Abstract—We consider a wireless cellular network with multiple amplify-and-forward (AF) relays in each cell, assisting the communication of multiple source-destination pairs with relay transmission beamforming. Our objective is to minimize the maximum interference power among all active receivers in a neighboring cell subject to per-relay power and minimum received SNR constraints. We propose an efficient algorithm to obtain the optimal relay beamforming vectors. We show that even though the optimization problem is non-convex, it has zero Lagrange duality gap and can be converted to a semi-definite programming problem. The performance of the proposed algorithm is studied numerically, both for the case where the interference channel information is exactly known and for the case of inaccurate channel information due to either limited feedback or channel estimation error. It is demonstrated that the min-max interference approach substantially outperforms the alternative where we simply minimize the maximum relay transmission power.

I. INTRODUCTION

Modern cellular systems suffer from inter-cell interference due to the small frequency reuse factor in a cell [1]. The other types of interference, intra-cell interference and co-antenna interference, can be avoided by orthogonal transmission of users in a cell such as time-division multiple access and orthogonal frequency-division-multiple-access (OFDMA). Hence, we focus on inter-cell interference in this paper. In particular, we study how to use multiple relays in beamforming to reduce such interference.

Wireless relaying has been a subject of many studies in the literature and is specified in standards such as LTE-Advanced [2] and WiMax [3]. The design of relay cooperative networks in interference limited environments has been considered under various criteria such as capacity, throughput, area spectral efficiency, and received signal-to-interference-plus-noise ratio (SINR) [4]–[8]. The objectives of these works do not include inter-cell interference reduction. Inter-cell interference mitigation techniques for relay networks with orthogonal-based transmission have been studied in [9]–[13]. The authors of [9] have proposed a radio resource management strategy for relay-user association, resource allocation, and power control. In [10], the performance of different relay strategies has been studied in interference-limited cellular systems. In [11], a joint subcarrier allocation, scheduling, and power control scheme has been proposed for OFDMA-based relay inter-cell interference limited networks. For relay-aided cellular OFDMA systems, the authors of [12] have proposed an interference coordination heuristic scheme. In [13], a game theoretic framework has been developed to mitigate interference in OFDMA relay networks. However, none of these works aims to directly minimize the inter-cell interference. Furthermore, none of them considers relay beamforming, which can lead to a complicated optimization problem.

In this work, we present a novel approach to optimally design relay beamforming in order to minimize inter-cell interference. We consider a cellular network where each cell has multiple single-antenna amplify-and-forward (AF) relays collaborating for the communication of multiple independent sources and destinations using orthogonal spectrum resource. The goal of this paper is finding the optimum relay beamforming in order to minimize the maximum received interference at the receivers in a neighboring cell. Our numerical results show that substantial reduction of interference can be achieved by using a moderate number of relays.

We first formulate the relay beamforming problem in order to minimize the maximum interference under minimum received SNR and per-relay power constraints. Then, the original non-convex problem is recast as a second-order-conic programming (SOCP) problem, through which we show that the original problem has zero Lagrange duality gap and hence the Lagrange dual method can be applied. We transform the dual problem into a semi-definite-programming (SDP) problem with much fewer variables and constraints compared to the original optimization problem. Hence, the computation complexity in finding the optimal beamweights is reduced significantly. Expressed in the SDP form, the dual problem can be solved by the interior-point methods having polynomial complexity. Three cases for the optimum dual variables are identified and the optimal relay beamweights are obtained accordingly. Furthermore, observing a form of the uplink-downlink duality [14]–[16], we derive a semi-closed form expression for the optimal relay beamweights. We evaluate the performance of min-max interference, both when true interference CSI is available and when there is only limited channel feedback or channel estimation error.

The rest of this paper is organized as follows: In Section II, the system model is described. The min-max interference problem is solved in Section III. Numerical results are presented in Section IV, and conclusions are drawn in Section V.

