Robust Streaming Codes based on Deterministic Channel Approximations

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University of Toronto

Joint Work with Ahmed Badr (Toronto),
Wai-Tian Tan (HP Labs) and John Apostolopoulos (HP Labs)

ISIT, 2013
July 9th 2013

Motivation - Delay Sensitive Communication

Delay is a central issue in many applications[1]

Application	Bit-Rate	MSDU (B)	Delay (ms)	Delay (pkts)	PLR
Video Conf.	2 Mbps	1500	100 ms	24	10^{-4}
Interactive Gaming	1Mbps	512	50 ms	12	10^{-4}
SDTV	4Mbps	1500	200 ms	60	10^{-6}

Communication Medium: Wireless Channel.

[1] IEEE Usage Model Proposal (doc.: IEEE 802.11-03/802r23)

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Prior Work - Real Time Streaming Communication

- Structural Theorems on Real-Time Encoders (Witsenhausen '79, Teneketzis '06)
- Tree Codes (Schulman '96, Sahai '01, Sukhavasi and Hassibi '11)
- Real-Time Scheduling (Hou and Kumar '11, Shakkottai and Srikanth '11)
- Low-delay Path Selection (Chen et. al.)

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Real-Time Streaming Model

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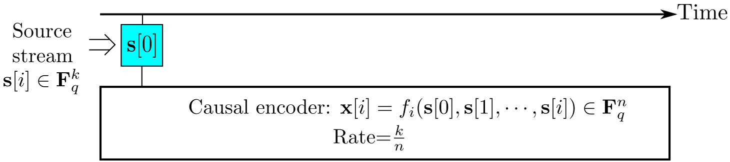

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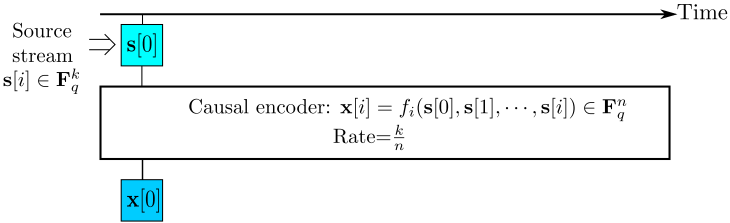

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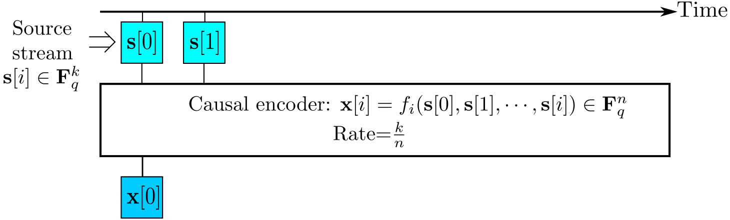

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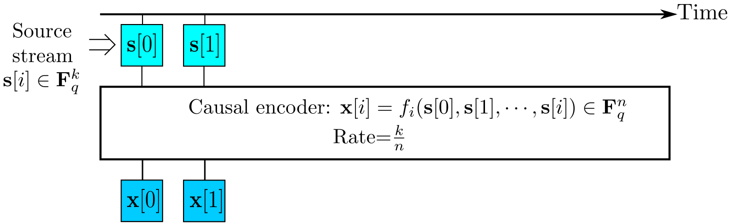

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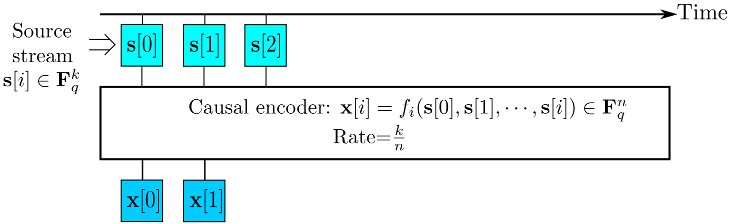

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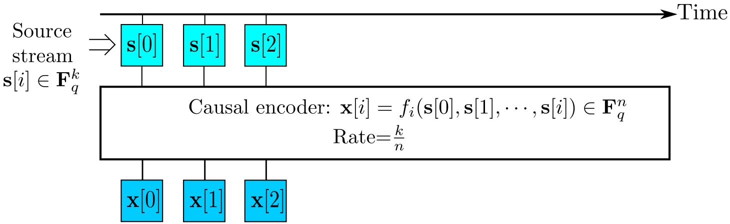

