Competition and Cooperation in Multiuser Communication Environments

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Introduction

- A multiuser communication environment is a competitive environment.

- What is the role of competition?
- What is the value of cooperation?
Multi-Antenna Wireless Communication

- Examples: broadcast wireless access (802.11), wireless local loop (WLL)

- Cooperation among multiple antennas within the same user
- Competition among the users
Digital Subscriber Lines (DSL), Ethernet

- DSL and Ethernet environments are interference-limited.

- Explore the benefit of cooperation.
- Manage the competition.
Goal

- To characterize channel capacity, optimum spectrum, and coding for

- Multiple Access

- Broadcast

- Interference

- assuming Gaussian noise.

- To illustrate the value of cooperation in these scenarios.
Gaussian Vector Channel

- Capacity: \( C = \max I(X; Y) \).

\[ W \in 2^{nC} \rightarrow X^n \rightarrow H \rightarrow Y^n \rightarrow \hat{W}(Y^n) \]

- Optimum Spectrum: Water-filling

\[
\begin{align*}
\text{maximize} & \quad \frac{1}{2} \log \frac{|HS_{xx}H^T + S_{zz}|}{|S_{zz}|} \\
\text{subject to} & \quad \text{tr}(S_{xx}) \leq P, \\
& \quad S_{xx} \geq 0.
\end{align*}
\]
Multiple Access Channel

- No transmitter coordination. Only receiver coordination.

- Capacity? Optimum Spectrum? Coding?
Capacity Region for Multiple Access Channel

\[
W_1 \in 2^{nR_1} \rightarrow X_1^n(W_1) \rightarrow H_1 \rightarrow Z^n \rightarrow Y^n \rightarrow \hat{W}_1(Y^n)
\]

\[
W_2 \in 2^{nR_2} \rightarrow X_2^n(W_2) \rightarrow H_2 \rightarrow Y^n \rightarrow \hat{W}_2(Y^n)
\]

- Capacity region:

\[
R_1 \leq I(X_1; Y|X_2);
\]

\[
R_2 \leq I(X_2; Y|X_1);
\]

\[
R_1 + R_2 \leq I(X_1, X_2; Y).
\]

- Ahlswede (’71), Liao (’72), Cover-Wyner (’73)
Coding for Multiple Access Channel

- Superposition coding and successive decoding achieves $I(X_1, X_2; Y)$:

\[
\begin{bmatrix}
  x_1 \\
  x_2
\end{bmatrix}
\xrightarrow{H}
\begin{array}{c}
  \text{Feedforward} \\
  \hat{x}_1 \\
  \hat{x}_2
\end{array}
\xleftarrow{\text{Feedback}}
\]

- Implementation: Generalized Decision-Feedback Equalizer (GDFE). Cioffi, Forney ('97), Varanasi, Guess ('97)
• Fix an input distribution $p(x_1)p(x_2)$, the capacity region is a pentagon.
Vector Multiple Access Capacity Region

\[
\max (R_1 + R_2) \iff \max I(X_1, X_2; Y) \text{ over all } p(x_1)p(x_2).
\]

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Uplink Power Control in Wireless Systems

- Successive decoding achieves capacity in multiple access channels.

- If channel state is known at the transmitter...
- What is the optimal power allocation?
Sum Capacity Maximization

\[ X_1 \sim \mathcal{N}(0, S_1) \rightarrow H_1 \]
\[ X_2 \sim \mathcal{N}(0, S_2) \rightarrow H_2 \]
\[ Z \sim \mathcal{N}(0, S_{zz}) \]

\[ Y \]

maximize

\[
\frac{1}{2} \log \left| \frac{H_1 S_1 H_1^T + H_2 S_2 H_2^T + S_{zz}}{|S_{zz}|} \right|
\]

subject to

\[ \text{tr}(S_i) \leq P_i, \quad i = 1, 2 \]
\[ S_i \geq 0, \quad i = 1, 2 \]

Maximizing a **concave** objective with **convex** constraints.
Competitive Optimum

- Optimum $S_1^*$ is a water-filling covariance against $S_2^*$.
- Optimum $S_2^*$ is a water-filling covariance against $S_1^*$.
- $(S_1^*, S_2^*)$ can be reached by each user iteratively water-filling against each other.

