EFFECTS OF MUTUAL COUPLING AND CHANNEL MISMATCH ON SPACE-TIME ADAPTIVE PROCESSING ALGORITHMS

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Abstract

The paper discusses the impact of mutual coupling on Space-Time Adaptive Processing (STAP) algorithms. To do so, simulations are used to illustrate the impact on the structure of the covariance matrix. Examples using measured data illustrate the effects of mutual coupling and channel mismatch.

INTRODUCTION

Space-Time Adaptive Processing (STAP) algorithms were originally developed mainly for proof-of-concept, assuming an idealized scenario. The main assumption is a linear array of equi-spaced, isotropic, point, sensors. Under this assumption, the array elements sample but do not re-radiate the incident fields. In addition, each element of the array is assumed to be exactly like every other element, i.e. the channels are perfectly matched. In the real world, each array element must have some physical size leading to mutual coupling between the elements. In addition, the channels are mis-matched due to manufacturing errors.

The effort of moving STAP from theory to practice has been significantly advanced by the availability of the Multi-Channel Airborne Radar Measurements (MCARM) database. The MCARM elements are reduced depth notch radiators. In applying STAP algorithms to measured data, researchers ignored the real world effects of mutual coupling and channel mismatch. However, electromagnetic analysis of antenna arrays shows that these effects severely degrade the performance of statistical [1] and direct data domain algorithms [2]. Reference [1] uses a simplistic analysis to illustrate the effects of mutual coupling. However the impact on the structure of the covariance matrix has not been studied. In the ideal case of a linear array of point sensors, the interference covariance matrix is Toeplitz (Toeplitz-block-Toeplitz in the space-time case). Furthermore, in the space-time case, the number of significant eigenvalues may be estimated a-priori. Both assumptions fail in the case of mutual coupling.

In applying STAP algorithms to measured data, another phenomenon, channel mismatch, must be taken into consideration. For example, under the ideal case the spatial steering vector forms a column of an appropriate DFT matrix. Certain algorithms therefore use a DFT to transform spatial data to the angle domain. In the real world, channel mismatch and mutual coupling affect the spatial steering vectors and a DFT is not optimal.

This paper uses both measured data and simulations to illustrate the effects of channel mismatch and mutual coupling on adaptive beamforming. For the simulations, the difference between the covariance matrices with and without mutual coupling is investigated. The simulations use a Method of Moments (MOM) analysis to evaluate the mutual coupling between the elements of a linear array of half-wavelength dipoles. For measured data, the algorithm investigated is the Joint Domain Localized (JDL) adaptive algorithm. Improved detection performance by accounting for channel mismatch and mutual coupling is illustrated.

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METHOD OF MOMENTS ANALYSIS OF MUTUAL COUPLING

In this work we use the MOM [3] to analyze the antenna and hence the mutual coupling. The MOM reduces to a matrix equation, the integral equation relating the incident field to the currents on the antenna. As we will show, the elements of the matrix quantize the mutual coupling between the antenna elements.

There are assumed to be Ne z-directed wires in the array, lying in the x - y plane. The array elements are of length L and radius a, with a \( \ll \) L. A linear polarized, narrowband, electric field, \( E^{inc} \), is incident on the array. Under the thin wire assumption, the current approximately flows only in the direction of the wire axes (here the z-direction), the surface current and charge densities on the wire can be approximated by line currents (I) and charge on the wire axes (they lie in the y = 0 plane) and the boundary condition can be applied to the axial component of \( E \) on the wire axes i.e. the boundary condition is applied to \( E_z \) on the wire axes. Using these assumptions, and the boundary condition that the total electric field on the axis of the wires must be identically zero, the integral equation that characterizes the behavior of the antenna array is

\[
E_z^{inc}(z) = j\omega \mu_0 \int_{axes} I(z') \frac{e^{-jkR}}{4\pi R} dz' - \frac{1}{j\omega \epsilon_0} \int_{axes} \frac{\partial I(z')}{\partial z'} \frac{e^{-jkR}}{4\pi R} dz', \quad \forall \in axes.
\]  

We solve this equation for the currents using the MOM. The basis functions used are the piecewise sinusoids as described by Strait et.al. [4]. The weighting functions are the same piecewise sinusoids i.e. a Galerkin formulation is used. This formulation is chosen because it yields analytic expressions for the elements of the matrix, hence eliminating the need for numerical integration. The resulting matrix equation can be written as

\[
V = ZI \Rightarrow I = YZ,
\]

where \( I \) is the MOM current vector with the coefficients of the expansion of the current in the above basis. \( Z \) is the MOM impedance matrix. \( Y \) is the MOM admittance matrix, the inverse of the impedance matrix. The matrices are of order \( N \times N \), where \( N \) is the number of unknowns used in the MOM formulation. The entries of \( V \) and \( Z \) are given by

\[
V_i = \frac{E_0 e^{j\theta} \cos \phi}{k \sin(k\Delta z) \sin^2 \theta} \left[ \cos(k\Delta z \cos \theta) - \cos(k\Delta z) \right],
\]

where \( \theta \) and \( \phi \) are the elevation and azimuth direction of arrival of the incident field.

