

# Optimal Relay-Subset Selection and Time-Allocation in Decode-and-Forward Cooperative Networks

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**Abstract**—We consider a half-duplex mesh network wherein a single source communicates to a destination with the help of  $N$  potential decode-and-forward relays. We develop the optimal selection of a relaying subset and allocation of transmission time. This resource allocation is found by maximizing over the rates achievable for each possible subset of active relays; in turn, the optimal time allocation for each subset is obtained by solving a linear system of equations. An assumed relay numbering imposes a causality constraint. We also present a recursive algorithm to solve the optimization problem which reduces the computational load of finding the required matrix inverses and the number of required iterations. We show that (i) optimizing transmission time significantly improves achievable rate; (ii) optimizing over the channel resources ensures that more relays are active over a larger range of signal-to-noise ratios; (iii) linear network constellations significantly outperform grid constellations; (iv) the achievable rate is robust to node ordering.

## I. INTRODUCTION

COOPERATION has become a popular technique to complement spatial diversity in the absence of multiple antennas at receiving and transmitting nodes [1]–[3]. Of specific interest here, resource allocation in cooperative networks has recently become an active research area and has been investigated under many scenarios and metrics. In this paper, we address the problem of resource allocation, in terms of transmission-time in multiple-relay networks with arbitrary connections. We describe the contributions of the paper in detail after a brief review of the pertinent literature.

For the single-relay case, several works have dealt with various aspects of resource allocation, in terms of power and/or bandwidth and time. Yao et al. determine the optimal power and time allocation for relayed transmissions specifically in the low-power regime [4]. Larsson and Cao present various strategies for allocating power and channel resources under energy constraints [5]. For the channel resource allocation problem, however, the authors consider selection combining

only and do not address the scenario of joint decoding of the source and relay signals. The works in [6]–[8] address the problem of power and channel resource allocation under sum average power constraints. In [9], the authors obtain the optimal time and bandwidth allocation, with power control, using instantaneous and average channel conditions is obtained. Channel resource allocation under fixed power is developed in [10].

In networks with multiple relays, the available literature can be classified into two groups: parallel-relay networks where relays do not communicate with one another and arbitrarily-connected networks without this restriction on relay communication. For the former, resource allocation has been addressed in [11]–[14]. Ibrahim and Liang develop the optimal power allocation for a multi-relay cooperative orthogonal frequency division multiple access amplify-and-forward (AF) system [11]. By maximizing the channel mutual information, Anghel et al. find the optimal power allocation for multiple parallel relays in AF networks [12], [13]. A more general solution is given in [14] where the authors develop the optimal power and channel resource allocation for a parallel-relay network with individual node-power constraints.

To the best of our knowledge, our problem of channel resource allocation for arbitrarily-connected networks and dedicated multiple access has not been addressed in the literature. In general, works in the area of multi-relay systems with arbitrary links neglect the bandwidth penalty arising from multiple hops by assuming either full-duplex nodes, a bandwidth-unconstrained system, or the availability of channel phase information at the transmitter [15]–[27].

These assumptions, however, are not realistic for practical wireless networks, where nodes are likely to be half-duplex, phase information is very difficult to obtain at the transmitter, and bandwidth is a scarce resource. To fill this void, in this paper we investigate resource allocation in a bandwidth-constrained, cooperative, decode-and-forward (DF), wireless network. We consider the most general setting where multiple relays cooperate with the source and with each other to transmit information between source and destination. We address the joint problem of optimal selection of a relaying subset and allocation of time resources to the selected relays. The resource allocation is framed in the context of mesh networks, thereby removing power allocation from the optimization. Resource allocation is, therefore, in terms of transmission time only; allowing only orthogonal transmissions further

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simplifies the problem. This also reduces node-complexity, allowing nodes to implement resource allocation simply by switching on and off. Our solution provides an upper bound on performance in a mesh network with orthogonal signaling.

To the best of our knowledge, no other work solves for time-allocation in an arbitrarily-connected cooperative network. The solution can be interpreted as a generalization of the opportunistic protocol of [6], where the relay is active only when it increases the outage rate. The resource allocation solution is a generalization, to networks of arbitrary size, of the solution in [10] which considers a three-node DF network under fixed power. Our problem and solution can also be interpreted as a generalization of single-node selection [3], [28]–[30] with relaxed transmission constraints, where multiple relays may be selected, transmission can occur on multiple time-slots, uses independent codebooks and relays can communicate with one another. A key contribution here is achieving this efficiently.

This paper is structured as follows. Section II describes the system model. In Section III and IV, respectively, we develop the proposed resource allocation scheme and present a significantly simplified recursive implementation. Simulation results are presented in Section V before conclusions in Section VI.

## II. SYSTEM MODEL

The system under consideration is a static, half-duplex, mesh network comprising a source node, a destination node and  $N$  potential relays. The inter-node channel powers are denoted as  $|a_{ij}|^2$ , where  $i$  and  $j$  represent the source node  $s$ , relay nodes  $r_k$ ,  $k = 1 \dots N$ , or the destination node  $d$ . The channel powers are assumed independent of each other and are modeled as flat, slowly-fading and exponential with parameter  $\lambda_{ij}$ .  $\lambda_{ij}$  is inversely proportional to the average channel power and is a function of inter-node distance,  $d_{ij}$ , through the path loss exponent  $p_a$ , e.g.,  $1/\lambda_{ij} = (1/d_{ij}^{p_a})$ . The fading model does not include shadowing, although this can easily be incorporated on an instantaneous basis. Note that this model is for simulations only and the theory does not assume a specific model.

If transmitting, each node in the network transmits its own time slot, thereby eliminating interference. Our resource allocation scheme assumes knowledge of all channel powers (although *not* channel phases). This is justified by the fact that the nodes are static and the channels gains, if not phases, are assumed to vary extremely slowly with time. These channel gains must be transmitted to a central processing node; however, how this information is communicated and the impact of the associated overhead are beyond the scope of this paper.

The relays are assumed to be numbered in some predetermined order such that relay  $r_j$  transmits after  $r_i$  if  $j > i$ ; e.g., the relays may be in a linear constellation as shown in Fig. 1; however the simulation results will show that the performance is robust to node ordering. DF cooperation uses *independent codebooks*, which allows for the optimization of transmission time (see [31] for an overview of current DF coding methods). Note that repetition coding *does not* allow for this form of resource allocation.

With these assumptions, the cooperation framework for the  $N$ -relay fully-connected network is as follows: the half-duplex constraint precludes the relays from transmitting and receiving simultaneously on the same channel and the unavailability of forward-channel phase information at transmitting nodes precludes simultaneous transmissions. The transmission between the source and destination is thus divided into  $N+1$  time-slots, of normalized duration  $t_0, t_1, \dots, t_N$ , with  $t_0 + t_1 + \dots + t_N = 1$ . In the first time-slot, of duration  $t_0$ , the source transmits its information to all the nodes. The first relay,  $r_1$ , decodes this information and the remaining  $N-1$  relays and the destination store the information for future processing. In the second slot, of duration  $t_1$ , the first relay re-transmits the information using an independent codebook; the second relay decodes the information using the signals from the first relay and the source, and the remaining  $N-2$  relays and the destination store the information for further processing. In general, each relay  $r_k$  decodes information from the source and from the previous relays  $r_1 \dots r_{k-1}$  using the signals received up to and including time-slot  $t_{k-1}$ . This process continues until all relays have transmitted and the destination attempts to decode the information.

