ use $\bar{\mathbf{X}}_{\tilde{r}_1}$ and $\bar{\mathbf{X}}_{\tilde{r}_2}$ symbols respectively. It can again be readily verified that any pair (d_0, d_1, d_2) that satisfies: $d_i \leq r_i - s$ and $d_0 \leq s - N_E$ is achievable.

For the converse we note that the constraint $d_0 + d_i \leq r_i - N_E$ follows from section V-E which does not use the mutual-privacy constraint. To establish the condition $d_i \leq r_i - s$, we assume that the messages (W_0, W_2) are revealed to the two receivers and only consider the transmission of message W_1. The resulting system reduces to a MIMO wiretap channel where the channel matrices $\mathbf{H}_1$ and $\mathbf{H}_2$ are known to the transmitter. As shown in [9] the high SNR capacity is achieved using the generalized singular value decomposition and the associated degree of freedom satisfies $d_1 \leq r_1 - s$. The upper bound $d_2 \leq r_2 - s$ can be established in an analogous manner. This completes the proof of the converse in Theorem 2.

VII. CONCLUSION

We study the achievable secrecy degrees of freedom region for a two-receiver MIMO broadcast wiretap channel where only the rank (or an upper bound on the rank) of the eavesdropper channel matrix is known to the legitimate terminals. While a direct extension of the single-user binning is sub-optimal, we show that the optimal degrees of freedom can be obtained by simultaneously diagonalizing the channel matrices of the legitimate receivers and carefully selecting the transmission on the resulting sub-channels in order to share a common fictitious message between the two receivers. We also extend the results to the case when an additional mutual-privacy constraint is imposed at the receivers.

While the present paper treats the case when the channels are static, extension to the case when the eavesdropper's

channel is time-varying, as considered in [15] in the single-user case, is left for future work. It will be also interesting to extend our result to the case of more than two receivers. This could perhaps require finding a suitable extension of the GSVD transform to more than two channel matrices. Another interesting future direction is to consider the case when the legitimate receiver's channel are also time-varying. One can also assume that the transmitter acquires CSI with a unit delay. A connection to the recent result in [24] appears natural and worth pursuing. Finally we note that this paper only considers the degrees of freedom of the broadcast network, which characterizes the pre-log of achievable rates. Such analysis is relevant if the network operates in the in the high SNR regime. The finer characterization of *constant-gap analysis* is left for future work .

REFERENCES

[1] C. E. Shannon, "Communication Theory of Secrecy Systems," *Bell System Technical Journal*, vol. 28, no. 4, pp. 656–715, September 1949.

[2] I. Csiszár and J. Körner, "Broadcast Channels with Confidential Messages," *IEEE Transactions on Information Theory*, vol. 24, no. 3, pp. 339–348, May 1978.

[3] E. Tekin and A. Yener, " The General Gaussian Multiple Access and Two-Way Wire-Tap Channels: Achievable Rates and Cooperative Jamming," *IEEE Transactions on Information Theory*, vol. 54, no. 6, pp. 2735–2751, June 2008.

[4] Y. Liang, G. Kramer, H. V. Poor, and S. Shamai, "Compound Wiretap Channels," *Eurasip Journal on Wireless Communication and Networking, Special issue in Wireless Physical Layer Security*, vol. 2009, Article ID 142374, 12 pages, 2009, doi:10.1155/2009/142374.

[5] E. Molavianjazi, "Secure Communication Over Arbitrarily Varying Wiretap Channels," *Master Thesis*, December 2009, available online at http://etd.nd.edu/ETD-db/theses/available/etd-12112009-112419/unrestricted/MolavianJaziE122009.pdf.

[6] L. Lai and H. El Gamal, "Cooperation for Secrecy: The Relay-Eavesdropper Channel," *IEEE Transactions on Information Theory*, vol. 54, no. 9, pp. 4005–4019, September 2008.

[7] E. Ekrem and S. Ulukus, "The Secrecy Capacity Region of the Gaussian MIMO Multi-receiver Wiretap Channel," *IEEE Transactions on Information Theory*, vol. 57, no. 4, pp. 2083–2114, April 2011.