Notation: We use $\| \cdot \|$ to denote the Euclidean norm of a vector, and $\odot$ to denote element wise multiplication. We use $(\cdot)^T$, $(\cdot)^H$, and $(\cdot)^\dagger$ to denote transpose, Hermitian, and matrix pseudo-inverse, respectively. The conjugate is represented by
We focus on the interference from the received signal at relay in the corresponding interfering channel vector over subchannel $i$. We consider a cellular system where each cell contains $M$ source-destination pairs and $N$ relays, and all nodes are equipped with a single antenna. A multichannel communication system (e.g., OFDMA) consisting of $M$ subchannels is used in each cell. Each source transmits data to its destination through the relays using a separate subchannel, and transmission among different pairs are orthogonal. We assume that the half-duplex AF protocol is used for relaying, and the direct path is ignored. Different subchannels of each relay may be assigned to multiple source-destination pairs. In this work, we study the interference caused by $N$ relays in one cell (desired) to the $M$ destinations in its neighboring cell.

Assume that the $m$-th source-destination pair communicate through $N$ relays over subchannel $m$. In the desired cell, the received signal at relay $i$ is given by $y_{m,i} = \sqrt{T_0} r_{m,i} s_m + n_{r,m,i}$, where $h_{m,i}$ is the subchannel $m$ between source $m$ and relay $i$, $s_m$ is the transmitted symbol with $\mathbb{E}[|s_m|^2] = 1$, $P_0$ is the transmission power, and $n_{r,m,i}$ is the AWGN with variance $\sigma^2$. Next, the relay $i$ multiplies the received signal over subchannel $m$ with a complex beamweight, denoted by $w_{m,i}$, for forwarding. Let $g_{m,i}$ denote the subchannel $m$ from relay $i$ to destination $m$. Then, the received signal at destination $m$ from all relays is given by

$$ r_m = \sqrt{T_0} g_m^T \hat{W}_m h_m s_m + g_m^T W_m n_{r,m} + n_m $$

(1)

where $h_m \triangleq [h_{m,1}, \cdots, h_{m,N}]^T$, $g_m \triangleq [g_{m,1}, \cdots, g_{m,N}]^T$, $W_m \triangleq \text{diag}(w_{m,1}, \cdots, w_{m,N})$, and $n_{r,m} \triangleq [n_{r,m,1}, \cdots, n_{r,m,N}]^T$ are the channel vectors at the first hop and second hop, the beamweight matrix, and the noise vector, through all relays for the $m$-th source-destination pair, respectively. In addition, $n_m$ is the AWGN at destination $m$ with variance $\sigma^2$. The received SNR at destination $m$ is given by

$$ \text{SNR}_m = \frac{P_0 w_m^H \hat{F}_m w_m}{w_m^H G_m w_m + \sigma^2} $$

(2)

where $w_m \triangleq \text{diag}(W_m)$, $F_m \triangleq (f_m f_m^H)^*$ with $f_m = g_m \circ h_m$, and $G_m \triangleq \sigma^2 \text{diag}((g_m g_m^H)^*)$.

Each transmitting relay causes interference to its neighboring cell. We focus on the interference from $N$ relays in one cell to the $M$ destinations in the neighboring cell. Let $g_m$ denote the corresponding interfering channel vector over subchannel $m$ from $N$ relays of the desired cell to destination $m$ of its neighboring cell. The received interference at destination $m$ of the neighboring cell is given by $\hat{r}_m = \tilde{g}_m^T \hat{W}_m (\sqrt{T_0} h_m s_m + n_r)$. The corresponding received interference power is given by $I_m \triangleq P_0 w_m^H \hat{F}_m w_m + w_m^H G_m w_m$, where $\hat{F}_m \triangleq (f_m f_m^H)^*$, $f_m \triangleq g_m \circ h_m$, and $G_m \triangleq \sigma^2 \text{diag}((g_m g_m^H)^*)$ are the interference corresponding to the forwarded signal and the amplified noise from the relays.