Assuming that each node uses power  $P$  and  $W$  Hz per transmission (noting that although each node transmits for a different length of time, the symbol durations and thus the corresponding bandwidth used by each node is the same), the signal-to-noise ratio (SNR) at node  $j$  resulting from transmission from node  $i$  can be written as  $\text{SNR}_{ij} = \frac{P}{N_0 W} |a_{ij}|^2$ , where  $N_0$  is the noise power spectral density. In the rest of the paper, we use the notation  $L_{ij}$  to denote  $\log_2(1 + \text{SNR}_{ij})$ , the capacity of the corresponding channel.

## III. OPTIMAL RESOURCE ALLOCATION AND RELAY SELECTION

In this section, we develop and solve the problem of joint resource allocation and relay selection for the network discussed above. Essentially, we obtain the optimum values of  $t_i$ ,  $i = 0 \dots N$ , such that the achievable rate between source and destination is maximized. We begin here with a fully-connected network, where each node is within the communication range of all other nodes.

### A. Fully Connected Network

For a fully connected network, assuming that each relay is active, the mutual information at each relay, and the destination, can be written as

$$I_1(t_0) = t_0 L_{sr_1}, \quad (1)$$

$$I_k(t_0, t_1, t_2, \dots, t_{k-1}) = t_0 L_{sr_k} + \dots + t_{k-1} L_{r_{k-1}r_k}, \quad (2)$$

$$I_D(t_0, \dots, t_k, \dots, t_{N-1}, t_N) = t_0 L_{sd} + \dots + t_N L_{r_N d}, \quad (3)$$

where  $I_k$  and  $I_D$  denote the mutual information at relay  $r_k$  and the destination, respectively.

With all  $N$  relays cooperating, the achievable rate under orthogonal transmissions is the minimum of the mutual information obtained at each relay node. The maximum of these

achievable rates is:

$$R_N = \max_{t_0, \dots, t_N} \min \{ I_1(t_0), I_2(t_0, t_1), \dots, I_k(t_0, \dots, t_{k-1}), \dots, I_N(t_0, \dots, t_{N-1}), I_D(t_0, \dots, t_N) \}, \quad (4)$$

such that  $t_i \geq 0, \forall i,$   
 $t_0 + t_1 + \dots + t_N \leq 1.$

The above expression is a straightforward generalization of the cut-set bound for the single-relay network. This generalization maintains orthogonal transmissions for each relay, a model which represents practical networks with simple nodes that cannot implement complex interference cancellation. We use this model as the basis of the optimization in the rest of this paper. We note, however, that because each relay transmits using an orthogonal channel,  $R_N$  is clearly not the channel capacity. For literature on the channel capacity of arbitrarily-connected networks, we direct the reader to [32]–[37] for full-duplex relays, and [38] for half-duplex relays.

For reasons that will soon be clear, consider the case with relay  $r_k$  removed from the network. The maximum achievable rate  $R_{N-1}^k$  becomes

$$R_{N-1}^k = \max_{t_0, \dots, t_{k-1}, t_{k+1}, \dots, t_N} \min \{ I_1(t_0), \dots, I_{k-1}(t_0, \dots, t_{k-2}), I_{k+1}(t_0, \dots, t_{k-1}), \dots, I_D(t_0, \dots, t_{k-1}, t_{k+1}, \dots, t_N) \}, \quad (5)$$

such that  $t_i \geq 0, \forall i,$   
 $t_0 + \dots + t_{k-1} + t_{k+1} + \dots + t_N \leq 1.$

Removing relay  $r_k$  is thus equivalent to removing  $t_k$  and  $I_k$  from the optimization. [The subscript in  $R_{N-1}^k$  denotes the maximum number of *potentially* active relays and the superscript denotes the relay(s) removed]. The maximum rate at which the source can transmit to the destination can thus be written as the maximum of the rate obtained by using all  $N$  relays and the rate obtained by successively removing each relay:

$$R_T = \max \{ R_N, R_{N-1}^1, R_{N-1}^2, \dots, R_{N-1}^N \}. \quad (6)$$

If  $R_T = R_{N-1}^k$ , the maximum rate can be obtained by iterating through (4) and (5), successively removing a relay each step. Note that obtaining  $R_{N-1}^k$  includes the cases where two or more relays are removed. In theory, therefore, all  $2^N$  possible cases must be checked. Since, even with reasonably low choices of  $N$ , the associated computation load would be impossible, in Section IV we develop an implementation with reduced complexity.

Let  $\mathbf{t}^* = (t_0^*, t_1^*, \dots, t_N^*)^T$  denote the resource allocation that solves the optimization problem. We begin an outline of the solution to the optimization problem in (4), (5) and (6) with the following proposition.

**Proposition 1:** With a maximum number of potential relays  $N$ , the maximum achievable rate  $R_T = R_N$  only if  $t_k^* \neq 0, \forall k$ . Otherwise, if  $t_k^* = 0, R_T = R_{N-1}^k$ .

**Proof:** With exactly  $N$  active relays, and with  $k < n < N$ , the resulting rate can be written explicitly as:

$$R_N = \max_{t_0, \dots, t_N} \min \{ (t_0 L_{sr_1}), (t_0 L_{sr_2} + t_1 L_{r_1 r_2}), \dots,$$

$$(t_0 L_{sr_k} + \sum_{i=1}^{k-1} t_i L_{r_i r_k}), (t_0 L_{sr_n} + \sum_{i=1}^{n-1} t_i L_{r_i r_n}), \dots, (t_0 L_{sd} + \sum_{i=1}^N t_i L_{r_i d}) \}. \quad (7)$$

Setting  $t_k = 0$  gives (8)–(11) on next page, since (10) has one fewer term in the minimization than (9). ■

To solve the optimization problem of (4) we thus require only the critical points for which  $t_k^* \neq 0, \forall k$ . In the following proposition, we show that for each  $R_N$ , i.e., given a set of potential relays, only one solution satisfies  $t_k^* \neq 0, \forall k$ .

**Proposition 2:** The unique solution to the minimization problem in (4) for which  $t_k^* \neq 0, \forall k$  is given by  $I_1(t_0) = I_2(t_0, t_1) = \dots = I_N(t_0, \dots, t_{N-1}) = I_D(t_0, t_1, \dots, t_N)$ .

**Proof:** We consider all possible critical points obtained from the optimization in (4). The points are obtained either by maximizing each individual term in (4) or at the intersection of all possible combinations of the terms in (4). We show that the only solution leading to non-zero solutions is at the intersection of all terms is in (4).

The critical points for the optimization problem can be obtained by solving the following:

- 1) Maximize the individual terms in (4) except  $I_D(t_0, \dots, t_N)$ :

$$\forall k \leq N, \max_{t_0, \dots, t_{k-1}} I_k(t_0, \dots, t_{k-1})$$

s.t.  $t_0 + \dots + t_{k-1} \leq 1. \quad (12)$

Because the optimization is not over  $t_m, \forall k \leq m \leq N$ , the solution to this problem clearly has all  $t_m = 0, \forall k \leq m \leq N$ , and thus cannot be a solution to the overall optimization problem.

- 2) Maximize  $I_D(t_0, \dots, t_N)$ :

$$\max_{t_0, \dots, t_N} I_D(t_0, \dots, t_N)$$

$$= \max_{t_0, \dots, t_N} \{ t_0 L_{sd} + \dots + t_N L_{r_N r_d} \},$$

s.t.  $t_0 + \dots + t_N \leq 1. \quad (13)$

In this case, all variables are included in the optimization. It is easy to show, however, that this function is maximized by selecting the largest  $L$  value, i.e., evaluating the Kuhn-Tucker conditions leads to a solution of the form  $t_m = 1, t_k = 0, \forall k \neq m$ , where  $m = \arg \max_k \{ L_{sd}, L_{r_1 d}, \dots, L_{r_k d}, \dots, L_{r_N d} \}$ . Therefore, this solution is also not a solution to the overall optimization problem.

