Adaptive multichannel filters for colour image processing

K.N. Plataniotis*, D. Androutsos, A.N. Venetsanopoulos

Department of Electrical and Computer Engineering, University of Toronto, 10 King's College Rd, Toronto, Ont., Canada M5S 3G4

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Abstract

This paper addresses the problem of noise suppression for multichannel data, such as colour images. The proposed filters utilize adaptive data dependent nonparametric techniques. Simulation results indicate that the new filters suppress impulsive as well as Gaussian noise and preserve edges and details. © 1998 Elsevier Science B.V.

Keywords: Colour image processing; Adaptive multichannel filters; Nonparametric techniques

1. Introduction

Filtering of multichannel images has received increased attention due to its importance in processing colour images. In different colour spaces, each image pixel is represented by three values. These three values can be considered as forming a vector. Therefore, a colour image is represented by a vector field, in which each vector's direction and length is related to the pixel's chromatic properties. The different filters applied to colour images are required to preserve edges and details and remove impulsive and Gaussian noise [12].

A number of multichannel filters have been proposed to date for image filtering. Among them are the Vector Median Filter (VMF), the Vector Directional Filter (VDF) [4, 11] and the Fuzzy Vector Directional Filter (FVF) [6, 7]. The large number of filters available poses some difficulties to the practitioner, since most of them are designed to perform well in a specific application and their performance deteriorates rapidly under different operational scenarios. Existing nonlinear filters, such as the GVDF [11] or the FVDF1 [6] obtain their best performance by utilizing a priori information regarding the noise characteristics to tune their parameters accordingly. However, such filters may not provide the expected filtering results in a realistic application scenario since in practice information regarding the actual noise characteristics is seldom available. Lately, heuristically derived filters, such as the so called Distance Dependent Multichannel Filter (DDMF) have also been introduced [2]. These 'averaging-type' filters utilize ad hoc defined parameters to generate the filtered output. Since, however, in an actual application the original noise-free image is never available for the post-processing analysis needed to 'guide' the selection of the ad hoc parameters, filters, such as the DDMF are of no use.

*Corresponding author. Now with the Department of Mathematics, Physics and Computer Science, Ryerson Polytechnic University, Toronto, Canada. E-mail: kplatani@acs.ryerson.ca; tel.: (416) 979-5000 ext 7062; fax: (416) 979-5064.
Thus, a nonlinear adaptive filter, which performs equally well in a wide variety of applications, is of great importance. Our goal is to devise a simple, computationally efficient and reliable filter structure, which will deliver acceptable results without making any assumption about signal or noise characteristics.

2. Adaptive multichannel filter

Consider the following model for the colour image degradation process:

\[ y_j = x_j + n_j, \quad (1) \]

where \( x_j \) is a three-dimensional uncorrupted image vector, \( y_j \) is the corresponding noisy vector to be filtered and \( n_j \) is an additive noise vector. In our analysis, it is assumed that the colour image vectors are unknown and that the noise vectors are uncorrelated at the different image locations and signal independent.

Let us denote with \( \hat{x}(y) \) the minimum variance estimator of the colour vector \( x \) given the noisy measurement vector \( y \). The expected square error in the filter when the image vectors are corrupted by additive noise as in Eq. (1), can be written as follows:

\[ V = \int_{-\infty}^{\infty} \left( \int_{-\infty}^{\infty} (x - \hat{x}(y))(x - \hat{x}(y))^T f(x|y) \, dx \right) f(y) \, dy, \quad (2) \]

where \( z^T \) denotes the transpose of \( z \). Since \( \hat{x}(y) \) does not enter into the outer integral and \( f(y) \) is always positive, it is sufficient for the optimal minimum variance estimator to minimize the expected value of the estimation cost (conditional Bayesian risk), given the observation \( y \). Thus, it is sufficient to minimize the quantity

\[ V_{BR} = \int_{-\infty}^{\infty} (x - \hat{x}(y))(x - \hat{x}(y))^T f(x|y) \, dx. \quad (3) \]