Let $P_r$ denote the total power available at each relay. Power allocation over each subchannel at relay $i$ should satisfy

$$ \sum_{m=1}^M |w_{m,i}|^2 |R_{y,m},i| \leq P_r, \quad \text{where } R_{y,m} \triangleq P_0 h_m h_m^H + \sigma^2 I $$

We assume perfect knowledge of CSI including the interfering channels in designing the relay beamweights of the desired cell. In Section IV, we further study through simulation the case where the interfering CSI is imperfect.

### B. Problem Formulation

Define $R_m \triangleq \text{diag}([R_{y,m},1], \cdots, [R_{y,m},N,N])$, and let $D_i$ denote the $N \times N$ diagonal matrix with 1 in the $i$-th diagonal and zero otherwise. Rewrite the interference power $I_m = w_m^H \tilde{B}_m w_m$, where $\tilde{B}_m \triangleq P_0 \hat{F}_m + G_m$, for $m = 1, \cdots, M$. Our goal is to design the relay beamweights of the desired cell to minimize the maximum interference to its neighboring cell, subject to per-relay power constraint and minimum SNR guarantee. The optimization problem is given by

$$ \min_{\tilde{\theta}} \quad \tilde{\theta} $$

subject to

$$ w_m^H \tilde{B}_m w_m \leq \tilde{\theta}, \quad m = 1, \cdots, M, $$

(3)

$$ \sum_{m=1}^M w_m^H R_m D_i w_m \leq P_r, \quad i = 1, \cdots, N, $$

(4)

$$ \frac{P_0 w_m^H \hat{F}_m w_m}{w_m^H G_m w_m + \sigma^2} \geq \gamma_m, \quad m = 1, \cdots, M. $$

(5)

We use $\tilde{\theta}^\circ$ to denote the minimum objective under the optimal solution.

### III. Minimizing Maximum Per-Subchannel Interference

The solution of the min-max interference problem (3) is provided in this section. We reformulate the problem which leads to a semi-closed-form solution in the Lagrange dual domain. In order to obtain the optimal $\{w_1, \cdots, w_M\}$, an SDP-based algorithm with polynomial worst-case complexity is proposed. Then three cases for the dual variables are studied and the optimal $\{w_1, \cdots, w_M\}$ are determined accordingly.

### A. Strong Duality

Since the SNR constraint (6) is not convex, the problem (3) is non-convex. In the following, we show that (3) has zero duality gap and can be solved in the Lagrange dual domain.

**Proposition 1:** Strong duality holds for the min-max interference problem (3).

**Proof:** We omit the details and provide an outline of the proof. We first show that the problem (3) can be reformulated as a second-order conic programming (SOCP) problem. It is

1We assume that the beamweights of the neighboring cell are also optimized such that the maximum interference in the desired cell is minimized. As future work, we may study how to set a pre-determined maximum interference power for the cells such that the minimum received SNR at the receivers is maximized.
known that the SOCP has zero conic duality gap [17]. Then we show that the Lagrange dual of the problem (3) and the Lagrange dual of the SOCP are equivalent.

Note that the problem (3) could be solved numerically by solving the above equivalent SOCP with $MN + 1$ variables and $2M + N$ constraints. To gain insight into the structure of the beamweights, an efficient algorithm using the Lagrange dual domain is proposed. In the following, we provide a semi-closed form solution of (3) using SDP. Through the proposed algorithm, the structure of the beamweights is derived and the computational complexity is reduced.

**B. The Semi-Closed Form Solution**

Using the results of Proposition 1, we can obtain the optimum solution of (3) through the dual problem. Let $\mu \triangleq [\mu_1, \cdots, \mu_M]^T$, $\lambda \triangleq [\lambda_1, \cdots, \lambda_N]^T$, and $\alpha \triangleq [\alpha_1, \cdots, \alpha_M]^T$ denote the Lagrange multipliers associated with the max interference constraint (4), per relay power constraint (5), and SNR constraint (6), respectively. The Lagrangian of (3) is given by

$$L(\{w_m\}, \theta, \lambda, \mu, \alpha) = \sum_{m=1}^M \alpha_m \sigma^2 + \theta(1 - \sum_{m=1}^M \mu_m) - P_r \sum_{i=1}^N \lambda_i + \sum_{m=1}^M w_m^H (K_m - \omega_m \sum_{m=1}^M f_m^H w_m) w_m,$$

where $K_m \triangleq R_m D_m + \mu_m B_m + \alpha_m G_m$.

and $D_m \triangleq \text{diag}(\lambda_1, \cdots, \lambda_N)$.