- 3) Maximize the function that results from the intersection of all possible combinations of the functions  $I_k$ . Let  $\mathcal{M}$  denote all possible subsets of  $\{1 \dots N\}$ .  $\mathcal{M}$  then contains  $2^N$  such subsets, i.e.,  $|\mathcal{M}| = 2^N$ . Consider one such subset  $\delta_k = (m_1, m_2, \dots, m_k)$ , with  $m_1 < m_2 < m_k$ . One critical point then is

$$\max_{t_0, \dots, t_{m_k-1}} I_{m_k}(t_0, \dots, t_{m_k-1}) \quad (14)$$

such that

$$I_{m_1}(t_0, \dots, t_{m_1-1}) = \dots = I_{m_k}(t_0, \dots, t_{m_k-1}). \quad (15)$$

This optimization then gets repeated for all sets  $\delta_k \in$

$$\begin{aligned}
R_N &= \max_{t_0, \dots, t_{k-1}, t_{k+1}, \dots, t_N} \min \\
&\{ (t_0 L_{sr_1}), (t_0 L_{sr_2} + t_1 L_{r_1 r_2}), \dots, (t_0 L_{sr_k} + \sum_{i=1}^{k-1} t_i L_{r_i r_k}), \\
&\left( t_0 L_{sr_n} + \sum_{i=1}^{k-1} t_i L_{r_i r_k} \right) + \sum_{i=k+1}^{n-1} t_i L_{r_i r_n}, \dots, \\
&\left( t_0 L_{sd} + \sum_{i=1}^{k-1} t_i L_{r_i d} \right) + \sum_{i=k+1}^N t_i L_{r_i d} \} \\
&\leq \max_{t_0, \dots, t_{k-1}, t_{k+1}, \dots, t_N} \min \{ (t_0 L_{sr_1}), (t_0 L_{sr_2} + t_1 L_{r_1 r_2}), \dots, \\
&(t_0 L_{sr_n} + \dots + t_{k-1} L_{r_{k-1} r_n} + t_{k+1} L_{r_{k+1} r_n} \dots t_{n-1} L_{r_{n-1} r_n}), \dots, \\
&(t_0 L_{sd} + \dots + t_{k-1} L_{r_{k-1} r_d} + t_{k+1} L_{r_{k+1} r_d} + \dots t_N L_{r_N r_d}) \} \\
&= R_{N-1}^{k-1},
\end{aligned} \tag{8}$$

$$\tag{9}$$

$$\tag{10}$$

$$\tag{11}$$

$\mathcal{M}$ . In all but one combination, this optimization is not over all the variables  $\{t_0, \dots, t_N\}$ . As in point (1), this maximization also leads to  $t_k = 0$  for some value of  $k$ .

4) Maximize the intersection of all terms in (4):

$$\begin{aligned}
I_1(t_0) = I_2(t_0, t_1) \dots &= I_N(t_0, \dots, t_{N-1}) \\
&= I_D(t_0, \dots, t_N). \tag{16}
\end{aligned}$$

This is the only case that leads to  $t_k \neq 0, \forall k = 0 \dots N$ .

Essentially, this proposition shows that if all  $N$  relays are to contribute, all terms in the minimization in (4) must be equal. This proposition applies to any value of  $N$ . Therefore, if the optimal solution has  $k < N$  relays, an expression like that in (4) can be written for those  $k$  relays.

### B. Optimal Solution

The linear system of equations in (16) has a simple solution. Setting each equation to a constant, solving for the vector of unknowns  $\mathbf{t} = [t_0, t_1 \dots t_N]^T$  and normalizing, we obtain

$$\begin{aligned}
\mathbf{L}_{N+1} \mathbf{t}_{N+1} &= \mathbf{1}_{N+1}, \\
\Rightarrow \mathbf{t}_{N+1} &= \frac{\mathbf{L}_{N+1}^{-1} \mathbf{1}_{N+1}}{\|\mathbf{L}_{N+1}^{-1} \mathbf{1}_{N+1}\|_1} = \frac{\mathbf{L}_{N+1}^{-1} \mathbf{1}_{N+1}}{\mathbf{1}_{N+1}^T \mathbf{L}_{N+1}^{-1} \mathbf{1}_{N+1}}, \tag{17}
\end{aligned}$$

where  $\|\mathbf{v}\|_1$  denotes the sum of the elements of  $\mathbf{v}$ , i.e., the 1-norm,  $\mathbf{1}_{N+1}$  is the length- $(N+1)$  vector of ones and  $\mathbf{L}_{N+1}$  is the  $(N+1) \times (N+1)$  rate matrix

$$\mathbf{L}_{N+1} = \begin{bmatrix} L_{sr_1} & 0 & 0 & \dots & 0 \\ L_{sr_2} & L_{r_1 r_2} & 0 & \dots & 0 \\ L_{sr_3} & L_{r_1 r_3} & L_{r_2 r_3} & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & 0 \\ L_{sd} & L_{r_1 d} & L_{r_2 d} & \dots & L_{r_N d} \end{bmatrix}. \tag{18}$$

The solution in (17) does not guarantee that the constraint  $t_k > 0 \forall k = 0 \dots N$  is satisfied. To ensure that only solutions for which this constraint is satisfied are considered, we again consider the set  $\mathcal{M}$ . Each entry in the set corresponds to a rate matrix,  $\mathbf{L}_m$ , similar to that in (18), formed using the relays in

that entry of the set. Furthermore, let  $|m|$  denote the size of the rate matrix  $\mathbf{L}_m$ . A relay set and its corresponding solution, denoted as  $\mathbf{t}_m$ , is included as a potential solution if  $\mathbf{t}_m$  satisfies the constraint

$$\mathbf{t}_m > \mathbf{0}_{|m|}, \tag{19}$$

where  $\mathbf{0}_{|m|}$  is the all-zero vector of size  $|m|$  and the inequality operates on an element-by-element basis. Let the set  $\mathcal{K}$  form the subset of  $\mathcal{M}$  that comprises all potential solutions. Let  $\mathbf{L}_k$ ,  $\mathbf{t}_k^*$  and  $|k|$  denote the rate matrix, its corresponding solution and size, respectively, for each entry of the set  $\mathcal{K}$ . Note that the number of active relays being considered in each entry is  $|k| - 1$ . Finally, the optimum solution,  $\mathbf{t}^*$ , can be obtained by solving (17) for all possible combinations of active relays in the set  $\mathcal{K}$  i.e.,

$$\mathbf{t}^* = \max_k \frac{\mathbf{L}_k^{-1} \mathbf{1}_{|k|}}{\mathbf{1}_{|k|}^T \mathbf{L}_k^{-1} \mathbf{1}_{|k|}}, \forall k = 1, \dots, |\mathcal{K}|. \tag{20}$$