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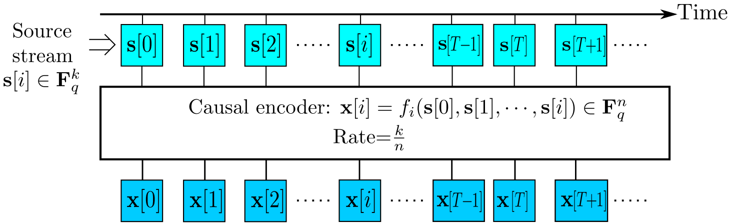

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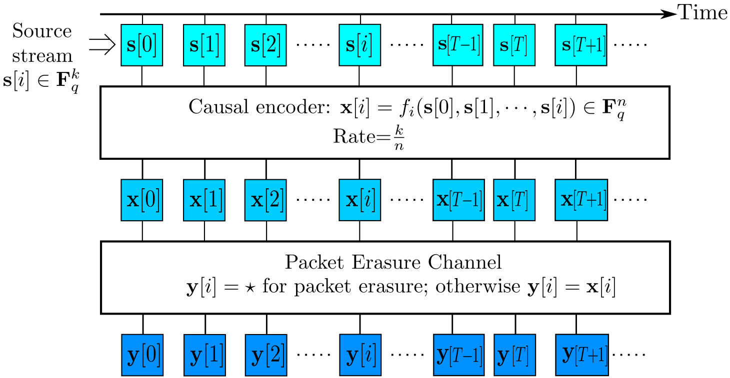

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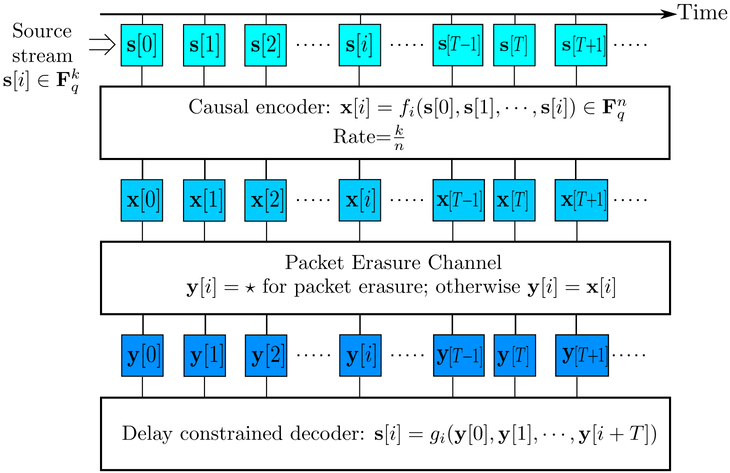

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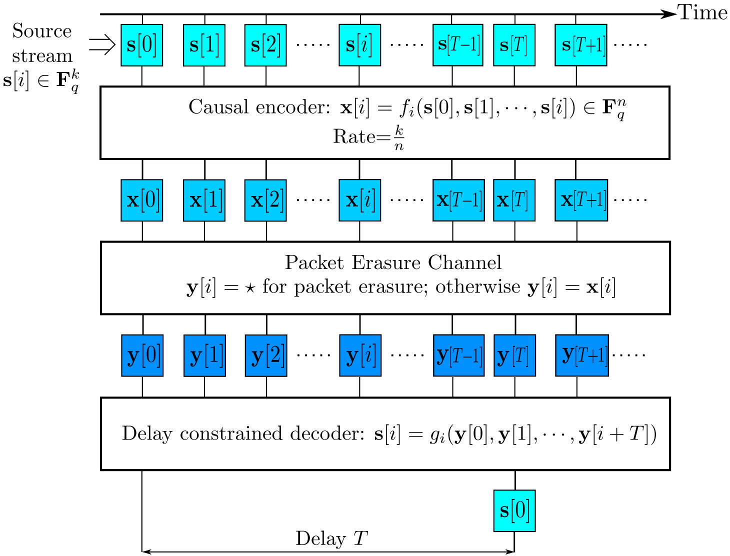