The dual problem of the problem (3) is given by

$$\begin{align*}
\max_{\lambda, \mu, \alpha} & \min_{w_m, \theta} L(\{w_m\}, \theta, \lambda, \mu, \alpha) \\
\text{subject to} & \lambda \succeq 0, \mu \succeq 0, \alpha \succeq 0.
\end{align*}$$

We observe that, after the inner minimization of (8), the dual problem (8) is equivalent to

$$\begin{align*}
\max_{\lambda, \mu, \alpha} & \sum_{m=1}^M \alpha_m \sigma^2 - P_r \sum_{i=1}^N \lambda_i \\
\text{subject to} & K_m \succeq \frac{\alpha_m P_0}{\gamma_m} f_m f_m^H, m = 1, \cdots, M \\
& \sum_{m=1}^M \mu_m \leq 1, \\
& \alpha_m \succeq 0.
\end{align*}$$

This is because the constraints (11) and (12) are implicit in the optimal solution of the problem (8). To see this, suppose one of the constraints (11) or (12) is not satisfied. Then there is some $\{w_m, \theta\}$ such that the inner minimization of (8) leads to $L(\{w_m\}, \theta, \lambda, \mu, \alpha) = -\infty$, which is not an optimum solution of (8).

In order to solve the dual problem (10), we first discuss the feasibility of constraint (11) in the following lemma.

**Lemma 1:** If either $\mu_m > 0$ or $\lambda > 0$, then $\alpha_m > 0$, i.e., the Lagrange dual variable associated with the SNR constraint (6) is strictly positive.

**Proof:** We can show that the constraint (11) is equivalent to $R_m D_m + \mu_m B_m + \alpha_m (G_m - \frac{\alpha_m P_0}{\gamma_m} f_m f_m^H) \succeq 0$ and $G_m - \frac{\alpha_m P_0}{\gamma_m} f_m f_m^H$ is an indefinite matrix.

The above lemma provides the condition under which the SNR constraint (6) for the $n$-th source-destination pair is met with equality. In the following, we provide the solution assuming $\alpha > 0$, i.e., the SNR constraint (6) is met with equality for all $m$. The solution for other cases is obtained similarly and is presented in Section III-D.

**Theorem 1:** If $\alpha > 0$, the optimum beamforming vector $w_m^o$ of the min-max interference problem (3) is given by

$$w_m^o = \zeta_m K_m^o f_m$$

where

$$\zeta_m \triangleq \frac{P_0}{\gamma_m} \left( \frac{|f_m^H K_m^o f_m|^2}{P_m^o} f_m^H K_m^o + G_m^o \right)^{\frac{1}{2}}$$

with $K_m^o$ obtained by substituting the optimum dual variables into (7).

**Proof:** See Appendix A.

Note that $w_m^o$ in (13) is a semi-closed form solution, because it still depends on the optimum dual variables $\{\lambda^o, \mu^o, \alpha^o\}$. In the next section, we provide an SDP-based numerical solution to find the dual variables.

**C. The Optimal Dual Variables Through SDP**

To determine the optimum $\{\lambda^o, \mu^o, \alpha^o\}$, instead of solving the dual problem (10) directly, we reformulate it into an SDP problem.

