Bandwidth-Efficient Coded Space-Time Multiple Access

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Background

- Space-time block coding (STBC) can be used in multi-user systems in several ways:
  - Orthogonal channeling e.g. CDMA, TDMA, OFDMA: bandwidth expansion necessary, conventional method.
  - Bandwidth-efficient two-dimensional spreading: Brehler/Varanasi, April 2003 (BV03) – N_t transmit antennas, use $N_t + 1$ time dimensions $\therefore$ minimal bandwidth expansion.
- BV03 shows that single-user performance is possible at high SNR for any no. of users, when spreading matrices are carefully designed.

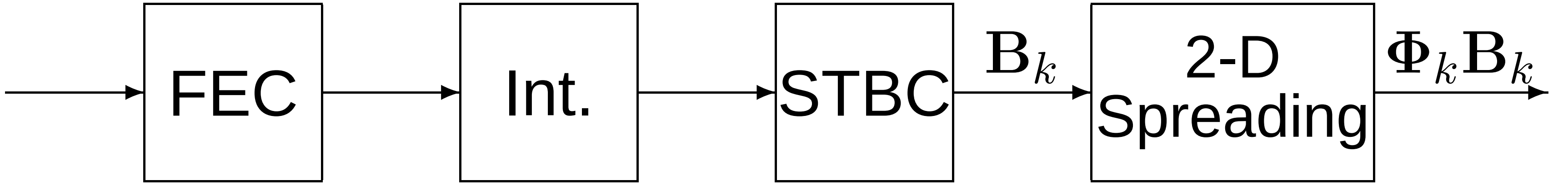
Our Contributions

- Show that with FEC, designing spreading matrices for asymptotic single-user performance with the Alamouti STBC is easier than without coding.
- Though above result is obtained assuming full-complexity ML decoding, showed through simulations that iterative multi-user decoding achieves near single-user performance at high SNR.

Transmitter (Uplink)

- User k encodes its information bits; code bits are interleaved, then modulated onto channel symbols; STBC is applied; an $N_t \times N_t$ linear transformation $\mathbf{\Phi}_k$ follows ($N_t =$no. of Tx antennas).
- If full-rate STBC is used (one symbol per channel use), this system uses the same bandwidth as the single-antenna system.

Transmitter k



Transmitter (Uplink)

- The ith row of $\mathbf{B}_k \in \mathbb{C}^{N_t \times D}$ is transmitted over antenna i over D symbol intervals.
- Therefore $\mathbf{\Phi}_k$ is $N_t \times N_t$, and the transmitted signal matrix is still $N_t \times D$, i.e. no bandwidth expansion due to the 2-D “spreading”.
- Question becomes: how do we design $\mathbf{\Phi}_k$, $k = 1, \ldots, K$, for any K, to get asymptotic single-user performance in fading channels with coding and interleaving?

Channel Side Information

- Flat Rayleigh fading channel linking each pair of Tx-Rx antennas.
- Channel known perfectly to receiver but transmitter does not make use of any channel knowledge.

Receiver

- We will implement an iterative decoder in the simulations.
- For the derivation of the asymptotic result, assume the receiver to be an ML decoder, jointly and optimally decoding both the FEC and the STBC.

Signal Model

- Drop user subscripts for neatness.
- Let frame length be L STBC symbols e.g. for Alamouti method, frame length = $2L$ data symbol intervals.
- The l-th STBC symbol is $\mathbf{B}(l) \in \mathbb{C}^{N_t \times D}$; the transmitted symbol is $\mathbf{S}(l) = \mathbf{\Phi}\mathbf{B}(l)$, where $\mathbf{\Phi}$ is the $N_t \times N_t$ spreading matrix.
- ML decoder detects super-symbol $\mathbf{B} = [\mathbf{B}(1), \ldots, \mathbf{B}(L)]$ by choosing from its code-constrained constellation $\mathcal{B}$.

Single-User Error Probability

- Goal: Derive expression for bit error probability in single-user Rayleigh flat fading channel at high SNR, with coding/interleaving.
- Why: Compare this with multi-user expression; if ratio $\rightarrow 1$ as SNR $\rightarrow \infty$, asymptotic single-user performance achieved.
- Spreading matrix in this single-user system must be unitary, so that $\mathbf{H}\mathbf{\Phi}$ is statistically identical to $\mathbf{H}$ (iid Rayleigh), or else performance loss.
- Any unitary $\mathbf{\Phi}$ will do – may as well assume $\mathbf{\Phi} = \mathbf{I}$.