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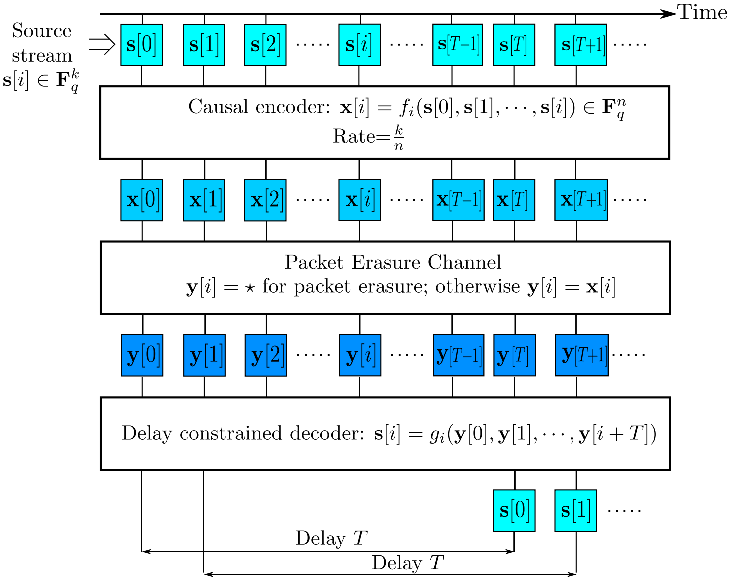

Problem Setup

- Assume $\mathbf{s}[t] \in \mathbb{F}_q^k$, i.i.d. uniform
- $\mathbf{x}[t] \in \mathbb{F}_q^n$. Causal Encoder.
- Rate: $R = \frac{k}{n}$

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  - A **burst** of maximum length B, or,
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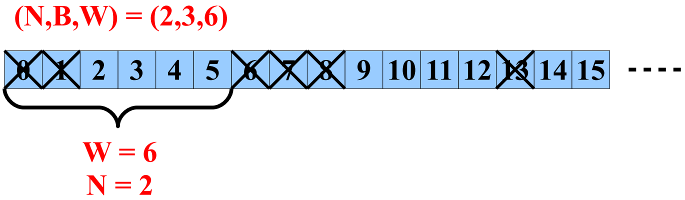

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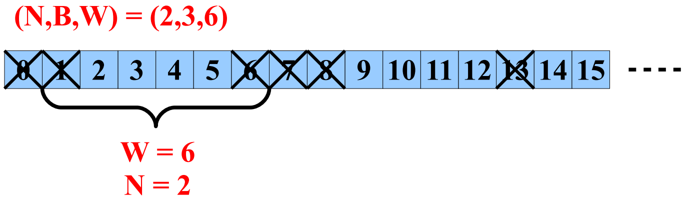

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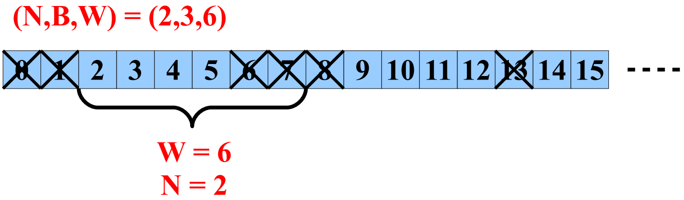

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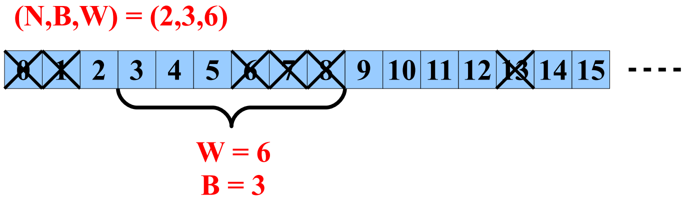

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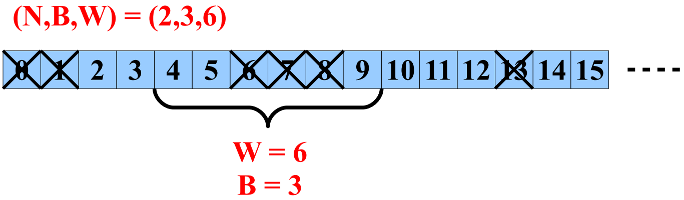

- Capacity $R(N, B, W, T)$

Main Result

Theorem

Consider the $\mathcal{C}(N, B, W)$ channel, with $W \geq B + 1$, and let the delay be T.

Upper-Bound*(Badr et al. INFOCOM'13) For any rate R code, we have:*

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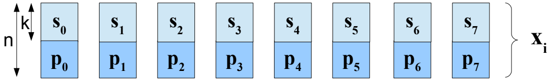
Lower-Bound: *There exists a rate R code that satisfies:*

$$\left(\frac{R}{1-R}\right) B + N \geq \min(W, T+1) - 1.$$

The gap between the upper and lower bound is 1 unit of delay.