**Proposition 2:** Denote $x \triangleq [\alpha^T, \lambda^T, \mu^T]^T$, $a \triangleq -[\sigma^2 1_M \times 1_N, P_r 1_{M \times 1}, 0_{1_M \times 1}]^T$ and $b \triangleq [0_{(M+N) \times 1}, 1_{M \times 1}]^T$. The dual problem (10) can be re-expressed as the following SDP

$$\begin{align*}
\min_x & a^T x \\
\text{subject to} & \sum_{i=1}^{2M+N} x_i \Psi_m \succeq 0, m = 1, \cdots, M, \\
& x \succeq 0, \ b^T x \leq 1
\end{align*}$$

where $\Psi_m = \sum_{m=1}^{2M+N} \frac{P_0}{\gamma_m} f_m f_m^H - G_m$, $\Psi_m, M + j = -R_m D_m$ for $j = 1, \cdots, N$, $\Psi_m, M + N = B_m$ for $m = 1, \cdots, M$, and all other $\Psi$ are zeros.

Standard SDP softwares such as SeDuMi can be used to solve (15). Note that the original problem (3) with $2M + N$ constraints and $MN + 1$ variables is converted to an SDP problem with $M + 2$ constraints and $2M + N$ variables. In addition to reducing the computation complexity, the semi-closed form solution (13) shows the structure of optimum beamweights.

**D. Three Cases of Dual Variables**

In the following, we partition the set of optimum dual variables in the dual problem (10) into three cases and propose an algorithm to obtain the optimum beamforming vectors (if existent) for each case.
1) Case 1: If \( \mu_m^o = 0 \), for \( m = 1, \ldots, M \), there is no solution for the original problem (3). In other words, there should be at least one active constraint (4). This case happens due to the infeasibility of (3), i.e., either the minimum SNR guarantees (6) cannot be achieved or per relay power exceeds the given threshold in (5).

In the following, we assume \( \mu_m^o > 0 \) for some \( m \).

2) Case 2: If \( \forall m \in \{1, \cdots, M\}, \mu_m^o > 0 \) or \( \lambda_i^o > 0 \), we have \( \alpha_m^o > 0 \) for \( m = 1, \cdots, M \) in (10). In other words, if \( K_m - \alpha_m^o G_m > 0 \), then \( \alpha_m^o > 0 \), for all \( m \), and the solution is given by Theorem 1.

3) Case 3: If \( \alpha_m^o = 0 \) for some \( m \), i.e., there exists \( m \) such that \( \mu_m^o = 0 \) and \( \lambda_i^o \neq 0 \) as shown in Lemma 1, we cannot use (13) for \( m = 1, \cdots, M \). Let \( \tilde{m} \) denote the pair with \( \mu_{\tilde{m}}^o > 0 \). For simplicity, suppose \( \mu_m^o = 0 \) for \( m \neq \tilde{m} \) and \( \lambda_i^o = 0 \) for some \( i \). Suppose that \( \alpha_{\tilde{m}}^o > 0 \) and \( \alpha_m^o = 0 \) for \( m \neq \tilde{m} \). We can simply extend our solution to the case in which \( \alpha_{\tilde{m}}^o > 0 \) for arbitrary \( m \)'s. Similar to the proof of Theorem 1, we have \( \frac{\alpha_{\tilde{m}}^o}{\Delta m} \sum M_i \frac{F_i}{m} f_{m} = 1 \). Hence, we can use the solution (13) to obtain the beamforming vector of \( \tilde{m} \).

Then assuming the original problem (3) is feasible, we have \( \tilde{\theta}^o = \alpha_{\tilde{m}}^o \sigma^2 - P_{\tilde{r}} \sum M_i \lambda_i^o \). Let \( M \Delta \{1, \cdots, M\} \{\tilde{m}\} \). In order to obtain the beamforming vectors for \( m \neq \tilde{m} \), we need to solve the following feasibility problem

\[
\begin{align*}
\text{find} & \quad w_1, \cdots, w_{\tilde{m}-1}, w_{\tilde{m}+1}, \cdots, w_M \\
\text{subject to} & \quad w_m^H \tilde{B}_m w_m < \tilde{\theta}^o, m \in M, \\
& \quad \sum M_i w_m^H R_m D_i w_m \leq P_r, i = 1, \cdots, N, \\
& \quad \frac{P_0 \sum M_i F_m w_m}{w_m^H G_m w_m + \sigma^2} \geq \gamma_m, m \in M.
\end{align*}
\]

Note that we can always scale \( w_m \) such that (20) meets with equality for \( m \neq \tilde{m} \). Among the infinite set of possible solutions of \( w_m \) for \( m \neq \tilde{m} \), we propose to extract one using the following algorithm. The essence of this algorithm is to remove the \( \tilde{m} \)-th pair from consideration and solve the resultant min-max interference problem to find the optimum beamweights associated with the other pairs.