Single-User Error Probability

- Received signal over lth STBC symbol is

$$\mathbf{Y}_l = \mathbf{H}_l \mathbf{B}(l) + \mathbf{N}_l \in \mathbb{C}^{N_r \times D}$$

where $\mathbf{H}_l$ is the $N_r \times N_t$ channel matrix at time l.

- ML decoding of the super-symbol $\mathbf{B}$:

$$\hat{\mathbf{B}} = \arg \min_{\mathbf{B} \in \mathcal{B}} \sum_{l=1}^{L} \|\mathbf{Y}_l - \mathbf{H}_l \mathbf{\Phi} \mathbf{B}(l)\|_F^2.$$

Single-User Error Probability

Pairwise error probability is thus

$$
\begin{aligned}
P[\mathbf{B} \to \mathbf{B}'] &= P\left[\sum_{l=1}^{L} \|\mathbf{Y}_l - \mathbf{H}_l \mathbf{B}(l)\|^2 > \sum_{l=1}^{L} \|\mathbf{Y}_l - \mathbf{H}_l \mathbf{B}'(l)\|^2 \middle| \mathbf{B} \right] \\
&> P\left[\|\mathbf{Y}_l - \mathbf{H}_l \mathbf{B}(l)\|^2 > \|\mathbf{Y}_l - \mathbf{H}_l \mathbf{B}'(l)\|^2 | \mathbf{B} \right] \\
& \quad \forall\, l \in \Xi(\mathbf{B}, \mathbf{B}') \\
&= \prod_{l \in \Xi(\mathbf{B}, \mathbf{B}')} P(\mathbf{B}(l) \to \mathbf{B}'(l)).
\end{aligned}
$$

where $\Xi(\mathbf{B}, \mathbf{B}') = \{l | \mathbf{B}(l) \neq \mathbf{B}'(l)\}$.

Single-User Error Probability

- Individual asymptotic PEP $P[\mathbf{B}(l) \rightarrow \mathbf{B}'(l)]$ has been derived (Brehler/Varanasi, IEEE Trans. IT, Sep. 2001).
- For Alamouti code (2 tx'ers, 1 rx'er), asymptotically

$$\prod_{l \in \Xi} P(\mathbf{B}(l) \rightarrow \mathbf{B}'(l)) = \binom{3}{2}^{|\Xi|} [d_P(\mathbf{B}, \mathbf{B}')]^{-2} \left(\frac{1}{\sigma^2}\right)^{-2|\Xi|}$$

where $d_P(\mathbf{B}, \mathbf{B}')$ is the product distance between $\mathbf{B}$ and $\mathbf{B}'$, and $\Xi = \Xi(\mathbf{B}, \mathbf{B}')$.

- Note that $|\Xi(\mathbf{B}, \mathbf{B}')|$ can be seen as the Hamming distance between $\mathbf{B}$ and $\mathbf{B}'$.

Single-User Error Probability

- At high SNRs, the lower bound is dominated by the term with smallest $|\Xi|$, i.e. the codewords separated by the smallest Hamming distance.
- Assuming the constellation $\mathcal{B}$ is symmetric,

$$\lim_{\sigma \to 0} P(\mathbf{B} \neq \mathbf{B}') = P(\mathbf{0} \to \mathbf{B}_{min}) > \prod_{l \in \Xi(\mathbf{0}, \mathbf{B}_{min})} P(\mathbf{0} \to \mathbf{B}_{min}(l))$$

where B_{min} is the super-symbol that is closest in Hamming distance to the all-zero symbol.

Single-User Error Probability

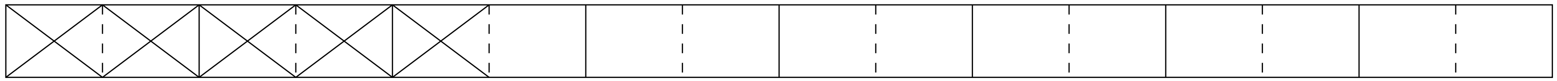
- Let minimum free distance of FEC code be d_{free}.
- Suppose the d_{free} codeword leads to $|\Xi| = l_{min}$.
- The interleaver that gives the minimum $|\Xi|$ leads to highest BEP, and $l_{min} = \lceil d_{free}/2 \rceil$ i.e. non-zero code bits occur in pairs.
- For this interleaver, information bit error prob. can be written

$$P_1^1(\sigma) > K_{su}\gamma^{-2l_{min}}$$

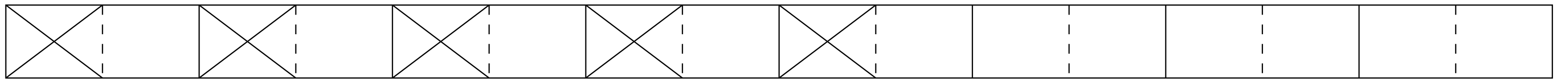
where γ denotes SNR.