The minimum variance estimator which minimizes the above cost is then known to be [10]

\[ \hat{x}(y)_{mv} = \int_{-\infty}^{\infty} x f(x|y) \, dx = \int_{-\infty}^{\infty} x f(x,y) \, dx, \quad (4) \]

with

\[ f(y) = \int_{-\infty}^{\infty} f(x,y) f(x) \, dx. \quad (5) \]

If the densities in (Eq. (4)) are known and a training record of the sample pairs \((x,y)\) is available, the minimum variance estimator can be derived. Unfortunately, in a realistic image processing scenario, no a priori knowledge about the noise process or the image itself is available. Thus, a non-parametric estimator must be utilized to approximate the probability density functions (PDF) in Eq. (4). Let us assume a window of finite length \( n \) centered around a noisy vector \( y \). Through this window a set of multivariate noisy samples \( W = (y_1, y_2, y_3, \ldots, y_n) \) become available. Based on the \( W \) set, an adaptive data dependent multivariate kernel estimator can be devised to approximate the densities in (4). The form of the adaptive kernel estimator selected, is as follows [1]:

\[ \hat{f}(x,y) = (n^{-1}) \sum_{i=1}^{n} (h_i)^{-\frac{M}{2}} K \left( \frac{y - y_i}{h_i} \right), \quad (6) \]

where \( y_i \) is the \( l \)th training vector, with \( l = 1, 2, 3, \ldots, n \), \( M = 3 \) is the dimensionality of the measurement space and \( h_i \) is the data dependent smoothing parameter which regulates the shape of the kernel. The variable kernel density estimator exhibits local smoothing which depends both on the point at which the density is evaluated and information local to each observation in the \( W \) set. The function \( h_i \) could be any function of the sample size \( n \) [3]. In this paper, the smoothing factor is defined as a function of the aggregate distance between the local observation under consideration and all the other vectors inside the \( W \) set, excluding the point at which the density is evaluated. Thus,

\[ h_i = n^{-k/M} A_i = n^{-k/M} \left( \sum_{j=1}^{n} |y_j - y_i| \right), \quad (7) \]

with \( y_j \neq y_i \) for \( \forall y_j \). \( j = 1, 2, \ldots, n \), \( |y_j - y_i| \) is the absolute distance (\( L_1 \) metric) between the two vectors and \( k \) is a parameter to be determined. The multiplier \( n^{-k/M} \) was adopted from [3]. A value of \( k = 0.33 \) in our simulation studies, where \( (0.5 > k > 0) \), guarantees the asymptotic consistency \( (\lim_{n \to \infty} (nh_i^2 M(n)) = \infty \), uniform consistency
and asymptotic unbiasedness \( \lim_{n \to +\infty} (h^M(n)) = 0 \) [3]. The selection of the \( A_t \) does not affect the asymptotic properties of the estimator in Eq. (7). However, for a finite number of samples, as in our case, the function \( A_t \) is the dominant parameter which determines the goodness of the approximation. The choice of the kernel function in Eq. (7) is not nearly as important as the smoothing factor. For the simulation studies in this paper the multivariate extension of the exponential kernel \( K(z) = \exp(-|z|) \) was selected [3].

Given Eqs. (4) and (7), the non-parametric estimator can be defined as

\[
\hat{x}(y)_{np} = \int_{-\infty}^{\infty} x f(x,y) \, dx = \sum_{l=1}^{n} x_l \left( \frac{(n-1)h^{-M}K((y-y_l)/h_l)}{\sum_{l=1}^{n} h_l^{-M}K((y-y_l)/h_l)} \right), \tag{8}
\]

\[
\hat{x}(y)_{np} = \sum_{l=1}^{n} x_l \left( \frac{h_l^{-1}MK((y-y_l)/h_l)}{\sum_{l=1}^{n} h_l^{-1}M K((y-y_l)/h_l)} \right), \tag{9}
\]

where \( y_l \in W \) and \( w_l(y) \) is a weighting function defined in the interval \([0,1]\).