Assuming that entry  $k^*$  corresponds to  $\mathbf{t}^*$ , the maximum achievable rate vector can thus be written as

$$\mathbf{L}_{k^*} \mathbf{t}^* = \mathbf{L}_{k^*} \frac{\mathbf{L}_{k^*}^{-1} \mathbf{1}_{|k^*|}}{\mathbf{1}_{|k^*|}^T \mathbf{L}_{k^*}^{-1} \mathbf{1}_{|k^*|}} = \frac{\mathbf{1}_{|k^*|}}{\mathbf{1}_{|k^*|}^T \mathbf{L}_{k^*}^{-1} \mathbf{1}_{|k^*|}}, \tag{21}$$

and the maximum achievable rate under our model,  $R^*$ , is

$$R^* = \frac{1}{\mathbf{1}_{|k^*|}^T \mathbf{L}_{k^*}^{-1} \mathbf{1}_{|k^*|}}, \tag{22}$$

Note that the solution described above is equivalent to the iterative maximization in (6), and that removing a relay  $r_k$  translates to removing the  $k^{\text{th}}$  row and  $(k+1)^{\text{th}}$  column from the rate matrix in (18). Removing the first relay, for example, reduces the rate matrix in (18) to

$$\mathbf{L}_N = \begin{bmatrix} L_{sr_2} & 0 & \dots & 0 \\ L_{sr_3} & L_{r_2 r_3} & \dots & 0 \\ \vdots & \vdots & \ddots & 0 \\ L_{sd} & L_{r_2 d} & \dots & L_{r_N d} \end{bmatrix}. \tag{23}$$

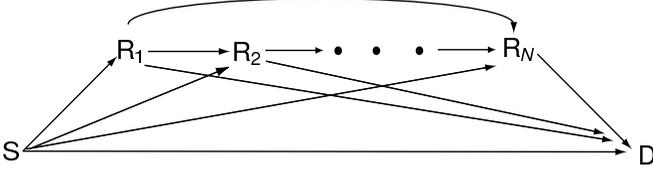


Fig. 1. Location of the relays with respect to the source and destination.

Since  $|\mathcal{M}| = 2^N$ , in theory  $2^N$  possible solutions must be tested to find the global optimum.

### C. Numbering

In Section III-A, the solution provided to the optimization problem assumes a predetermined ordering, e.g., as in Fig. 1. The numbering of the relay nodes impacts performance through causality: relay  $r_k$  decodes information from relay  $r_{k-1}$ , but not vice-versa. A complete solution to the optimization problem must therefore take into account an optimal numbering scheme. In the worst case (in terms of computational power), an optimal solution can be obtained for a specific numbering scheme, and the truly optimal solution can be maximized over all possible numbering schemes.

Clearly, such an approach is impractical. Although a search for an optimal or effective sub-optimal solution is beyond the scope of this paper, we study the effects of numbering on the solution and resulting rate by considering some numbering schemes based on heuristics. We consider two approaches: numbering based on average channel conditions, and numbering based on instantaneous channel conditions.

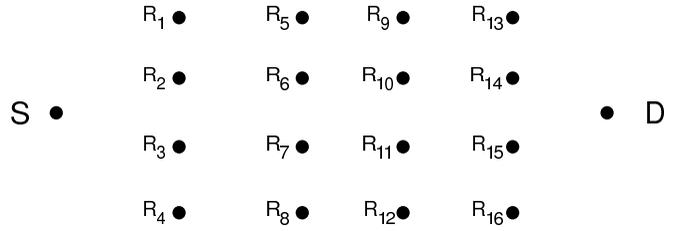
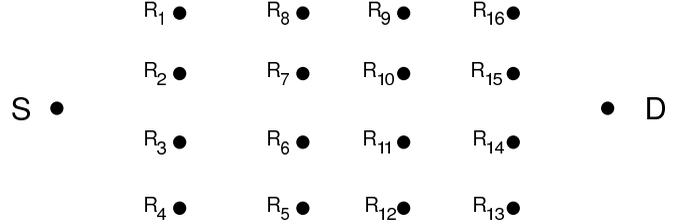
#### 1) Numbering Based on Average Channel Conditions:

In the case of the linear network in Fig. 1, the numbering is trivial: node numbers increase away from the source and towards the destination. In the case of square network with nodes arranged in a grid, we consider two numberings which we refer to as **Average Descending Numbering** and **Average Linear Numbering**, shown in Figs. 2 and 3, respectively, for a  $4 \times 4$  network.

- **Average Descending numbering:** node numbers increase towards the destination and downwards,
- **Average Linear numbering:** node numbers increase towards the destination but vertical numbering ensures that nodes closest to each other retain close numbering.

#### 2) Numbering Based on Instantaneous Channel Conditions:

- **Instantaneous  $s - r_k$  numbering** : node numbers increase with increasing source-relay channels. The first node has the best source-relay channel, the second node has the second-best source-relay channel, etc.
- **Instantaneous  $r_k - r_m$  numbering** : nodes are numbered to maximize the channel between adjacent nodes. The first relay has the best source-relay channel. The second relay has the strongest  $r_1$ -relay channel. Numbers are assigned in this process to unoccupied relays. This heuristic is based on the notion that we should maximize the capacity of each  $(R_k, R_{k+1})$  hop.
- **Random numbering** : nodes are numbered randomly. This case evaluates the worst-case scenario and tests the robustness of the optimization to numbering.

Fig. 2. Numbering in a square  $4 \times 4$  network in descending order.Fig. 3. Numbering in a square  $4 \times 4$  network in linear order.

These schemes are evaluated via simulations in Section V. As we will see, the achievable rate is remarkably robust to the chosen numbering scheme.

### D. Partially Connected Network

In this section we briefly discuss the more practical case of a partially connected network in which some links between the nodes in the network are unavailable. This is a generalization of the fully-connected network discussed in Section III-A. Such a network is more likely to represent a large scale network where, in any case, the solution in (20) would be computationally infeasible.

As an example, consider the two-relay network with the link between  $r_1$  and  $r_2$  is removed. The rate matrix thus becomes

$$\mathbf{L}_3 = \begin{bmatrix} L_{sr_1} & 0 & 0 \\ L_{sr_2} & 0 & 0 \\ L_{sd} & L_{r_1d} & L_{r_2d} \end{bmatrix}. \quad (24)$$

Removing the link thus reduces the rank of this matrix by one, and the rate matrix is now non-invertible, eliminating the solution defined by  $I_1 = I_2 = I_3$ , where both relays are active. The optimal solution in this case is thus to select  $r_1, r_2$ , or not to relay. Note, however, that removing a link does not automatically lead to a non-invertible rate matrix. Consider, for example, the three-relay network with the link between  $r_1$  and  $r_3$  removed. The corresponding rate matrix

$$\mathbf{L}_4 = \begin{bmatrix} L_{sr_1} & 0 & 0 & 0 \\ L_{sr_2} & L_{r_1r_2} & 0 & 0 \\ L_{sr_3} & 0 & L_{r_2r_3} & 0 \\ L_{sd} & L_{r_1d} & L_{r_2d} & L_{r_3d} \end{bmatrix} \quad (25)$$

is full-rank and invertible.

The approach to the optimization problem for the case of the arbitrary connected network is that the same as for the fully-connected network, with the exception that the rate matrix  $\mathbf{L}_{N+1}$  may not be invertible, in which case the corresponding solution is inadmissible. The remaining steps remain unchanged.

#### IV. IMPLEMENTATION WITH REDUCED COMPLEXITY

The solution to the optimization problem in (4), (5) and (6) involves checking  $2^N$  potential solutions. Although the process is conceptually simple, each solution involves the inverse of a rate matrix. In this section, we show how the optimization problem in the previous section can be significantly simplified using a recursive solution. This solution, which exploits the special structure of the rate matrix, greatly simplifies the matrix inversion, as well as reduces the number of possible solutions to check. Essentially, while the solution in Section III-A was a top-down approach, the approach we suggest here is bottom-up.