[8] A. Khisti and G. Wornell, "Secure Transmission with Multiple Antennas-I: The MISOME Wiretap Channel," *IEEE Transactions on Information Theory*, vol. 56, no. 7, pp. 3088–3104, July 2010.

[9] ——, "Secure Transmission with Multiple Antennas-II: The MIMOME Wiretap Channel," *IEEE Transactions on Information Theory*, vol. 56, no. 11, pp. 5515–5532, November 2010.

[10] R. Liu, T. Liu, and H. V. Poor, "Multiple-input Multiple-output Gaussian Broadcast Channels with Confidential Messages," *IEEE Transactions on Information Theory*, vol. 56, no. 9, pp. 4215–4227, September 2010.

[11] M. Kobayashi, Y. Liang, S. Shamai, and M. Debbah, "On the Compound MIMO Broadcast Channels with Confidential Messages," in *IEEE International Symposium on Information Theory*, June 2009.

[12] A. Khisti and D. Zhang, "Artificial-Noise Alignment for Secure Multicast using Multiple Antennas," IEEE Communications Letters, To Appear

[13] J. Xie and S. Ulukus, "Secure degrees of freedom of the Gaussian wiretap channel with helpers and no eavesdropper CSI: Blind cooperative jamming," CISS, Mar. 2013

[14] J. Xie and S. Ulukus, "Unified Secure DoF Analysis of K-User Gaussian Interference Channels", ISIT, Istanbul, Turkey, July 2013.

[15] X. He and A. Yener, "MIMO Wiretap Channels with Arbitrarily Varying Eavesdropper Channel States," submitted to the IEEE Transactions on Information Theory, July, 2010, available online at http://arxiv.org/abs/1007.4801.

[16] X. He, A. Khisti and A. Yener, "MIMO Multiple Access Channel With an Arbitrarily Varying Eavesdropper: Secrecy Degrees of Freedom," IEEE Transactions on Information Theory, Vol. 59, No. 8, pp. 4733-4745, Aug. 2013

[17] X. He, A. Khisti, and A. Yener, "MIMO Broadcast Channel with Arbitrarily Varying Eavesdropper Channel: Secrecy Degrees of Freedom," in *IEEE Global Telecommunication Conference*, December 2011.

[18] N. Cai and R. W. Yeung, "Secure Network Coding," in *IEEE International Symposium on Information Theory*, June 2002.

[19] D. Silva and F. R. Kschischang, "Universal Secure Network Coding via Rank-Metric Codes," *IEEE Transactions on Information Theory*, vol. 57, no. 2, pp. 1124–1135, 2011.

[20] A. Khisti, D. Silva, and F. Kschischang, "Secure Broadcast Codes over linear demintic channels," in *IEEE International Symposium on Information Theory*, May 2010.

[21] M. Maddah-Ali, "On the degrees of freedom of the compound MISO broadcast channels with finite states," in ISIT, 2010, pp. 2273—2277

[22] H. Weingarten, S. Shamai, and G. Kramer, On the compound MIMO broadcast channel, in Proceedings of Annual Information Theory and Applications Workshop UCSD, Jan 2007.

[23] T. Gou, S. Jafar, and C.Wang, "On the degrees of freedom of finite state compound wireless networks - Settling a conjecture by Weingarten et al." 2009 [Online]. Available: http://arxiv.org/abs/0909.4177

[24] S. Yang, M. Kobayashi, P. Piantanida, and S. Shamai, "Secrecy Degrees of Freedom of MIMO Broadcast Channels with Delayed CSIT," submitted to IEEE Transactions on Information Theory, December, 2011, available online at http://arxiv.org/abs/1112.2306.

[25] E. Ekrem and S. Ulukus, "Degrees of Freedom Region of the Gaussian MIMO Broadcast Channel with Common and Private Messages," in *IEEE Global Telecommunication Conference*, December 2010.

[26] C. C. Paige and M. A. Saunders, "Towards a Generalized Singular Value Decomposition," *SIAM Journal on Numerical Analysis*, vol. 18, no. 3, pp. 398–405, 1981.