*Multiple Access Channel Sum Capacity = Competitive Optimum*
Theorem 1. The iterative water-filling process, where each user water-fills against the combined interference and noise, converges to the sum capacity of a Gaussian vector multiple access channel.

\[ S_1^{(0)} \rightarrow S_2^{(0)} \rightarrow S_1^{(1)} \rightarrow S_2^{(1)} \rightarrow \cdots \rightarrow (S_1^*, S_2^*) \]
Related Works

- Multiple access channel with ISI: Cheng and Verdu ('93).

- Multiple access fading channel:
  - Single-antenna: Knopp and Humblet ('95), Hanly and Tse, ('98).
  - Multi-antenna (asymptotic): Viswanath, Tse, Anantharam ('00)
  - Multi-path fading channels: Medard ('00)
  - CDMA channels: Viswanath and Anantharam ('99), Yates, Rose ('00)

- Iterative water-filling is a generalization for vector multi-access channels
Multi-user Diversity in Wireless Systems

- Solves the power control problem for multi-antenna fading channels:

  - Single receive antenna: one user should transmit at the same time.
  - Multiple receive antennas: multiple users transmit at the same time.
Results So Far

Vector Channel

\[ C = \max I(X; Y) \]

Water-filling

Vector Coding

Multiple Access

\[ C = \max I(X_1, X_2; Y) \]

Iterative Water-filling

GDFE

Broadcast

??

??

??

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Broadcast Channel

- Coordination at transmitter. No receiver coordination.

- Capacity? Optimum Spectrum? Coding?
Broadcast Channel Capacity

• Introduced by Cover ('72)
  – Superposition coding: Cover ('72).
  – Degraded broadcast channel: Bergman ('74), Gallager ('74)
  – Coding using binning: Marton ('79), El Gamal, van der Meulen ('81)
  – Sum and product channels: El Gamal ('80)
  – Gaussian vector channel, $2 \times 2$ case: Caire, Shamai ('00)

• General capacity region is a well-known open problem.
  – We focus on a non-degraded Gaussian vector broadcast channel.
  – Simultaneous and independent work was done by Vishwanath, Jindal, Goldsmith, and Viswanath, Tse.
Gaussian Vector Broadcast Channel

\[ X_1 \sim \mathcal{N}(0, S_1) \quad \text{and} \quad X_2 \sim \mathcal{N}(0, S_2) \]

\[ Z_1 \]
\[ Z_2 \]

\[ Y_1 \]
\[ Y_2 \]

\[ H_1 \]
\[ H_2 \]

- Superposition coding gives:

\[ R_1 = I(X_1; Y_1) = \frac{1}{2} \log \frac{|H_1S_1H_1^T + H_1S_2H_1^T + S_{z_1z_1}|}{|H_1S_2H_1^T + S_{z_1z_1}|} \]

\[ R_2 = I(X_2; Y_2) = \frac{1}{2} \log \frac{|H_2S_2H_2^T + H_2S_1H_2^T + S_{z_2z_2}|}{|H_2S_1H_2^T + S_{z_2z_2}|} \]
Gaussian Channel
\[ Z \sim \mathcal{N}(0, S_{zz}) \]

... with Transmitter Side Information
\[ S \sim \mathcal{N}(0, S_{ss}) \quad \text{and} \quad Z \sim \mathcal{N}(0, S_{zz}) \]

\[ C = \frac{1}{2} \log \frac{|S_{xx} + S_{zz}|}{|S_{zz}|} \]

\[ C = \frac{1}{2} \log \frac{|S_{xx} + S_{zz}|}{|S_{zz}|} \]

- Capacities are the same if \( S \) is known non-causally at the transmitter.
  - Based on Gel’fand and Pinsker (’80), Heegard and El Gamal (’83).
  - Generalizations: Cohen, Lapidoth (’01), Erez, Zamir, Shamai (’01).
New Achievable Region

$$W_1 \in 2^{nR_1} \rightarrow X_1^n(W_1, X_2^n) \rightarrow H_1 \rightarrow X^n \rightarrow Y_1^n \rightarrow \hat{W}_1(Y_1^n)$$

$$W_2 \in 2^{nR_2} \rightarrow X_2^n(W_2) \rightarrow H_2 \rightarrow X^n \rightarrow Y_2^n \rightarrow \hat{W}_2(Y_2^n)$$

$$R_1 = I(X_1; Y_1|X_2) = \frac{1}{2} \log \frac{|H_1 S_1 H_1^T + S_{z_1z_1}|}{|S_{z_1z_1}|}$$

$$R_2 = I(X_2; Y_2) = \frac{1}{2} \log \frac{|H_2 S_2 H_2^T + H_2 S_1 H_2^T + S_{z_2z_2}|}{|H_2 S_1 H_2^T + S_{z_2z_2}|}$$
Converse

