A scattered signal has the form

\[ x_n(t) = \sum_{i = 1}^{\infty} \alpha_i \cos(2\pi f_i t + \theta_i) \]

\( \alpha_i \) are random variables with a uniform distribution in [0, 2\( \pi \)].

\[ S_n(f) = \int_{-\infty}^{\infty} \frac{1}{2\pi} \left| \sum_{i = 1}^{\infty} \alpha_i e^{j2\pi f_i t} \right|^2 dt \]

\[ S_n(f) = \sum_{i = 1}^{\infty} \frac{\alpha_i^2}{2\pi} \delta(f - f_i) \]

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\[ S_n(f) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left( \sum_{i = 1}^{\infty} \alpha_i \alpha_j e^{j2\pi (f_i - f_j) t} \right) dt \]

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2. A transmitter transmits a sinusoidal signal with frequency $f_0$ to a receiver that is travelling through an environment that has a large number of scatterers distributed uniformly over an angle of 360 degrees. The receiver antenna is a two-element array with element spacing of $\frac{\lambda}{2}$ (where $\lambda$ is the wavelength) and direction of motion perpendicular to the array. The receiver has a velocity $v$ with respect to the transmitter and the receiving antenna has an amplitude gain that is proportional to $\cos(\theta)$ where $\theta$ is the angle of arrival of a signal component relative to the direction of motion.

\[
\begin{align*}
\frac{\lambda}{2} & \quad \theta \\
& \quad v
\end{align*}
\]

a) If the power of the received signal is $P$, find the power spectral density of the received signal.

b) Find the autocorrelation function of the complex envelope of the received signal.

c) Assuming that the coherence time for the channel is defined using a correlation value of 0.5 (absolute value of the correlation of the complex envelope) give the coherence time for the channel.

3) In a given environment a scattered signal is being received at two stationary antennas. The scattered signal has multiple components uniformly distributed over an angle of 180 degrees ($|\theta| \leq \frac{\pi}{2}$ in the following Fig.).

\[
\begin{align*}
A_1 & \quad \theta \\
& \quad d \\
& \quad A_2
\end{align*}
\]

plane wave

a) Determine the correlation of the complex envelope of the received signal at the two antennas as a function of the antenna spacing $d$. 

b) Determine the coherence distance for this antenna configuration based on a correlation factor of 0.5 (absolute value of correlation of complex envelope).

4) A given channel has a delay spread given by the function $p(\tau) = \frac{2}{T} \left( 1 - \frac{\tau}{T} \right)$.

a) Determine the spaced frequency correlation function for this channel (based on complex envelopes).

b) Based on a correlation value of 0.5 (absolute value of correlation of complex envelope) determine the coherence bandwidth of this channel.

5) A receiver travels through an environment with velocity $v$. The environment causes a delay spread given by $p(\tau)$ (in 4)) and causes multi-path components to arrive with an angle that is uniformly distributed over a range of 0-360 degrees. Assume that the center frequency of the received signal is $f_0$.

For each of the following four cases

i) $T = 0.1 \ \mu s, \ v = 10 \ Km/H$

ii) $T = 10 \ \mu s, \ v = 10 \ Km/H$

iii) $T = 0.1 \ \mu s, \ v = 100 \ Km/H$

iv) $T = 10 \ \mu s, \ v = 100 \ Km/H$

a) Plot the joint delay spread - Dopper spread for this channel for the following cases:

b) Plot the joint spaced-time spaced-frequency autocorrelation function for this channel. Show your plots as 3-D plots and contour plots.