[27] S. Goel and R. Negi, "Guaranteeing Secrecy using Artificial Noise," *IEEE Transactions on Wireless Communications*, vol. 7, no. 6, pp. 2180–2189, June 2008.

[28] R. G. Gallager, *Information theory and reliable communication*. John Wiley & Sons, Inc. New York, NY, USA, 1968.

[29] T. M. Cover and J. A. Thomas, *Elements of Information Theory*. Wiley-Interscience New York, 1991.

[30] A. El Gamal and Y. H. Kim, *Network Information Theory*. Cambridge, U.K., Cambridge University Press, 2012

[31] Y. Sun, C. Emre Koksal and N. B. Shroff, "Capacity of Compound MIMO Channels with Additive Uncertainty," CoRR abs/1208.4656, 2012, http://arxiv.org/abs/1208.4656

[32] D. N. C. Tse and P. Viswanath, *Fundamentals of Wireless Communication*. Cambridge University Press, 2005

Xiang He (S'08, M'10) received B.S. and M.S. degrees in Electrical Engineering from Shanghai Jiao Tong University, Shanghai, China in 2003 and 2006 respectively. His master study is about high speed FPGA implementation of channel encoder, decoder and MIMO detectors. He received his Ph.D. degree in 2010 from the Department of Electrical Engineering at the Pennsylvania State University and joined Microsoft in that year. In 2010, he received Melvin P. Bloom Memorial Outstanding Doctoral Research Award from the Department of Electrical Engineering at the Pennsylvania State University and the best paper award from the Communication Theory Symposium in IEEE International Conference on Communications (ICC). In 2011, he was named as one of the exemplary reviewers by IEEE Communication Letters. His research interests include information theoretic secrecy, coding theory, queuing theory, optimization techniques, distributed detection and estimation.

Ashish Khisti (S'02, M'09) is an Assistant Professor in the Electrical and Computer Engineering (ECE) Department and a Canada Research Chair (Tier II) in Network Information Theory at the University of Toronto, Toronto, Ontario, Canada. He received his BASc degree in Engineering Sciences from University of Toronto in 2002 and his S.M. and Ph.D. degrees from the Massachusetts Institute of Technology (MIT), Cambridge, MA, USA in 2004 and 2008, respectively. He has been with the University of Toronto since 2009. His research interests span the areas of information theory, wireless physical layer security and streaming communication systems. During his graduate studies, Professor Khisti was a recipient of the NSERC postgraduate fellowship, Harold H. Hazen Teaching award and the Morris Joseph Levin Masterworks award. At the University of Toronto he is a recipient of the Ontario Early Researcher Award (2012) and a Hewlett-Packard IRP award (2011, 2012). He is an associate editor of the IEEE TRANSACTIONS ON COMMUNICATIONS.

Aylin Yener (S'91-M'00) received the B.Sc. degree in electrical and electronics engineering, and the B.Sc. degree in physics, from Bogazici University, Istanbul, Turkey; and the M.S. and Ph.D. degrees in electrical and computer engineering from Wireless Information Network Laboratory (WINLAB), Rutgers University, New Brunswick, NJ. Commencing fall 2000, for three semesters, she was a P.C. Rossin Assistant Professor at the Electrical Engineering and Computer Science Department, Lehigh University, PA. In 2002, she joined the faculty of The Pennsylvania State University, University Park, PA, where she was an Assistant Professor, then Associate Professor, and is currently Professor of Electrical Engineering since 2010. During the academic year 2008-2009, she was a Visiting Associate Professor with the Department of Electrical Engineering, Stanford University, CA. Her research interests are in information theory, communication theory and network science, with recent emphasis on green communications and information security. She received the NSF CAREER award in 2003.

Dr. Yener previously served as a technical program chair or co-chair for various conferences for the IEEE Communications Society, as an associate editor for the IEEE Transactions on Communications, as an associate editor and an editorial advisory board member for the IEEE Transactions on Wireless Communications. She served as the student committee chair for the IEEE Information Theory Society 2007-2011, and was the co-founder of the Annual School of Information Theory in North America co-organizing the school in 2008, 2009 and 2010. Dr. Yener currently serves on the board of governors of the IEEE Information Theory Society as its treasurer.