To obtain the required estimate we must assume that, in the absence of noise, discrete sample vectors \( x_l \) are available. This is not a severe restriction since in many cases such samples may be obtained by a calibration procedure in a controlled environment, perhaps at a very high signal-to-noise ratio. In a real time image processing application, however, that is not the case. Therefore, alternative suboptimal solutions are introduced. In a first approach, we substitute the vectors \( x_l \) in Eq. (9) with their noisy measurements. The resulting adaptive nonparametric multichannel filter (hereafter ANMF1) is solely based on the available noisy vectors and the form of the minimum variance estimator. Thus, the form of the ANMF1 is as follows:

\[
\hat{x}(y)_{ANMF1} = \sum_{l=1}^{n} x_l \left( \frac{h_l^{-1}M K((y-y_l)/h_l)}{\sum_{l=1}^{n} h_l^{-1}M K((y-y_l)/h_l)} \right), \tag{11}
\]

A different form of the adaptive nonparametric estimator can be obtained if a reference vector \( x_{lr} \) is used instead of the actual noisy measurement in Eq. (10). The ideal reference vector is of course the actual value of the multidimensional signal in the specific location under consideration. However, since the \( x_l \) vector is not available a robust estimate of the location, usually evaluated in a smaller subset of the input vector set, is utilized instead. Usually the median is the preferable choice since it smooths out impulsive noise and preserves edges and details. However, unlike scalars, the most central vector in a set of vectors can be defined in more than one way. Thus, the Vector Median Filter (VMF) or the marginal median filter (MAMF) operating in a \( 3 \times 3 \) window centered around the current pixel can be used to provide the requested reliable reference. In this paper, the VMF evaluated in a \( 3 \times 3 \) window was selected to provide the reference vector. The new Adaptive Nonparametric Multichannel Filter (hereafter ANMF2) has the following form:

\[
\hat{x}(y)_{ANMF2} = \sum_{l=1}^{n} x_l^{VMF} \left( \frac{h_l^{-1}M K((y-y_l)/h_l)}{\sum_{l=1}^{n} h_l^{-1}M K((y-y_l)/h_l)} \right). \tag{12}
\]

The ANMF2 can be viewed as a double-window two stage estimator. First the original image is filtered by a multichannel median filter in a small processing window in order to reject possible outliers and then an adaptive nonlinear filter with data dependent coefficients defined in Eq. (9) is utilized to provide the final filtered output. The ANMF2 filter can be viewed as an extension to the multichannel case, of the double-window (DW) filtering structures extensively used for gray-scale image processing [5]. As in gray-scale processing, with this adaptive filter, we can distinguish between two operators: (i) the computation of the median in a, usually, smaller window; and (ii) the adaptive averaging in a second processing window.

3. Application to colour images

The new filters are compared quantitatively with the widely used vector median filter (VMF), the arithmetic vector mean filter (AVMF) and
chromaticity based filters, such as the basic vector directional filter (BVDF), the generalized vector directional filter (GVDF) [11], the distance directional filter (DDF) [4] and the fuzzy vector directional filter (FVDF3) of [7]. The RGB colour test image 'Lenna' has been contaminated using various noise source models in order to assess the performance of the filters under different noise distributions (see Table 1). In addition, the filters are compared with the heuristically derived DDMF1 filter discussed in [2]. Given the fact that for such filters no justification for the selection of the design parameters is available we select the DDMF1 with the artificially defined parameters $\alpha = 5$, $\beta = 0.005$ and $d_{\text{max}} = 176.67$ which seems to provide the best results, from this class of filters, for the different noise scenarios summarized in Table 1.

The normalized mean square error (NMSE) has been used as quantitative measure for evaluation purposes. It is computed as

$$\text{NMSE} = \frac{\sum_{i=0}^{N_1} \sum_{j=0}^{N_2} \| \mathbf{x}(i,j) - \hat{\mathbf{x}}(i,j) \|^2}{\sum_{i=0}^{N_1} \sum_{j=0}^{N_2} \| \mathbf{x}(i,j) \|^2},$$

(13)

where $N_1$, $N_2$ are the image dimensions, and $\mathbf{x}(i,j)$ and $\hat{\mathbf{x}}(i,j)$ denote the original image vector and the estimation at pixel $(i,j)$, respectively. Table 2 summarizes the results obtained for the test image 'Lenna' for a $3 \times 3$ filter window. The results obtained using a $5 \times 5$ filter window are given in Table 3.