Consider a set of  $p$  relays,  $\mathcal{P} = \{r_{(1)}, r_{(2)}, \dots, r_{(p)}\}$ ,  $p \geq 0$ , with  $r_{(k)} < r_{(k+1)}$ , and its corresponding rate matrix  $\mathbf{L}_{p+1}^{\mathcal{P}}$ , solution vector  $\mathbf{t}_{p+1}^{\mathcal{P}}$  and maximum rate (if available)  $R^{\mathcal{P}}$ . Note that if  $p = 0$  and the set is empty, the rate matrix and solution vector are constants,  $L_{sd}$  and 1, respectively. Denote as  $\mathcal{P}'$  the set  $\mathcal{P}$  appended with another relay, i.e.,  $\mathcal{P}' = \{r_{(1)}, r_{(2)}, \dots, r_{(p)}, r_{(p+1)}\}$  with  $r_{(p)} < r_{(p+1)}$ . Also, denote as  $\mathbf{L}_{p+2}^{\mathcal{P}'}$ ,  $\mathbf{t}_{p+2}^{\mathcal{P}'}$ , and  $R^{\mathcal{P}'}$  the matrix, solution vector and rate corresponding to set  $\mathcal{P}'$ .

**Proposition 3:** Given  $(\mathbf{L}_{p+1}^{\mathcal{P}})^{-1}$ ,  $(\mathbf{L}_{p+2}^{\mathcal{P}'})^{-1}$  can be obtained with computational complexity order of  $O(p^2)$

**Proof:** For  $p \geq 0$ , the rate matrix  $\mathbf{L}_{p+2}^{\mathcal{P}'}$  can be written as

$$\mathbf{L}_{p+2}^{\mathcal{P}'} = \left[ \begin{array}{c|c} \mathbf{L}_{p+1}^{\mathcal{P}}(1:p, 1:p) & \mathbf{0}_{p \times 2} \\ \hline \mathbf{F}_{2 \times p} & \mathbf{T}_2 \end{array} \right], \quad (26)$$

where  $\mathbf{L}_{p+1}^{\mathcal{P}}(1:p, 1:p)$  denotes the first  $p$  rows and columns of the rate matrix  $\mathbf{L}_{p+1}^{\mathcal{P}}$ ,  $\mathbf{0}_{p \times 2}$  is a  $(p \times 2)$  matrix of zeros,  $\mathbf{T}_2$  is a  $(2 \times 2)$  lower-triangular matrix, and  $\mathbf{F}_{2 \times p}$  is a  $(2 \times p)$  fully-loaded matrix. Note that  $\mathbf{L}_{p+1}^{\mathcal{P}}(1:p, 1:p)$  is triangular.

Using the inverse of a partitioned matrix [39],  $(\mathbf{L}_{p+2}^{\mathcal{P}'})^{-1}$  is given by (27) on next page.

Here  $(\mathbf{L}_{p+1}^{\mathcal{P}}(1:p, 1:p))^{-1}$  is the inverse of a partition of the triangular matrix  $\mathbf{L}_{p+1}^{\mathcal{P}}$ . Using the inverse of a partitioned matrix one more time, however, it is easy to see that

$$(\mathbf{L}_{p+1}^{\mathcal{P}}(1:p, 1:p))^{-1} = (\mathbf{L}_{p+1}^{\mathcal{P}})^{-1}(1:p, 1:p), \quad (28)$$

and thus (29) (see top of next page), and hence obtaining  $(\mathbf{L}_{p+2}^{\mathcal{P}'})^{-1}$  is an  $O(p^2)$  operation. ■

Using this proposition, the solution vector  $\mathbf{t}_{p+2}^{\mathcal{P}'}$  of  $\mathbf{L}_{p+2}^{\mathcal{P}'}$  can be obtained from the solution vector  $\mathbf{t}_{p+1}^{\mathcal{P}}$  of  $\mathbf{L}_{p+1}^{\mathcal{P}}$  as:

$$\mathbf{t}_{p+2}^{\mathcal{P}'} = \frac{(\mathbf{L}_{p+2}^{\mathcal{P}'})^{-1} \mathbf{1}_{p+2}}{\mathbf{1}_{p+2}^T (\mathbf{L}_{p+2}^{\mathcal{P}'})^{-1} \mathbf{1}_{p+2}} = \begin{bmatrix} \mathbf{t}_{p+1}^{\mathcal{P}}(1:p) \\ \mathbf{t}_{p+2}^{\mathcal{P}'}(p+1) \\ \mathbf{t}_{p+2}^{\mathcal{P}'}(p+2) \end{bmatrix}, \quad (30)$$

where  $\mathbf{t}_{p+1}^{\mathcal{P}}(1:p)$  represents the first  $p$  entries of the solution vector  $\mathbf{t}_{p+1}^{\mathcal{P}}$  already-calculated.  $\mathbf{t}_{p+2}^{\mathcal{P}'}(p+1)$  and  $\mathbf{t}_{p+2}^{\mathcal{P}'}(p+2)$  are the last two entries of the solution vector  $\mathbf{t}_{p+2}^{\mathcal{P}'}$  that remain to be calculated.  $R^{\mathcal{P}'} = \left[ \mathbf{1}_{p+2}^T (\mathbf{L}_{p+2}^{\mathcal{P}'})^{-1} \mathbf{1}_{p+2} \right]^{-1}$  is the maximum achievable rate obtained using the set  $\mathcal{P}'$  of relays. The last two entries of the solution vector  $\mathbf{t}_{p+2}^{\mathcal{P}'}(p+1)$  and  $\mathbf{t}_{p+2}^{\mathcal{P}'}(p+2)$  can be written as (31) (see top of next page). With a corresponding achievable rate  $R^{\mathcal{P}'}$  (see (32) on next page),

where we use  $\sum_{i,j} \mathbf{A}(i,j)$  to denote the summation over all the elements of matrix  $\mathbf{A}$ .

Using the discussion above, the optimization problem for a network of  $N$  potential relays can be solved recursively as follows:

- 1) Determine the set of all potential relay combinations. Sequence the set as:

$$\begin{aligned} \mathcal{M} = \{ & (r_1), (r_1, r_2), (r_1, r_2, r_3), \dots, (r_1, r_2, \dots, r_N), \\ & (r_1, r_3), (r_1, r_3, r_4), \dots, (r_1, r_3, \dots, r_N), \\ & \dots \dots \dots \\ & (r_1, r_N), \\ & (r_2), (r_2, r_3), (r_2, r_3, r_4), \dots, (r_2, r_3, \dots, r_N), \\ & (r_2, r_4), (r_2, r_4, r_5), \dots, (r_2, r_4, \dots, r_N), \\ & \dots \dots \dots \\ & (r_2, r_N), \\ & \dots \\ & (r_{N-1}, r_N) \}. \end{aligned}$$

Note that each ‘‘row’’ of  $\mathcal{M}$  is a subset of relay combinations in which each element is formed from the previous element by adding a relay. We had used this approach earlier to form  $\mathcal{P}'$  from  $\mathcal{P}$ .