Error Correction: Baseline Techniques



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Erasure Codes:

- Random Linear Codes
- Strongly-MDS Codes (Gabidulin'88, Gluesing-Luerssen'06)

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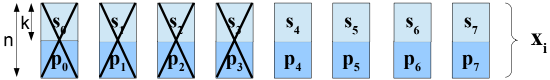

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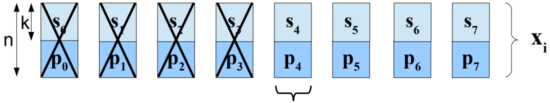

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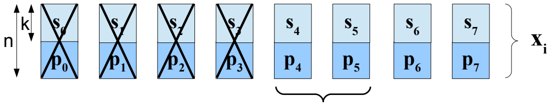

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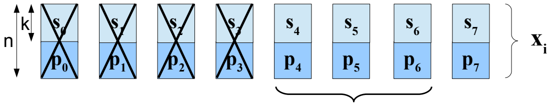

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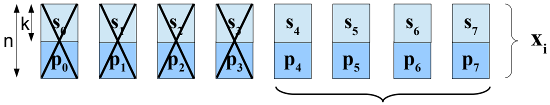

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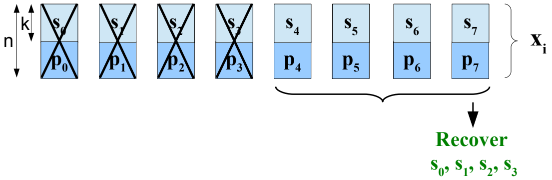

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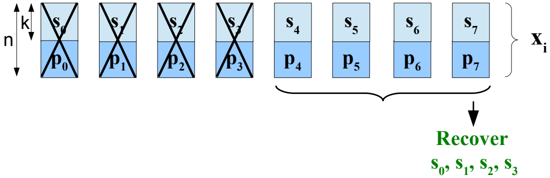

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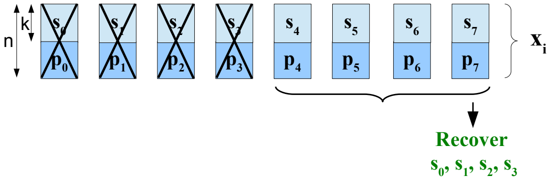

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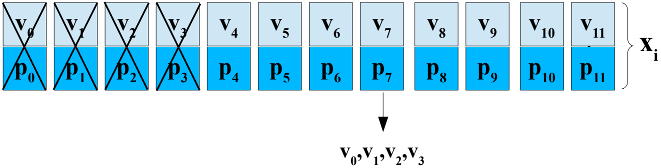
$$\begin{bmatrix} \mathbf{p}_4 \\ \mathbf{p}_5 \\ \mathbf{p}_6 \\ \mathbf{p}_7 \end{bmatrix} = \underbrace{\begin{bmatrix} \mathbf{H}_4 & \mathbf{H}_3 & \mathbf{H}_2 & \mathbf{H}_1 \\ \mathbf{H}_5 & \mathbf{H}_4 & \mathbf{H}_3 & \mathbf{H}_2 \\ 0 & \mathbf{H}_5 & \mathbf{H}_4 & \mathbf{H}_3 \\ 0 & 0 & \mathbf{H}_5 & \mathbf{H}_4 \end{bmatrix}}_{\text{full rank}} \begin{bmatrix} \mathbf{s}_0 \\ \mathbf{s}_1 \\ \mathbf{s}_2 \\ \mathbf{s}_3 \end{bmatrix}$$

Streaming Codes - Burst Erasure Channel

$N = 1$, $B = 4$, $T = 8$

Rate $1/2$ Baseline Erasure Codes, $T = 7$



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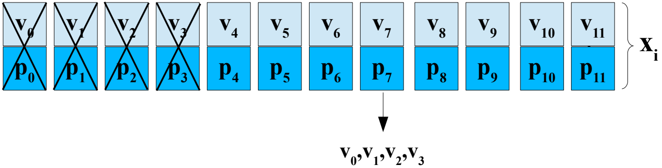
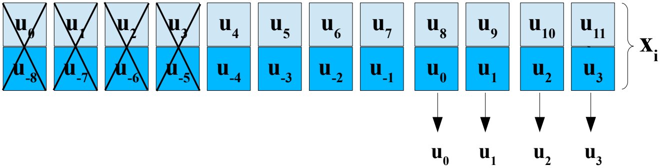