In many application areas, such as multimedia, telecommunications (e.g. TV), production of motion pictures, printing industry and graphic arts, greater emphasis is given to perceptual image quality. Consequently, the perceptual closeness (alternatively perceptual difference or error) of the filtered image to the uncorrupted original image is ultimately the best measure of the efficiency of any colour image filtering method.

This leads us to the question of how to estimate the perceptual error between two colour vectors. Precise quantification of the perceptual error between two colour vectors is one of the most important and open research problems. RGB is the most popular colour space used conventionally to store,
The human perception of colour cannot be described using the RGB model since it is far from exhibiting perceptual uniformity. Therefore, measures such as the normalized mean square error (NMSE) defined in the RGB colour space are not appropriate to quantify the perceptual error between images. Thus, it is important to use colour spaces, which are closely related to the human perceptual characteristics and suitable for defining appropriate measures of perceptual error between colour vectors. A number of such colour spaces are used lately in areas such as computer graphics, motion pictures, graphic arts, and printing industry. Among these, perceptually uniform colour spaces are the most appropriate to define simple yet precise measures of perceptual error. The Commission Internationale de l’Eclairage (CIE) standardized two colour spaces, L*u*v* and L*a*b*, as perceptually uniform. The L*a*b* colour space is chosen for our analysis [8, 9].

In a uniform colour space, such as L*a*b*, we computed the normalized colour difference (NCD) which is estimated according to the following formula:

\[
NCD = \frac{\sum_{i=0}^{N1}\sum_{j=0}^{N2}||\Delta E_{ab}||}{\sum_{i=0}^{N1}\sum_{j=0}^{N2}||E^*_{ab}||},
\]

(14)

where \(E^*_{ab}\) is the square of the norm or magnitude of the uncorrupted original image pixel vector in the \(L^*a^*b^*\) space and \(\Delta E_{ab}\) is the difference between the original image and the filtered result at the specific image location \((i,j)\) defined as follows:

\[
\Delta E_{ab}(i,j) = (L(i,j) - \hat{L}(i,j))^2 + (a(i,j) - \hat{a}(i,j))^2 + (b(i,j) - \hat{b}(i,j))^2.
\]

(15)

Tables 4 and 5 summarize the results obtained for the test image 'Lenna'.

In addition to the quantitative evaluation presented above, a qualitative evaluation is necessary since the visual assessment of the processed images is, ultimately, the best subjective measure of the efficiency of any method. Therefore, we present sample processing results in Figs. 1 and 2. Fig. 1 shows the colour image 'Lenna', corrupted with additive (4%) impulsive noise. Fig. 2 presents the result using the ANMF2 with a 3 x 3 processing window.

From the results listed in the tables we can conclude that our adaptive designs attenuate both impulsive and additive Gaussian noise with or without outliers present in the test image. It must be noted that no assumption about the noise characteristics is needed in their derivation and that the ANMF filters generate the best results using no
information about the type and the degree of noise corruption. The new adaptive filters outperform filters, such as the GVDF which uses a priori knowledge about the actual noise characteristics to optimize its performance [6] and greatly outperform heuristically derived filters, such as the DDF1, preserving at the same time the chromaticity component as we can see from the chromaticity related errors summarized in Tables 4 and 5.

4. Conclusions

Adaptive multichannel filters based on non-parametric density estimators were introduced in this paper. The filters smooth noise under different scenarios, outperforming other widely used multichannel filters. The adaptive filters can effectively remove impulses, smooth out nominal noise and keep edges and details unchanged. Moreover, the proposed adaptive filters are suitable for perceptually lossless image processing applications since the filtered output is perceptually close to the uncorrupted original.

References

Fig. 2. ANMF2 of (1) with a $3 \times 3$ window.