Example ($d = 5$)

$|\Xi| = 3$



$|\Xi| = 5$



Asymptotic Multi-User Performance

- Using a similar technique, we can bound the asymptotic BEP of any user in a K-user i.i.d. channel, $P_K^k(\sigma, L)$.
- The ratio of the bounds on $P_K^k(\sigma, L)$ to $P_1^1(\sigma, L)$ is ϱ.
- If $\lim_{\sigma\to 0} \varrho = 1$, the system is said to achieve asymptotic single-user performance with ML decoding.

Theorem 1 (Uplink)

In a multi-access channel in which each user encodes its information with the same channel encoder, interleaves its code bits using a unique interleaver, performs Alamouti space-time encoding, and then space-time spreading with matrix Φ_k,

$$\mathbf{\Phi}_k^H \mathbf{\Phi}_k = \mathbf{I} \Rightarrow \lim_{\sigma \to 0} \varrho = 1$$

when the frame length $L \to \infty$.

Implications of Theorem 1

- Spreading matrices only have to be unitary for asymptotic single-user performance, regardless of no. of users.
  - Uplink, so each user's channel is independent of others'. Channel matrices can be used to distinguish users.
  - Infinite SNR, so capacity is infinite too.
  - Does not imply that all unitary sp. matrices give same performance at all SNRs. Scope for finding best design.

Theorem 2 (Downlink)

For downlink transmission, sufficient conditions for asymptotic SU performance are:

$$\mathbf{\Phi}_k^H \mathbf{\Phi}_k = \mathbf{I}; \tag{1}$$

$$\text{rank}\left(\sum_{p=1}^{k}\sum_{q=1}^{k}[\mathbf{B}_p(l) - \mathbf{B}'_p(l)]^H \mathbf{\Phi}_p^H \mathbf{\Phi}_q [\mathbf{B}_q(l) - \mathbf{B}'_q(l)]\right) \geq 1 \tag{2}$$

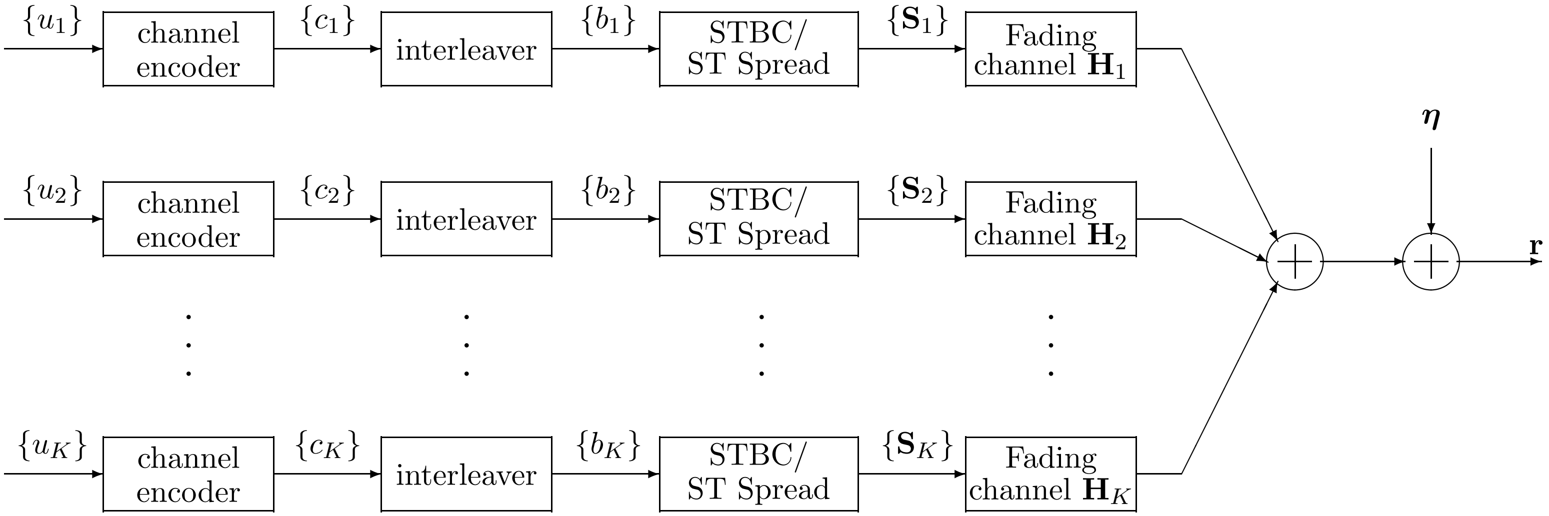
for all $k = 1, \ldots, K$, where B_p denotes the p-th user's STBC matrix.