\[
Z_{il} = \int_{s_{p-1,m}}^{s_{p+1,m}} I_{q,m}(z) \left\{ \int_{s_{p-1,m}}^{s_{p+1,m}} I_{q,n}(z') e^{-jkR} 4\pi R dz' - \frac{1}{j\omega \epsilon_0} \int_{s_{p-1,m}}^{s_{p+1,m}} \frac{\partial I_{q,n}(z')}{\partial z'} e^{-jkR} 4\pi R dz' \right\} dz,
\]

where \( i = [(m - 1)P + q] \), \( l = [(n - 1)P + p] \) and \( I_{q,m} \) is the \( q \)-th basis function on the \( m \)-th element.

Note that the entries of the voltage vector are directly related to the incident field and are hence free of the effects of mutual coupling. The entries of the impedance matrix are the interaction between the field due to the current source \( I_{p,n} \) at the location corresponding to the basis function \( I_{q,m} \). Therefore, by their very nature, the entries of the impedance matrix are a measure of the mutual coupling between the sections of the array.

Using the MOM admittance matrix and the voltage vector, we can show that the voltages measured at the ports of the array are given by

\[
V^{\text{meas}} = Z_L Y_{\text{port}} V
\]

where, \( Y_{\text{port}} \) is the \( Ne \times Ne \) matrix of the rows of \( Y \) that correspond to the ports of the \( Ne \) elements. \( Z_L \) is the \( Ne \times Ne \) diagonal matrix with the port loads as its entries.

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Simulated Data

In the case of an ideal array, the spatial interference covariance matrix is Toeplitz. In the space-time case, the space-time data vector can be arranged such that the covariance matrix is Toeplitz-Block-Toeplitz. However, this is true only under the ideal case. Under the ideal case each element of the array is independent of the others and the correlation between elements is only a function of the distance between them. However, in the true case,
each element in the array sees a different environment. The correlation between two elements is a function of the distance between them and also the location of the elements in the array. Solving for the adaptive weights using a Toeplitz solver will lead to incorrect solutions.

To illustrate this point, Table 1 presents the magnitude of the covariance matrix for the case with and without mutual coupling. The array is made up of 7 elements spaced λ/2 apart. Each element is of length λ/2 and radius λ/λ0. The clutter is 25dB above the noise floor. 28 secondary data vectors are used to estimate the covariance matrix. The covariance matrices are normalized to the maximum absolute value. The “Non-Toeplitzity” of the matrix is defined as mean squared summed error between the matrix and its Hermitian.

For an ideal array, the number of significant clutter eigenvalues is set by the number of pulses in a coherent pulse interval (CPI), number of elements and speed of the platform. In general [5],

\[ N_{\text{sig}} \approx N + \beta(M - 1), \]

where \( \beta \) is the number of half inter-element spacings covered in a single pulse period. In this example, the seven element array of Table 1 is used in conjunction with three pulses forming the CPI. The speed of the aircraft is set such that \( \beta = 1 \). The number of eigenvalues is therefore approximately 9. We use 84 secondary data vectors to estimate the 21 × 21 covariance matrix. The eigenvalue spread for the case without and with mutual coupling is given in Fig.1. As can be seen, the eigenvalue plot for the case without mutual coupling shows a sharp cut off at the 9th eigenvalue. For the case with mutual coupling, the cut off is significantly less.

**Measured Data**

In using STAP algorithms to detect weak signals in interference, the effects of mutual coupling and channel mismatch are usually ignored. However, as we shall see, both have significant effects on the performance of STAP algorithms. Here the effect is illustrated on the JDL algorithm which adaptively processes data within a Localized Processing Region (LPR) in angle-Doppler space.

The MCARM database is a collection of measured data taken over several flights with several acquisitions.
per flight. Also provided with the database is a set of measured steering vectors. These steering vectors account for the mutual coupling and channel mismatch and must be used to transform the space-time data to the angle-Doppler domain. In the ideal case, the magnitude of the steering vector at each element is constant. In the measured case, as shown in Fig. 2, the magnitude may fluctuate significantly.

In past applications of JDL to the MCARM database, the data is transformed to the angle-Doppler domain using the measured steering vectors, but the space-time steering vector is not. It is assumed that the steering vector follows the ideal case. Figure 3 illustrates the performance of the STAP algorithm for the case that the array is assumed to be ideal. The data is from acquisition 575 on flight 5. A weak target is injected in range bin 290. There are many false alarms and the target cannot be easily detected. Figure 4 illustrates the performance after accounting for the non-ideal array. As is seen, the false alarms are significantly reduced and the weak target can be detected.

CONCLUSIONS

This paper provides a brief review of the effects of mutual coupling and channel mismatch on the performance of STAP algorithms. Accounting for the non-ideal array can significantly improve STAP performance.

References


