Analysis of LDPC Decoding for Correlated and Uncorrelated Block Fading Channels

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Abstract — **This paper presents a density evolution analysis of the sum-product algorithm used for channel estimation and decoding of low density parity check (LDPC) codes on correlated and uncorrelated two-state block fading channels. The channels under consideration use binary symmetric channels and binary-input Gaussian channels as components, and the thresholds for regular LDPC codes on these channels are calculated. The analysis shows that for both correlated and uncorrelated block fading channels, the threshold increases with the memory length. If the memory length is fixed, introducing correlation between successive blocks increases the threshold; as the memory length increases, this effect diminishes.**

The focus of this paper is joint channel estimation and decoding using LDPC codes [1] on *block fading channels*. Also known as *block interference channel*, the block fading channel is a channel with memory in which the channel state remains fixed over a *block* of given size [2]. It was assumed that the channel states from one block to the next were independent in [2]. This paper considers a more general model where it permits state correlation between consecutive blocks.

Joint LDPC decoding and channel estimation on a two-state block fading channel was first investigated in [4]. Analysis of joint LDPC decoding and channel estimation on the Gilbert-Elliott channel was done using the density evolution [3] technique in [5]. The density evolution algorithm is generalized to block fading channels in this work and is used to assess the thresholds for $(3,6)$ regular LDPC codes for the following two-state channels:

- (1a) a block fading channel comprised of two binary symmetric channels, with state independence between blocks;
- (1b) a block fading channel comprised of two binary symmetric channels, with state correlation between blocks;
- (2a) a block fading channel comprised of two binary-input Gaussian channels, with state independence between blocks.

For channel 1b, defining the *memory* between states as $\mu = 1 - b - g$. In the experiments, $\mu = 0.96$. To make channel 1a comparable to channel 1b, we set $p_B = b/(b+g)$ and $p_G = g/(b+g)$ - i.e., both channels have the same marginal distribution on good and bad states. The inversion probability in the "bad" state is set at $\eta_B = 0.5$. Here b and g denote the transition probabilities between good state and bad state, and p_G (p_B) denotes the probability of a channel state being in a good (bad) state.

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Table 1: Thresholds (η_G^*) for block fading channels with $\eta_B = 0.5$, $p_B = 0.1$, $p_G = 0.9$, $b = 0.004$, $g = 0.036$

	$h=5$	$h=10$	$h=15$	$h=20$
uncorrelated	0.052	0.059	0.062	0.064
correlated	0.066	0.067	0.067	0.067

Table 2: Thresholds (σ^*, in dB) for Gaussian block fading channel with $p_B = 0.4$, $p_G = 0.6$, $\alpha_B = 0.45$, $\alpha_G = 1.18$

	$h=5$	$h=10$	$h=15$	$h=20$
uncorrelated	2.92	2.80	2.70	2.66

Table 1 presents the threshold results for channels 1a and 1b with different value of block memory, denoted by h. Table 2 presents threshold results for the block fading channel comprised of two binary-input Gaussian channels with multiplicative amplitude fading; for these results, we used $p_B = 0.4$ and $p_G = 0.6$ and fading levels $\alpha_B = 0.45$ and $\alpha_G = 1.18$.

The results show that the threshold improves as the memory length h increases. Introducing memory between component channels results in better thresholds. However, as h increases, this difference in threshold decreases. Intuitively, this is because the channel estimation obtained using just the information within a block is quite good when h is large, so the additional channel information provided by the correlation between component channels is of diminishing importance.

References

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