Implications of Theorem 2

- Design of spreading matrices for downlink is complicated by the rank requirement.
- Φ_k's need to be "sufficiently different" to ensure separability of users, because can no longer rely on channels to distinguish users.
- Systematic design still not found, but when interleaver length is large, simulations show that almost any unitary matrix will do.

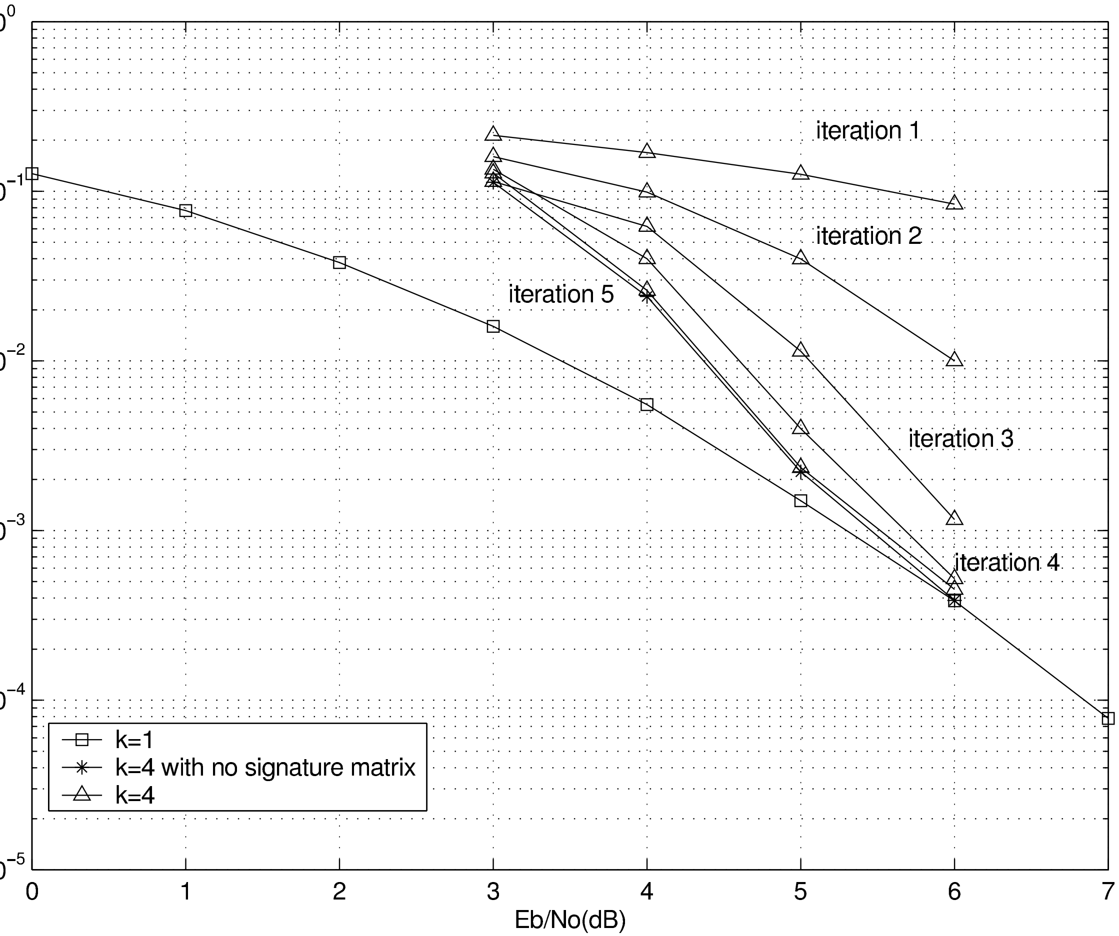
Practical Receiver Structure

System is very similar to coded/interleaved CDMA channel.