Rate $1/2$ Repetition Code, $T = 8$



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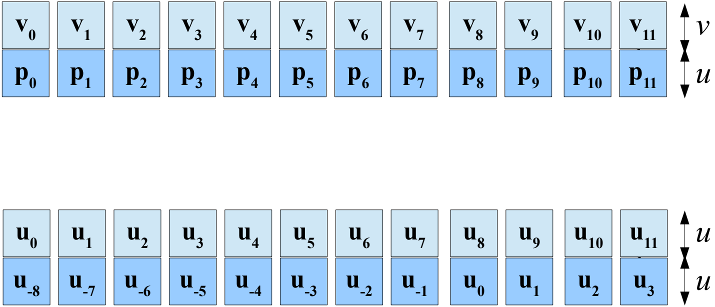

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$$R = \frac{u+v}{3u+v} = \frac{1}{2}$$

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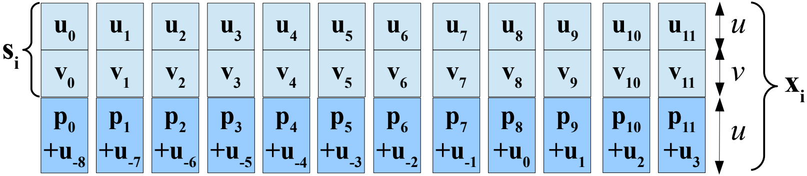

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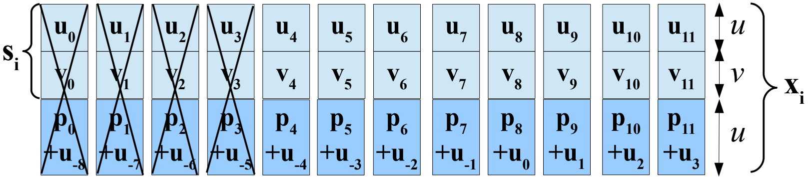

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Encoding:

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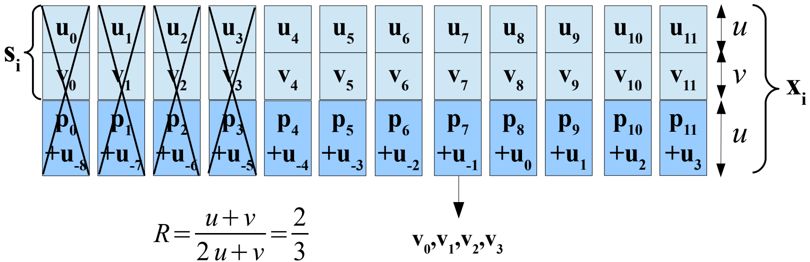

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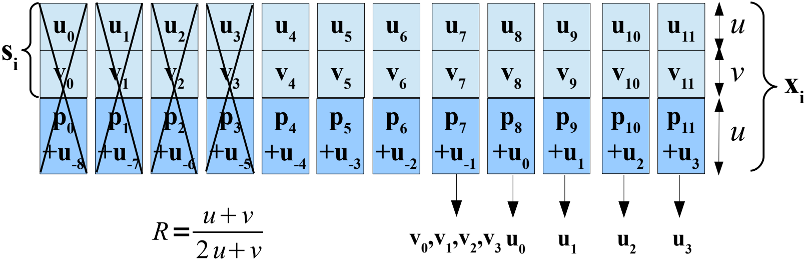

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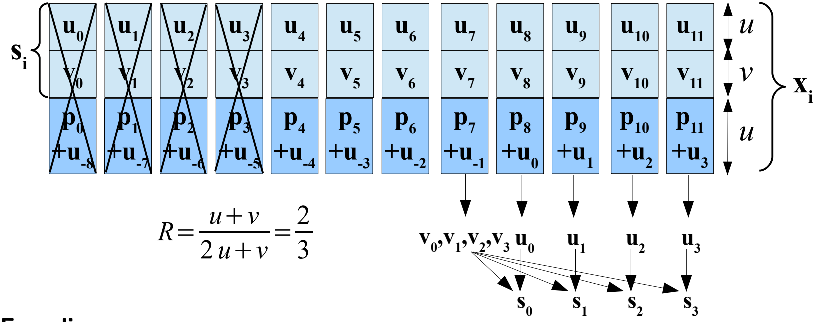

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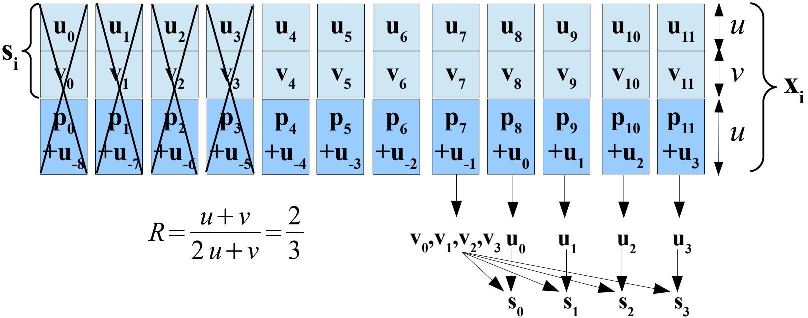