- 2) In each ‘‘row’’, obtain the rate matrix, its respective optimized time allocation vector and achievable rate for each element (i.e., relay combination) recursively using (29), (30), (31) and (32).
- 3) Check that for each particular set  $\mathcal{P}$  of  $p$  relays, the solution  $\mathbf{t}_p$  and achievable rate  $R_p$  satisfies the constraints:

$$R^{\mathcal{P}} \geq 0, \quad (33)$$

$$\mathbf{t}_{p+1}^{\mathcal{P}} > \mathbf{0}_{p+1}. \quad (34)$$

- If both constraints are satisfied, place the solution in the potential set of valid solutions  $\mathcal{K}$ , advance elements and return to step (1).
- If (34) is not satisfied, check which element of the the allocation vector  $\mathbf{t}_p$  does not satisfy the constraint.
  - If any of the first  $(p-1)$  entries of  $\mathbf{t}_p$  are less than zero, i.e.,  $\mathbf{t}_p(1:p-1 < \mathbf{0}_{p-1})$ , this constraint will not be satisfied for any other relay combinations in this ‘‘row’’. Advance rows and return to item (1).
  - If the constraint is not satisfied by either of the last two items in the solution vector, discard the solution but check the other elements in the ‘‘row’’.

- 4) From the set  $\mathcal{K}$ , pick the highest achievable rate and its corresponding time allocation.

The recursive algorithm given above simplifies the optimization problem in two ways:

- 1) It reduces the computation load of determining successive matrix inverses by writing each matrix inverse as a function of another, already known, matrix inverse, and two other matrices obtained through simple matrix multiplication.

$$\left(\mathbf{L}_{p+2}^{\mathcal{P}'}\right)^{-1} = \left[ \begin{array}{c|c} \left(\mathbf{L}_{p+1}^{\mathcal{P}}(1:p, 1:p)\right)^{-1} & \mathbf{0}_{p \times 2} \\ \hline -\mathbf{T}_2^{-1}\mathbf{F}_{2 \times p}\left(\mathbf{L}_{p+1}^{\mathcal{P}}(1:p, 1:p)\right)^{-1} & \mathbf{T}_2^{-1} \end{array} \right] \quad (27)$$

$$\left(\mathbf{L}_{p+2}^{\mathcal{P}'}\right)^{-1} = \left[ \begin{array}{c|c} \left(\mathbf{L}_{p+1}^{\mathcal{P}}\right)^{-1}(1:p, 1:p) & \mathbf{0}_{p \times 2} \\ \hline -\mathbf{T}_2^{-1}\mathbf{F}_{2 \times p}\left(\mathbf{L}_{p+1}^{\mathcal{P}}\right)^{-1}(1:p, 1:p) & \mathbf{T}_2^{-1} \end{array} \right] \quad (29)$$

$$\begin{bmatrix} \mathbf{t}_{p+2}^{\mathcal{P}'}(p+1) \\ \mathbf{t}_{p+2}^{\mathcal{P}'}(p+2) \end{bmatrix} = R^{\mathcal{P}'} \left[ \begin{array}{c|c} -\mathbf{T}_2^{-1}\mathbf{F}_{2 \times p}\left(\mathbf{L}_{p+1}^{\mathcal{P}}\right)^{-1}(1:p, 1:p) & \mathbf{T}_2^{-1} \end{array} \right] \mathbf{1}_{(p+2) \times 1}, \quad (31)$$

$$\begin{aligned} R^{\mathcal{P}'} &= \frac{1}{\mathbf{1}_{p+2}^T \left(\mathbf{L}_{p+2}^{\mathcal{P}'}\right)^{-1} \mathbf{1}_{p+2}} = \left( \sum_{i,j} \left(\mathbf{L}_{p+2}^{\mathcal{P}'}\right)^{-1}(i,j) \right)^{-1}, \\ &= \left( \sum_{i,j} \left(\mathbf{L}_{p+1}^{\mathcal{P}}\right)^{-1}(i,j) - \sum_{i,j} \mathbf{T}_2^{-1}\mathbf{F}_{2 \times p}\left(\mathbf{L}_{p+1}^{\mathcal{P}}\right)^{-1}(1:p, 1:p)(i,j) + \sum_{i,j} \mathbf{T}_2^{-1}(i,j) \right)^{-1} \end{aligned} \quad (32)$$

- 2) It may eliminate infeasible solutions by discarding relay combinations which do not satisfy constraints. For example, if the relay combination  $(r_1, r_2, r_3)$  does not satisfy the constraints, the combination  $(r_1, r_2, r_3, r_4)$  can be automatically discarded.

#### A. Complexity and Number of Operations

In this section we calculate this complexity, which also quantifies the computational savings of the recursive scheme presented above in Section IV.

The complexity of the recursive scheme is bounded by complexity of matrix multiplication. The number of operations (multiplications and additions) required in the product of two matrices of size  $(m, n)$  and  $(n, p)$  is  $2mpn - mp$  [40], and the number of operations required for the product of a matrix of size  $(m, n)$  with a square, size- $n$  diagonal matrix is

$$m \left( \sum_{k=0}^{n-1} k + \sum_{k=1}^n k \right) = mn^2. \quad (35)$$

We now calculate the number of operations required for each rate matrix of size  $(q+1)$ , corresponding to the set  $\mathcal{Q}'$  of  $q$  relays. The calculation of the matrix fundamental to the recursive algorithm,

$$\left(\mathbf{L}_{q+1}^{\mathcal{Q}'}\right)^{-1} = -\mathbf{T}_2^{-1}\mathbf{F}_{2 \times (q-1)}\left(\mathbf{L}_q^{\mathcal{Q}}\right)^{-1}(1:q-1, 1:q-1). \quad (36)$$

requires a total of  $2q^2 + 2q + 1$  operations, broken down as:

- 1)  $-\mathbf{T}_2^{-1} \rightarrow 5$  operations,
- 2)  $-\mathbf{T}_2^{-1}\mathbf{F}_{2 \times (q-1)} = \mathbf{A}_{2 \times (q-1)} \rightarrow 6(q-1)$  operations using  $2mpn - mp$ ,
- 3)  $\mathbf{A}_{2 \times (q-1)}\left(\mathbf{L}_q^{\mathcal{Q}}\right)^{-1}(1:q-1, 1:q-1) \rightarrow 2(q-1)^2$  operations, using (35).

From (32), the number of operations required to calculate  $R^{\mathcal{Q}'}$  is  $q^2 + 2q + 4$ . Using (31), the number of operations required to update the solution vector is  $1 + 2(q+1) = 2q + 3$ . Summing the above, we obtain the total number of operations required in one iteration of the resource allocation algorithm:

$$\begin{aligned} \text{Op}(q) &= (2q^2 + 2q + 1) + (q^2 + 2q + 4) + (2q + 3) \\ &= 3q^2 + 6q + 8. \end{aligned} \quad (37)$$

Note that the complexity order of calculating each rate and solution vector is  $O(q^2)$ . Without the recursion, this complexity is of order  $O(q^3)$ , resulting from the inverse of the rate matrix. The recursion thus introduces significant savings in terms of complexity.

We now calculate the worst-case total number of operations required by the resource allocation algorithm. In the worst case, the algorithm cycles through  $2^N$  operations consisting of  $\binom{N}{q}$  sets of  $q$  relays which require  $3q^2 + 6q + 8$  operations. The total worst case number of operations is therefore

$$\sum_{q=0}^N \binom{N}{q} (3q^2 + 6q + 8). \quad (38)$$

This calculation could be rendered more precise if it were possible to account for the savings obtained in Section IV which eliminates some infeasible solutions a priori by discarding relay combinations known to not satisfy the constraints. The probability of this occurring for particular channel realizations is unfortunately very difficult to compute, and we thus develop only the worst-case result in closed form.