Denote \( w_m^o \tilde{H} R_m D_i, w_m^o \Delta e_i \) for \( i = 1, \cdots, N \). We can solve the following problem

\[
\begin{align*}
\text{min} & \quad \delta \\
\text{subject to} & \quad w_m^H \tilde{B}_m w_m \leq \delta, m \in M, \\
& \quad \sum M_i w_m^H R_m D_i w_m \leq P_r - e_i, \forall i, \\
& \quad \frac{P_0 \sum M_i F_m w_m}{w_m^H G_m w_m + \sigma^2} \geq \gamma_m, m \in M.
\end{align*}
\]

Let \( \delta^o \) denote the optimum value of (21), and suppose \( \alpha_{\tilde{m}}^o > 0 \). If \( \delta^o < \tilde{\theta}^o \), then we can find \( w_m^o \). If \( \delta^o \geq \tilde{\theta}^o \), then (3) is infeasible. In the following, the SDP to obtain the optimum dual variables of (21) is summarized. Define

\[
\begin{align*}
\tilde{a}_1 & \triangleq [\tilde{a}_1^T, P_r - e_1, \cdots, P_r - e_N, \tilde{a}_2^T]^T, \\
\tilde{b}_1 & \triangleq [\tilde{b}_1^T (M_i+N+1)^+ 	ilde{b}_1^T]^T
\end{align*}
\]

where \( \tilde{a}_1 \in \mathbb{R}^{M \times 1}, \tilde{a}_2 \in \mathbb{R}^{M \times 1}, \) and \( \tilde{b}_1 \in \mathbb{R}^{M \times 1} \) are obtained by substituting \( a_{\tilde{m}} = 1, a_{M_i+N+\tilde{m}} = 1 \) (or any arbitrary positive value), and \( b_{M_i+N+\tilde{m}} = 0 \), respectively.

The dual problem is equivalent to

\[
\begin{align*}
& \min_x \tilde{a}^T x \\
& \text{subject to} \quad \sum_{i=1}^{2M+N} x_i \Psi_{m,i} \leq 0, m = 1, \cdots, M. \\
& \quad x \succeq 0, \tilde{b}^T x \leq 1
\end{align*}
\]

Algorithm 1 Minimizing the maximum interference

1: Solve the SDP problem (15) finding \( \alpha^o, \mu^o, \lambda^o \)

2: Obtain \( \tilde{\gamma} \{m \mid \alpha_m^o > 0\} \)

3: if \( \tilde{\gamma} = \{1, \cdots, M\} \) then

4: Compute \( K_m^o \) (7)

5: Compute the coefficient \( \zeta_m \) (14) and \( w_m^o \) (13), \( \forall m \)

6: else

7: Find \( w_m^o \) (13) for all \( m \in \gamma \)

8: Update \( \tilde{a} \) (25), \( \tilde{b} \) (26), and \( \Psi_{m,i} \)

9: Solve (27) finding \( l \in \{1, \cdots, M\} \backslash \gamma \) with \( \alpha_l^o > 0 \)

10: Compute \( \zeta_l, w_l^o \) (13), and update \( \gamma = \gamma \cup \{l\} \)

11: end if

IV. NUMERICAL RESULTS

In this section, we provide simulations results to evaluate the performance of the proposed min-max interference algorithm. In the simulations, we set \( \sigma_2^2 = \sigma_2^2 = 1, P_0/\sigma_2^2 = 10 \) dB, and \( P_r/\sigma_2^2 = 20 \) dB. The number of feasible realizations is set to 500. The minimum SNR targets are set to \( \gamma_m = 5 \) dB for \( m = 1, \cdots, M \). The channel vectors \( h_m \) and \( \{\gamma_m, \gamma_m\} \) are assumed i.i.d. zero-mean Gaussian with variance 1.