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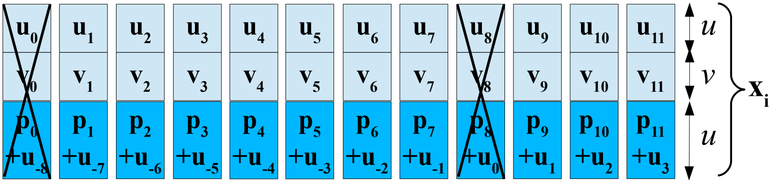
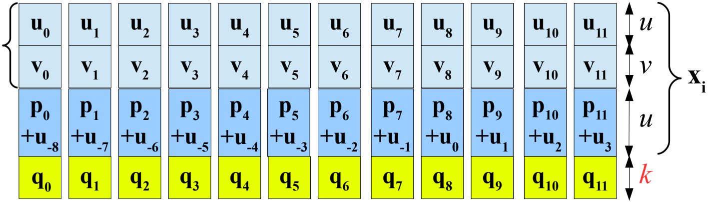
Isolated Erasures

$N \geq 2$

$T = 8$



- Erasures at time $t = 0$ and $t = 8$
- $\mathbf{u}_0$ cannot be recovered due to a repetition code



Layered Code Design

- Burst-Erasure Streaming Code $\mathcal{C}_1 : (\mathbf{u}_i, \mathbf{v}_i, \mathbf{p}_i + \mathbf{u}_{i-T})$
- Erasure Code: $\mathbf{q}_i = f_i(\mathbf{u}_0, \ldots, \mathbf{u}_{i-1}) \in \mathbb{F}_q^k$
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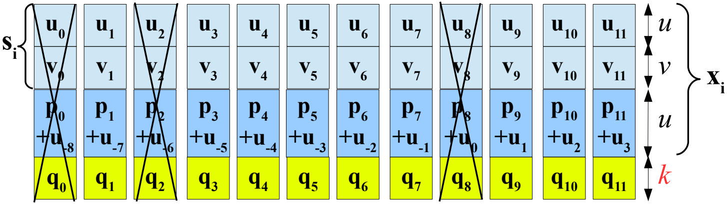
$$R = \frac{u+v}{2u+v+k}, \qquad k = \frac{N}{T-N+1}B$$



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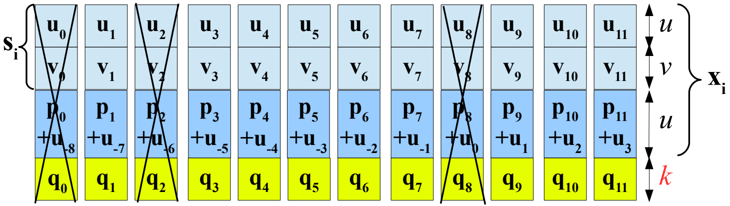
Proposed Approach: Layering

$N \geq 2$



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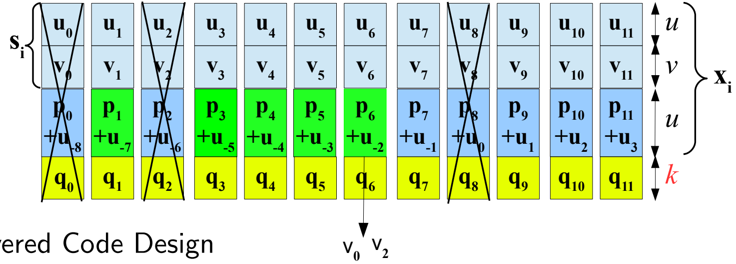

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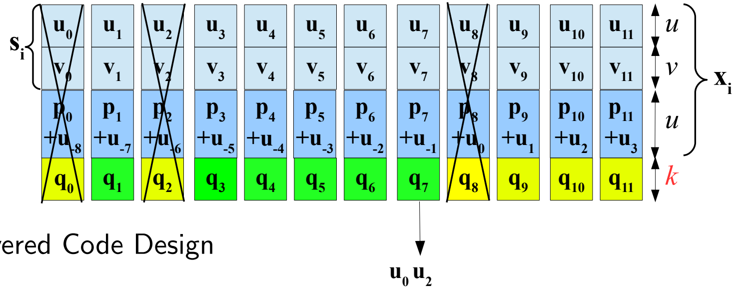