$\Rightarrow$ turbo multiuser detection for near-optimal performance with low(ish) complexity.

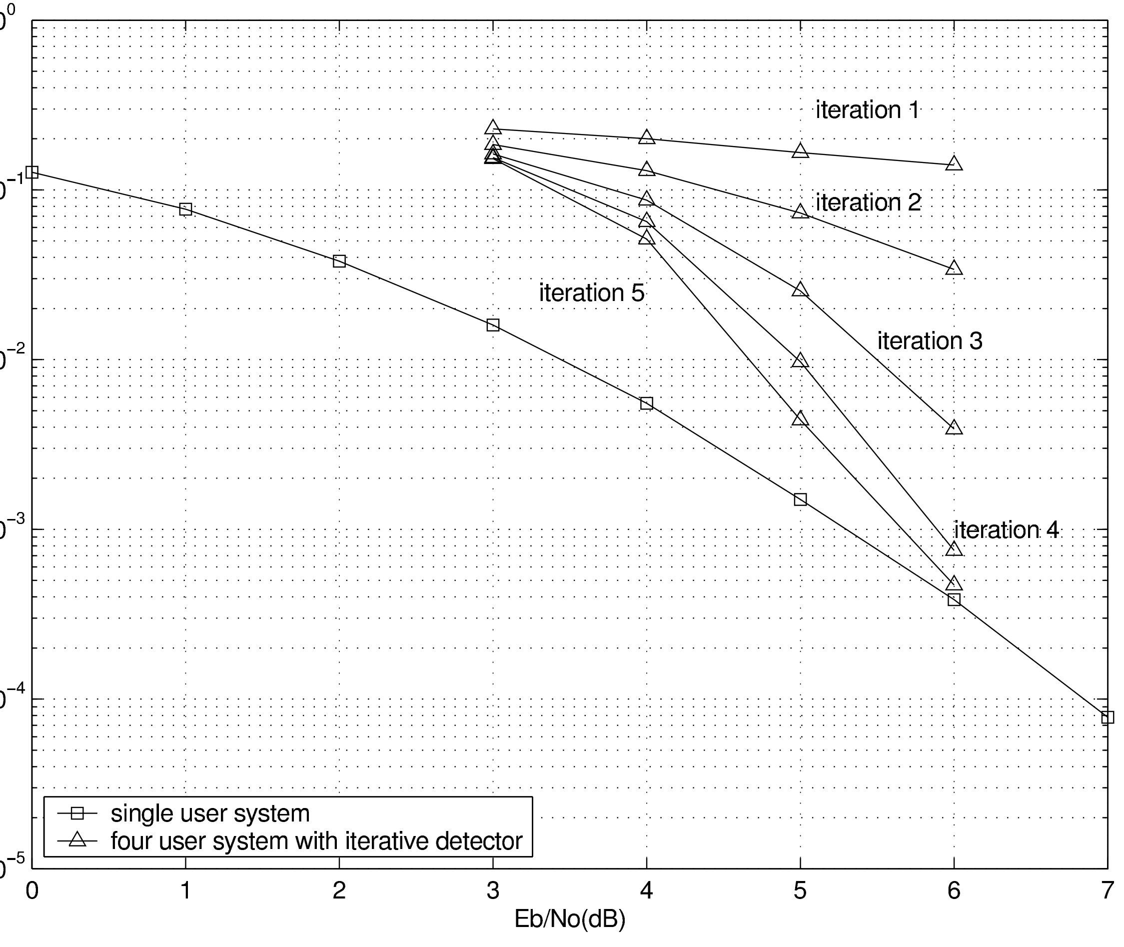
Uplink Simulation



<u>Full-complexity turbo MUD</u>

4 users, independent Rayleigh block fading channels. Unitary spreading matrices give single-user performance at high SNR, as we had hoped.

Downlink Simulation



Full-complexity turbo MUD

4 users, independent Rayleigh block fading channels.
Unitary spreading matrices chosen to satisfy conditions of Theorem 2.

Conclusions

- Analyzed coded space-time spreading with Alamouti STBC.
- Derived sufficient conditions for asymptotic single-user performance.
- Under assumption of independent fading channels for all users, uplink does not require any space-time spreading – channels themselves can be used for user separation.
- For downlink, a sort of cross-correlation rank criterion must be satisfied, but that is not difficult either.