## V. SIMULATIONS

This section results of simulations testing the resource allocation scheme discussed in Section III. Two networks

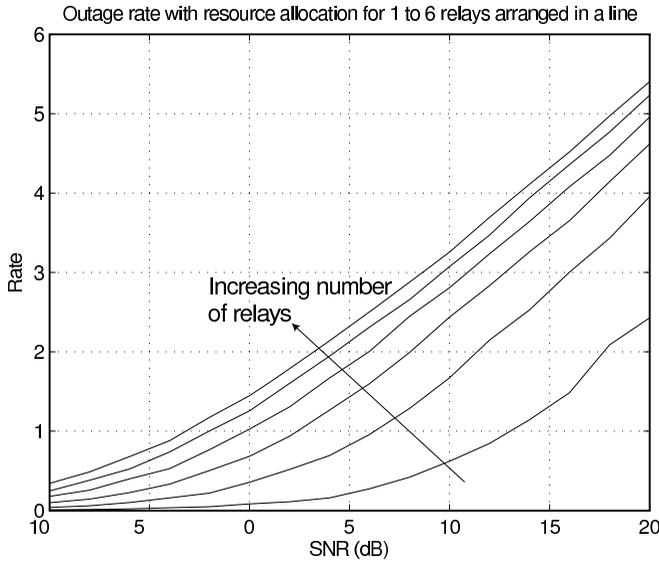


Fig. 4. Outage rate vs. SNR using 1, ... 6 potential relays and with resource allocation.

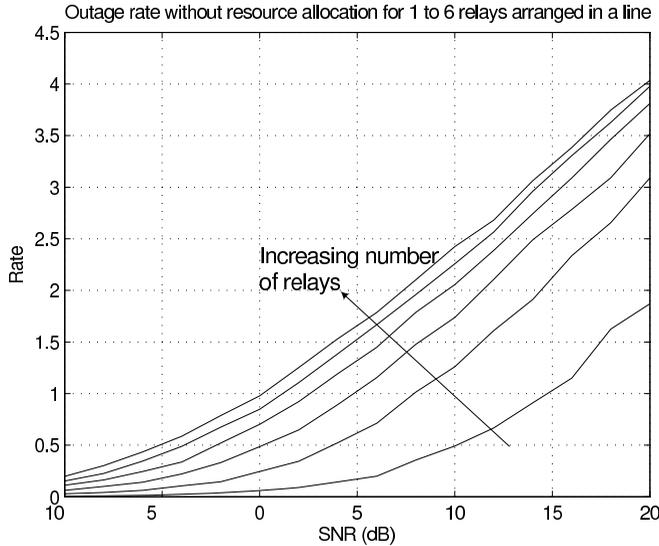


Fig. 5. Outage rate vs. SNR using 1, ... 6 potential relays and without resource allocation.

considered have 1 to 6 relays arranged linearly, and 4 and 9 nodes arranged in a grid. The figure of merit is the achievable rate  $R_a$  with an outage probability of  $10^{-3}$ , i.e.,  $\Pr[R^* < R_a] = 10^{-3}$ . A closed form expression for the outage probability of optimized cooperation is very complicated and beyond the scope of the paper. The outage probability and rate are thus obtained numerically.

The relays are equispaced on a line between the source and destination, as in Fig. 1, and we use an attenuation exponent of  $p_a = 2.5$ . This choice is motivated by the application of static mesh-nodes installed on posts; transmissions between such nodes should undergo little shadowing and a lower attenuation exponent. From 60000 fading realizations we obtain the cumulative density function of the instantaneous rate  $F_R(r)$ . The outage rate is the rate for which the probability of outage is  $10^{-3}$ , i.e.,  $F_R^{-1}(10^{-3})$ .

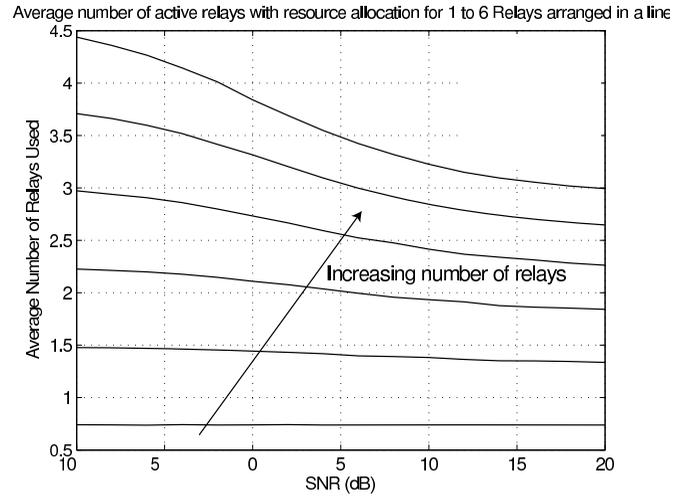


Fig. 6. Average number of active relays with 1, ... 6 potential relays and with resource allocation.

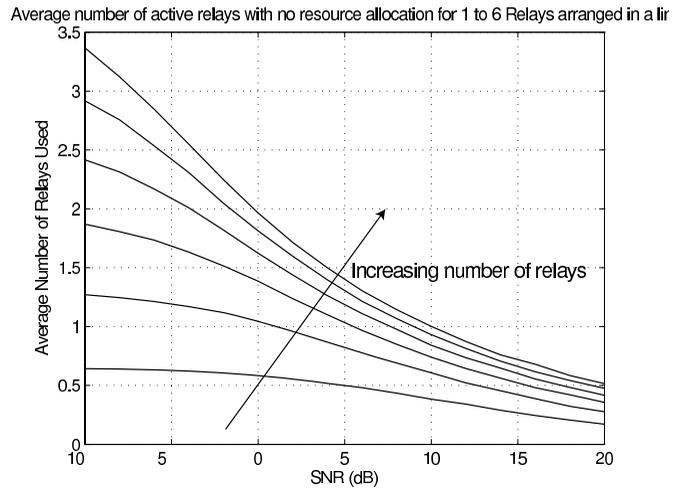


Fig. 7. Average number of active relays with 1, ... 6 potential relays and without resource allocation.

Figs. 4 and 5 plot the outage rate as a function of the average end-to-end SNR,  $\frac{P}{N_0W}$ , for optimized and non-optimized cooperation, respectively. The rate for the optimized cooperation is obtained from (20). Non-optimized cooperation uses equal time allocation, i.e., the rate for a particular relay set is simply the minimum of the mutual information at each node. Non-optimized cooperation, however, does optimally select relays by choosing the best, in terms of outage rate, of the  $2^N$  relay combinations. Comparing Fig. 4 and Fig. 5 shows that optimizing resources increases rates significantly, as expected. The outage rate increases as a function of nodes available to relay. We also note the typical phenomenon of decreasing marginal returns: the gains of adding each additional relay decreases with increasing number of relays.