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Related Works

- Streaming Codes for Burst Erasure Channel (Block Code + Interleaving): Martinian and Sundberg (IT-2004), Martinian and Trott (ISIT-2007)

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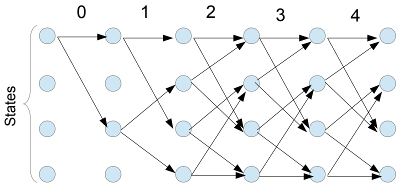
- Tree Codes: Schulman (IT 1996), Sahai (2001), Martinian and Wornell (Allerton 2004), Sukhavasi and Hassibi (2011)

Distance and Span Properties

MiDAS $\rightarrow$ (Near) Maximum Distance And Span tradeoff

Consider (n, k, m) Convolutional code: $\mathbf{x}_i = \sum_{j=0}^{m} \mathbf{s}_{i-j}\mathbf{G}_j$



Trellis Diagram

Distance and Span Properties

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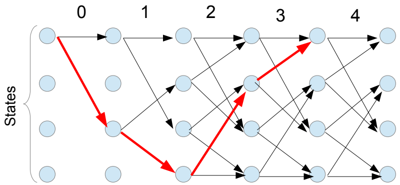

Trellis Diagram – Free Distance

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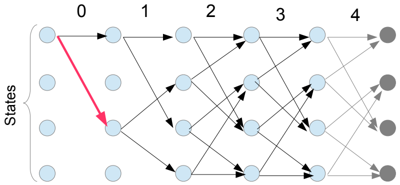

Column Distance in [0,3]

Column Distance: d_T

$$d_T = \min_{\substack{[\mathbf{s}_0,\ldots,\mathbf{s}_T] \\ \mathbf{s}_0 \neq 0}} \mathrm{wt}\left(\begin{bmatrix}\mathbf{s}_0 & \ldots & \mathbf{s}_T\end{bmatrix} \begin{bmatrix} \mathbf{G}_0 & \mathbf{G}_1 & \ldots & \mathbf{G}_T \\ 0 & \mathbf{G}_0 & \ldots & \mathbf{G}_{T-1} \\ \vdots & & \ddots & \vdots \\ 0 & & \ldots & \mathbf{G}_0 \end{bmatrix} \right)$$

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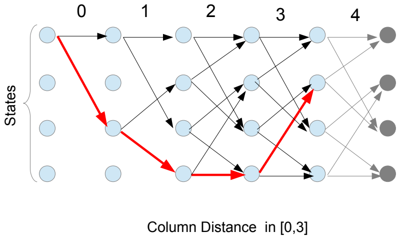

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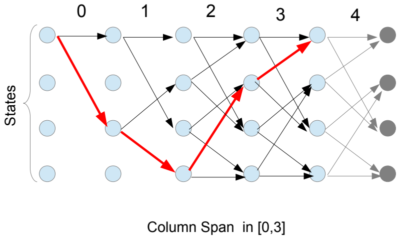

Column Span: c_T

$$c_T = \min_{\substack{[\mathbf{s}_0,\ldots,\mathbf{s}_T] \\ \mathbf{s}_0 \neq 0}} \text{span} \left(\begin{bmatrix} \mathbf{s}_0 & \ldots & \mathbf{s}_T \end{bmatrix} \begin{bmatrix} \mathbf{G}_0 & \mathbf{G}_1 & \ldots & \mathbf{G}_T \\ 0 & \mathbf{G}_0 & \ldots & \mathbf{G}_{T-1} \\ \vdots & & \ddots & \vdots \\ 0 & & \ldots & \mathbf{G}_0 \end{bmatrix} \right)$$

Column-Distance & Column Span Tradeoff

Theorem

Consider a $\mathcal{C}(N, B, W)$ channel with delay T and $W \geq T + 1$. A streaming code is feasible over this channel if and only if it satisfies: $d_T \geq N + 1$ and $c_T \geq B + 1$

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Theorem

For any rate R convolutional code and any $T \geq 0$ the Column-Distance d_T and Column-Span c_T satisfy the following:

$$\left(\frac{R}{1-R}\right) c_T + d_T \leq T + 1 + \frac{1}{1-R}$$

There exists a rate R code (MiDAS Code) over a sufficiently large field that satisfies:

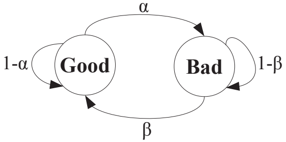
$$\left(\frac{R}{1-R}\right) c_T + d_T \geq T + \frac{1}{1-R}$$