- Broadcast capacity does not depend on noise correlation: Sato ('78).

\[
\begin{align*}
&\begin{array}{c}
\begin{array}{c}
\begin{array}{c}
\begin{array}{c}
 x_1 \hspace{1cm} z_1 \hspace{1cm} y_1
\end{array}
\begin{array}{c}
\begin{array}{c}
\begin{array}{c}
 x_2 \hspace{1cm} z_2 \hspace{1cm} y_2
\end{array}
\end{array}
\end{array}
\end{array}
\end{array}
= \begin{array}{c}
\begin{array}{c}
\begin{array}{c}
\begin{array}{c}
 x_1 \hspace{1cm} z_1' \hspace{1cm} y_1
\end{array}
\begin{array}{c}
\begin{array}{c}
\begin{array}{c}
 x_2 \hspace{1cm} z_2' \hspace{1cm} y_2
\end{array}
\end{array}
\end{array}
\end{array}
\end{array}
\end{array}
\end{array}
\end{align*}
\]

\[
\text{if } \begin{cases} 
 p(z_1) = p(z_1') \\
 p(z_2) = p(z_2') 
 \end{cases}, \text{ not necessarily } p(z_1, z_2) = p(z_1', z_2').
\]

- Thus, sum-capacity \( C \leq \min_{S_{zz}} \max_{S_{xx}} I(X; Y). \)
Least Favorable Noise

• Fix Gaussian input $S_{xx}$:

$$\begin{align*}
\text{minimize} & \quad \frac{1}{2} \log \frac{|HS_{xx}H^T + S_{zz}|}{|S_{zz}|} \\
\text{subject to} & \quad S_{zz} = \begin{bmatrix} S_{zz11} & * \\ * & S_{zz22} \end{bmatrix} \\
& \quad S_{zz} \geq 0
\end{align*}$$

• Minimizing a **convex** function over **convex** constraints.

• Optimality condition: $S_{zz}^{-1} - (HS_{xx}H^T + S_{zz})^{-1} = \begin{bmatrix} \Psi_1 & 0 \\ 0 & \Psi_2 \end{bmatrix}$.

  – if $S_{zz} > 0$ at minimum.
GDFE Revisited

- Least Favorable Noise $\iff$ Feedforward filter is diagonal!
- Decision-feedback may be moved to the transmitter by precoding.

$$R = \min_{S_{zz}} I(X; Y) \ (i.e. \ with \ least \ favorable \ noise) \ is \ achievable.$$
Gaussian Broadcast Channel Sum Capacity

- Achievability: \[ C \geq \max_{S_{xx}} \min_{S_{zz}} I(X; Y). \]

- Converse (Sato): \[ C \leq \min_{S_{zz}} \max_{S_{xx}} I(X; Y). \]

- (Diggavi, Cover '98): \[ \min_{S_{zz}} \max_{S_{xx}} I(X; Y) = \max_{S_{xx}} \min_{S_{zz}} I(X; Y). \]

**Theorem 2.** *Gaussian vector broadcast channel sum capacity is:*

\[
C = \max_{S_{xx}} \min_{S_{zz}} \frac{1}{2} \log \frac{|HS_{xx}H^T + S_{zz}|}{|S_{zz}|}
\]

*whenever \( S_{zz} > 0 \) at the saddle-point.*
Gaussian Mutual Information Game

$X \sim \mathcal{N}(0, S_{xx})$ → $H$ → $Y$

$Z \sim \mathcal{N}(0, S_{zz})$

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Objective</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Signal Player</strong></td>
<td>max $I(X; Y)$</td>
</tr>
<tr>
<td>${S_{xx} : \text{trace}(S_{xx}) \leq P}$</td>
<td></td>
</tr>
<tr>
<td><strong>Fictitious Noise Player</strong></td>
<td>min $I(X; Y)$</td>
</tr>
<tr>
<td>$\left{ S_{zz} : S_{zz} = \begin{bmatrix} S_{zz1} &amp; * \ * &amp; S_{zz2} \end{bmatrix} \geq 0 \right}$</td>
<td></td>
</tr>
</tbody>
</table>

Competitive equilibrium exists.
Saddle-Point is the Broadcast Capacity

- The optimum $S_{xx}^*$ is a water-filling covariance against $S_{zz}^*$.
- The optimum $S_{zz}^*$ is a least-favorable noise for $S_{xx}^*$.