Figs. 6 and 7 show the average number of relays that are active from the set of potential relays for optimized and non-optimized cooperation as a function of SNR. For each network size, this number is a decreasing function since, with increasing SNR, each node can communicate with a node further away. Interestingly, the number of active relays

Outage rate with resource allocation for 4 and 9 relays arranged in a grid and in a line

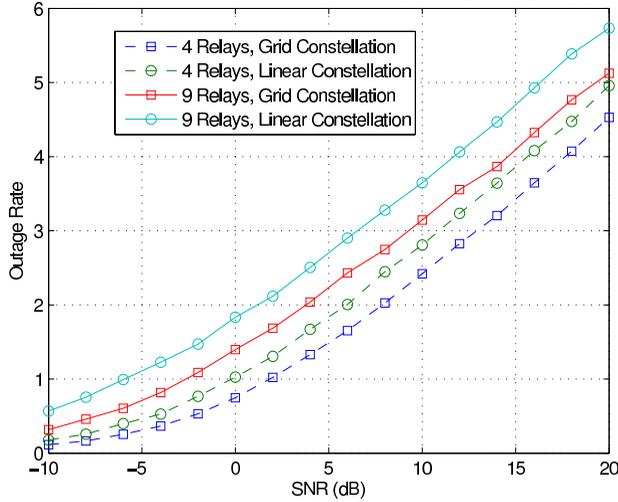


Fig. 8. Outage rate vs. SNR using resource allocation and for 4 and 9 relays arranged in a grid and in a line.

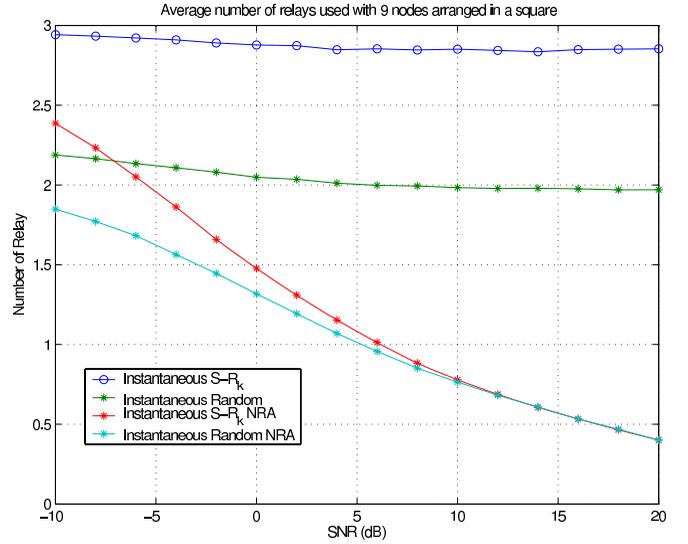


Fig. 10. Average number of active relays using resource allocation and with various numbering schemes for 9 potential nodes arranged in a grid.

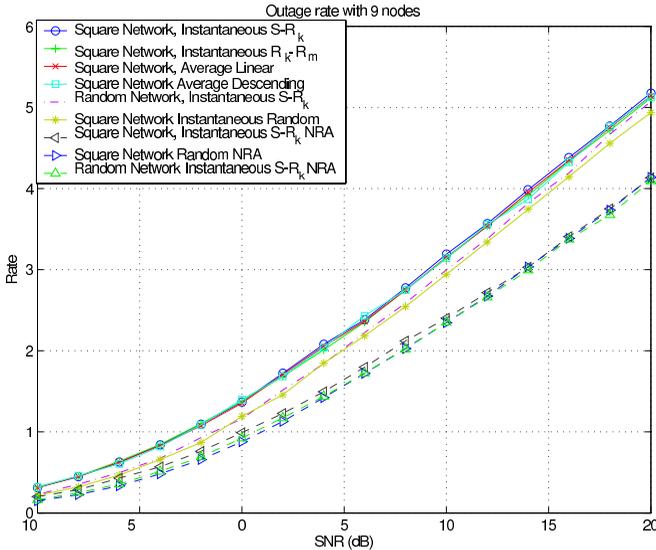


Fig. 9. Outage rate vs. SNR using resource allocation and for various numbering schemes for 9 potential nodes arranged in a grid.

decreases much faster for non-optimized as compared to optimized cooperation, suggesting that optimizing resources distributes the relaying burden more effectively and equitably.

To test the effect of geometry on the outage rate, we compare the rates obtained by optimizing resources and the placing relays on a line, as in Fig. 4, to those obtained by placing the relays on a regular square grid. We number the relays in the grid in ascending order downwards and towards the source; a derivation of the optimal numbering is beyond the scope of this paper. The results are demonstrated in Fig. 8, where we place 4 and 9 relays on a  $2 \times 2$  and  $3 \times 3$  square grid. As shown in the figure, the rate for the linear constellation is significantly higher than that obtained by the grid constellation, suggesting that the path-loss incurred by traversing all the nodes laterally results in non-negligible performance loss.

We evaluate the performance of the numbering schemes

discussed in Section III-C in Fig. 9. The four schemes, including two based on average channel conditions and two based on instantaneous channel conditions, exhibit indistinguishable performance in terms of rate. There is an expected drop in rate with random numbering, though, note that this drop is no more than approximately 0.25 bits/channel use. The algorithm is thus quite robust to numbering schemes.

Fig. 9 also shows the outage rate for a network with randomly placed nodes. Here the node locations are chosen from a uniform distribution over an area equivalent to that of the square grid. The internode channels are obtained as before. This example eliminates possible dependencies of the results obtained earlier on the chosen array geometry. The numbering here is based on the instantaneous  $S-R_k$  channels. In such a random network, as expected, the available outage rate is lower than in a square grid network; however, at higher SNR levels this difference disappears. Again, the significant gains due to resource allocation are clear.

In Fig. 9 we also compare the effect of numbering when used without resource allocation, and show only the case of instantaneous  $s-r_k$  numbering and random numbering. The improvement from instantaneous over random numbering in this case is less than 0.1 bits/channel use. The robustness of the numbering scheme thus increases by eliminating time optimization. To gain insight into this phenomenon, in Fig. 10 we plot the average number of active users for the instantaneous and random numbering schemes with and without resource allocation.

We first observe that the instantaneous numbering scheme uses more relays than the random numbering scheme when resource allocation is used, and that this difference is constant over the SNR region of interest. Without resource allocation, on the other hand, the number of relays used when using instantaneous and random numbering decreases quickly and is constant for SNR values higher than 10 dB. It is clear from this figure that the difference in rate performance between instantaneous and random numbering is an increasing function

of the number of selected relays. Because so few relays are selected without resource allocation, the effect of the numbering scheme is negligible. The influence of the numbering scheme increases when time allocation is introduced, increasing the number of relays used for both numbering schemes and increasing the sensitivity to the numbering scheme. This sensitivity increases slowly, however, and is negligible for the various numbering schemes based on heuristics.

## VI. CONCLUSIONS

In this paper, we determined the optimal channel resource allocation, in terms of transmission-time allocation, for the  $N$ -node cooperative diversity, multihop network using DF. Time-allocation requires use of independent codebooks. For a particular network, i.e., set of potential relays, the unique solution for a particular relay numbering scheme is obtained by taking the inverse of the triangular rate matrix. The optimal solution overall is found by choosing the network size with the maximum rate for each possible sub-network. One requirement assumed here is that there is an ordering of relays such that relay  $r_{k+1}$  transmits after relay  $r_k$ , but not vice-versa. Not explored here is an optimal ordering of relays using the channel values; through simulations, however, the optimization is shown to be robust to the numbering scheme. In the second phase of the paper, we showed that by exploiting the special structure of the rate matrix, the optimization can be performed in a recursive fashion which decreases the computation load of the rate matrix inverse and the number of required iterations.

Multiple-node selection, a generalization of the single-node selection in [3], [28]–[30], is inherent to the optimization strategy. Simulation results show significant gains in achievable rate due to resource allocation, but diminishing marginal returns as a function of network size. Furthermore, we show a significant benefit to arranging the nodes in a linear, as opposed to a grid, constellation.

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