Simulation Results

Gilbert-Eliott Channel $(\alpha, \beta) = (5 \times 10^{-4}, 0.5)$, $T = 12$ and $R = 12/23$

Gilbert Elliott Channel

- Good State: $\Pr(\text{loss}) = \varepsilon$
- Bad State: $\Pr(\text{loss}) = 1$



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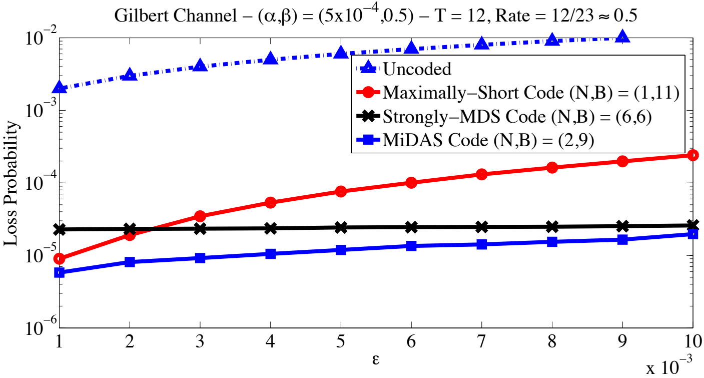
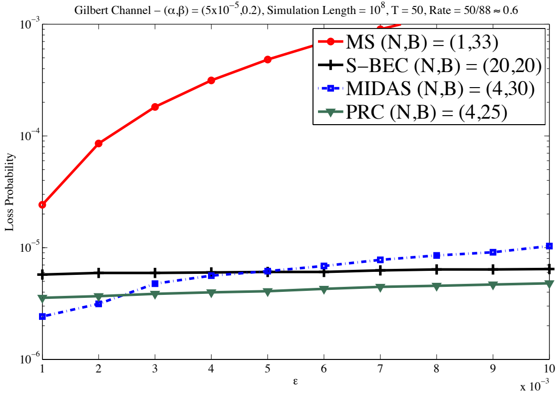


Code	N	B	Code	N	B
Strongly MDS	6	6	MiDAS	2	9
Burst-Erasure	1	11			

Simulation Results - II

Gilbert-Eliott Channel $(\alpha, \beta) = (5 \times 10^{-5}, 0.2)$, $T = 50$ and $R \approx 0.6$



Code	N	B	Code	N	B
Strongly MDS	20	20	MiDAS	4	30
Burst-Erasure	1	33	PRC	4	25

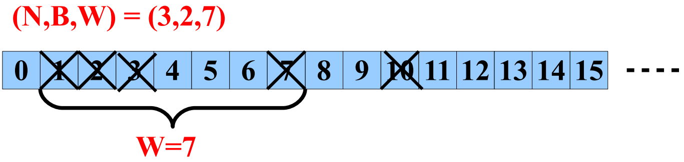
Conclusions

- Error Correction Codes for Real-Time Streaming
- Deterministic Channel Models $\mathcal{C}(N, B, W)$
- Tradeoff between achievable N and B
- MiDAS Constructions
- Column-Distance and Column-Span Tradeoff
- Partial Recovery Codes for Burst + Isolated Erasures

Burst plus Isolated Erasures

$\mathcal{C}_{II}(N, B, W)$ that in a window of length W introduces

- A burst erasure of length B *plus* one isolated erasure
- Upto N isolated erasures



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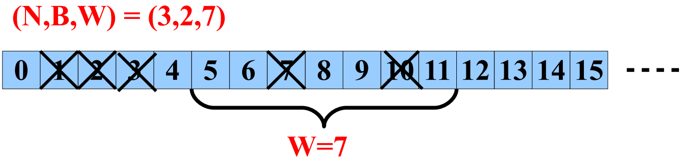

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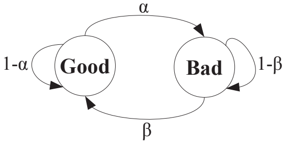

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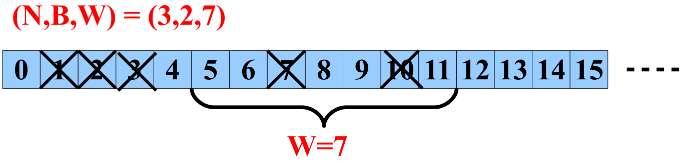

Partial Recovery Codes

- Layered Construction
- Partial Recovery for burst + isolated patterns