Broadcast Channel Sum Capacity = Competitive Equilibrium
The Value of Cooperation

\[
\begin{align*}
\max_{S_{xx}} & I(X; X + Z) \\
\text{s.t.} & \quad \text{trace}(S_{xx}) \leq P
\end{align*}
\]

\[
\begin{align*}
\max_{S_{xx}} & I(X; X + Z) \\
\text{s.t.} & \quad \text{trace}(S_{xx}) \leq P
\end{align*}
\]

\[
\begin{align*}
\min_{S_{zz}} & \max_{S_{xx}} I(X; X + Z) \\
\text{s.t.} & \quad \text{trace}(S_{xx}) \leq P
\end{align*}
\]

\[
\begin{align*}
S_{xx} & = \begin{bmatrix} S_1 & 0 \\ 0 & S_2 \end{bmatrix}, \\
S_{zz} & = \begin{bmatrix} S_{z1z1} & * \\ * & S_{z2z2} \end{bmatrix}
\end{align*}
\]
Application: Multi-line Transmission in DSL

- With coordination, crosstalk can be “pre-subtracted”.
  - Practical pre-subtraction: Tomlinson precoding.
Performance: Vector DSL/Ethernet

- 20 VDSL lines
- Large improvement
  - for short loops
- Courtesy:
  - George Ginis
Results So Far

<table>
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<tr>
<th>Multiple Access</th>
<th>Broadcast</th>
<th>Interference</th>
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<td>$C = \max I(X_1, X_2; Y)$</td>
<td>$C = \max \min I(X; Y)$</td>
<td>??</td>
</tr>
<tr>
<td>Iterative Water-filling</td>
<td>Minimax</td>
<td>??</td>
</tr>
<tr>
<td>GDFE</td>
<td>GDFE Precoder</td>
<td>??</td>
</tr>
</tbody>
</table>

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Interference Channel

- No transmitter coordination. No receiver coordination.

- Capacity is a difficult open problem.
Near-far problem: The closer user emits too much interference.

- Power back-off is necessary.
- Current system imposes a maximum power-spectral-density limit.
Power Control Problem

- Find an optimum \((P_1(f), P_2(f))\) to maximize:

\[
R_1 = \int_0^W \log \left( 1 + \frac{|H_{11}(f)|^2 P_1(f)}{N_1(f) + |H_{21}(f)|^2 P_2(f)} \right) df,
\]

\[
R_2 = \int_0^W \log \left( 1 + \frac{|H_{22}(f)|^2 P_2(f)}{N_2(f) + |H_{12}(f)|^2 P_1(f)} \right) df.
\]

s.t. \(\int_0^W P_1(f) df \leq P_1\), \(\int_0^W P_2(f) df \leq P_2\)

- Finding the global optimum is computationally difficult.
• Each user maximizes its *own* data rate regarding other users as noise.
  – Non-cooperative game.
Iterative Water-filling

\[ P_1^{(0)}(f) \rightarrow P_2^{(0)}(f) \rightarrow P_1^{(1)}(f) \rightarrow P_2^{(1)}(f) \rightarrow \cdots \]

**Theorem 3.** Under a mild condition, the two-user Gaussian interference game has a competitive equilibrium. The equilibrium is unique, and it can be reached by iterative water-filling.
Distributed Power Control for DSL

\[ P_1 = P_1 - \Delta \]

Yes

\[ R_1 \geq \text{target?} \]

No

\[ P_1 = P_1 + \Delta \]

\[ P_2 = P_2 - \Delta \]

Yes

\[ R_2 \geq \text{target?} \]

No

\[ P_2 = P_2 + \Delta \]

Iterative

Water-filling

Water-filling

Control \((P_1(f), P_2(f))\) by setting \((P_1, P_2)\).
Performance

- 4 VDSL lines at 3000ft
  - rates: 6.7Mbps

- 4 VDSL lines at 1000ft
  - 10Mbps $\Rightarrow$ 21Mbps.

*Competitive optimal points are much better than existing methods.*
Conclusion

Multiple Access

Iterative water-filling achieves sum capacity.

Broadcast

Sum Capacity is a saddle-point of a mutual information game.

Interference

Competitive optimum is a desirable operating point.
Future Work

• Network Information Theory

• Multi-antenna/Multi-line Signal Processing

• Multiuser system design: physical layer vs network layer

• Applications to broadband access networks:
  – Wireless Local Area Networks
  – High-speed Ethernet
  – Digital Subscriber Lines
  – Computer Interconnects