To study the behavior of the maximum interference as the number \( N \) of relays increases, we plot the CDF of the maximum interference power under various \( N \) in Fig. 1. Also shown in Fig. 1 is the maximum interference under the optimization problem where the objective is to minimize the maximum transmission power over all relays while meeting the minimum SNR guarantees. The min-max relay power problem can be solved using a similar technique to the one proposed in this paper. The number of antennas are chosen...
as $N = 2^i$ for $i \in \{0, \cdots, 5\}$. It can be noticed that as $N$ increases, the maximum interference CDF curves are shifted to the left for both optimization objectives. It can be seen that the curves for the min-max interference do not converge as $N$ becomes very large. In fact, those curves are uniformly shifted to the left. Note that the min-max interference approach outperforms the per-relay power approach for each $N$, and the performance gap increases as $N$ increases.

So far, true interference CSI is assumed to be known perfectly at the relays. In practice, obtaining such interference CSI may not be possible. In order to observe how robust the proposed algorithm is with respect to imperfect CSI, we consider the following scenarios with two types of imperfect CSI, i.e., limited number of CSI feedback (FB) bits and channel estimation error.

In Scenario 1, the receiver knows the interference CSI perfectly. However, the FB bits to the relays are limited. We consider equiprobable quantization of channel values. Let $B$ denote the number of available FB bits. Every real and imaginary part of a channel is quantized with equal probability according to the channel.

In Scenario 2, the channels are estimated at the receiver with error and the exact estimated channel is fed back to the relays. In order to model the channel estimation error, let us define $\tilde{h} = h + \alpha \hat{h}$, where $h$ is the true channel coefficient, $\hat{h}$ is the estimated channel coefficient used for optimization, and $\hat{h} \sim \mathcal{CN}(0, 1)$ is the error. The coefficient $\alpha$ is set to adjust the variance of error.

In Fig. 2, the CDF of the maximum received interference under true interference CSI is compared with that of Scenario 1 with $B = 6$. It can be seen the performance of the maximum received interference in Scenario 1 is very close to the that of true CSI even for $N = 8$. As expected, the performance gap between the limited FB case and the true CSI case increases as $N$ increases. In addition, the min-max interference under limited FB still outperforms the min-max per-relay objective in terms of maximum received interference.

Finally, Fig. 3 shows the CDF of the maximum received interference under true interference CSI as compared with that of Scenario 2 with estimation error for $\alpha = 0.01$. The performance under Scenario 2 is very close to that of true CSI. As expected, performance degrades as $\alpha$ increases, and is very poor when $\alpha = 0.5$, i.e., estimation error’s variance is half of the variance of true CSI. In addition, we see that the performance gap between Scenario 2 and the true CSI case increases as $N$ increases.

V. CONCLUSION

In this paper, we have considered a multi-relay cellular network, where each cell has multiple source-destination pairs communicating in orthogonal channels with assistance from the relays. In order to manage inter-cell interference, we have formulated the min-max interference problem, under per-relay power and guaranteed received SNR constraints. We have shown that the strong duality property holds for this non-convex problem. Using the Lagrange dual domain, an efficient SDP-based algorithm has been proposed to find the optimum relay beamweights.

APPENDIX A

PROOF OF THEOREM 1

Since $\alpha > 0$, we have $K_m > 0$ for $m = 1, \cdots, M$. Using [15, Lemma 1] and rewriting the expression of the matrix
Note that $\gamma_m > 0$, the SNR constraint (6) is met with equality based on the slackness condition. Substituting $w_\bar{o}$ into the equation $P_0 w_\bar{o}^H \gamma_m \otimes w_\bar{o} + \sigma_m^2 = \gamma_m$ and after some manipulations, (14) is obtained and the proof is complete.

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