A Cyber-Enabled Stabilizing Control Scheme for Resilient Smart Grid Systems
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Abstract—A parametric controller is proposed for transient stability of synchronous generators after the occurrence of a disturbance in the power grid. The proposed controller based on feedback linearization control theory relies on receiving timely phasor measurement unit (PMU) information from selected parts of the power grid to employ fast acting flywheels that are situated near synchronous generators. The local storage devices aim to balance a swing equation model of the synchronous generator to drive the associated rotor speed to stability. The advantages of the proposed controller include that it is tunable and integrates well with existing governor controls in contrast to other forms of PMU-based control. Further, a comparison is drawn between the proposed controller and recently proposed nonlinear controllers for transient stabilization. Numerical results show the effectiveness and robustness of the proposed controller when applied to the 39-bus 10-generator New England power system.

Index Terms—Cyber-physical systems, distributed control, phase cohesiveness, smart grid, system resilience, transient stability.

I. INTRODUCTION

SMART GRID systems employ advanced data acquisition, communications, and control to enable increased efficiency, capacity, and consumer centricty of power delivery. In addition, they facilitate the integration of new forms of generation including renewable sources thus helping to lower society’s carbon footprint. Given this modern vision for power delivery, natural questions arise as to how the effects of disturbances on system operation should be mitigated. Specifically, the greater dependence on lower inertia renewable sources makes the power grid more susceptible to incidental disturbances in the form of common system faults and natural disasters [1]–[3]. Moreover, the greater dependence on information systems increases opportunities for cyber-attack and coordinated cyber-physical disturbances [4]–[6].

In this paper, we investigate how distributed storage units, advanced sensors and communications can be leveraged for improving system resilience through advanced control. These entities represent a valuable and expanding asset base within smart grid systems that can be leveraged to better mitigate both natural (physical) faults and intentional (cyber) attacks.

We assert that these cyber-physical disturbances must be addressed through a defense-in-depth paradigm whereby prevention, detection, and reaction approaches for protection are simultaneously employed at various levels. Preventative approaches aim to obstruct the impact of a disturbance by making it impossible to be carried out, as for example, in the case of a cyber-attack, or by immediately isolating the associated fault. Examples of preventative strategies are encryption and secure communication protocols that represent an initial level of security against cyber-intrusions [4], [7]. Relays and circuit breakers are also a form of initial defense to prevent the propagation of a severe fault [8], [9]. Detection is employed when prevention is unsuccessful in thwarting a disturbance; these strategies make use of system measurement and models of (ab)normal behavior for the identification of unwanted anomalies. Such techniques can be used to detect the occurrence of an unwanted system state [10], successful cyber-attack [11], or a combination of both [12]. Reaction entails strategies to recover from a disturbance and include approaches to control system operation [13]. In this paper, we focus on this last approach, specifically, to enhance system resilience through the use of distributed control.

Recently, a distributed control paradigm based on flocking theory was proposed by Wei et al. [14]–[16] and Wei and Kundur [17]. The framework represents an analogy for transient stabilization providing a rich theoretical foundation upon which to prove stability under model assumptions. However, the controller can be costly computationally and demonstrates a graceful yet slow time scale for stabilization as communication latency grows. Consequently, questions arise as to whether more aggressive strategies exist that can drive the power system to stability in a shorter period of time.

This paper proposes an agile low-complexity tunable distributed controller that easily integrates with generator governors. When the power system undergoes transient instability, the proposed solution utilizes state information to execute a feedback linearization controller that synchronizes the generators more aggressively. Feedback linearization is a well-known approach that converts a nonlinear system (plant) into an equivalent linear system through controller design that aims to cancel out all (or part) of the nonlinear dynamics. Previously, it has been investigated in [18] to control the excitation system of the generators; however, our proposed...
solution utilizes external storage to stabilize the rotor speed and achieve phase cohesiveness among the generators.

The rest of this paper is organized as follows. The problem setting is presented in Section II and the proposed controller is detailed in Section III. Section IV investigates the performance of the proposed controller. A study of control performance under practical limitations of the communication and information system is presented in Section V followed by the conclusion in Section VI.

II. DISTRIBUTED CONTROL SETTING

We assume that the smart grid is comprised of $N$ agents whereby each agent is comprised of the following.

1) A synchronous generator.
2) An associated phasor measurement unit (PMU) that provides measurements of the generator rotor angle and speed.
3) A distributed controller that processes PMU data from local and neighboring agents.
4) A fast-acting storage device that can inject or absorb real power in the system depending on the value of the control signal, specifically, the controller actuates the local fast-acting storage entity such as a flywheel or other distributed storage source.

A communication network connects the PMUs and distributed controllers.

We consider the physical dynamics of each agent to depend on its own state (specifically, the state of its synchronous generator) as well as the states of other agents in the overall multiagent system. In such setting, a centralized controller would require that all agent states be transmitted to a common location for processing and decision-making requiring significant communication overhead raising scalability issues. In contrast a decentralized controller would only require the state of its own agent eliminating the need for significant communication; however, such an approach may experience long convergence times for the controller tasks. In this paper, we consider a distributed control paradigm where each controller makes use of its own local state and those of its agent-neighbors that represent a subset of the remaining $N - 1$ agents of the system. We assert that such a system balances the communication requirements with convergence speed. Mathematically, distinctions amongst the three approaches can be represented as [19]

$$U_i = \begin{cases} U_i(\Sigma_j) & \text{centralized control} \\ U_i(x_i, \Sigma_i) & \text{distributed control} \\ U_i(x_i) & \text{decentralized control} \end{cases}$$

where $U_i$ is the output of the controller at agent $i$, $x_i$ is the state of agent $i$, $\Sigma_j$ is the state of the neighbor agents of agent $i$, and $\Sigma_i$ is the state of all agents in the system.

The overall multiagent system is considered to be cyber-physical in nature whereby the PMUs, distributed controllers and associated communication infrastructure form the cyber-resources and the synchronous generators and associated power system devices including fast-acting sources and storage represent the physical elements. The physical-to-cyber interface occurs at the sensors that convert physical measurements to digital information while the cyber-to-physical junction is between the controllers and fast-acting sources.

As an example, we consider the New England 10-generator 39-bus (physical) power system with associated (cyber) infrastructure as shown in Fig. 1(a). It should be noted that although $N = 10$ for this system, generator 10 at bus 39 represents an aggregation of a large number of power generators. The generator parameters are defined in Table I, where $\delta_i$ is expressed in radians, $M_i$, $T_i$, and $D_i$ are expressed in seconds, and the remaining are in per units quantities.

We employ the swing equation model to describe physical synchronous generator dynamics. The rotor angle and speed states of such a model enable the study of transient stability. To address the physically networked nature of the power system,
we make use of Kron reduction to reduce the order of the interconnections and determine effective mutual couplings between the synchronous generators. Kron reduction is a graph-based technique used in power systems to reduce the order of an interconnected system [20]. In this process, the Kron reduction transforms a complex power system into an equivalent grid between the generators of the power system.

The relative normalized rotor angular speed of generator $i$ is defined as $\omega_i = (\omega_i^{act} - \omega_i^{nom})/\omega_i^{nom}$, where $\omega_i^{nom}$ is the nominal angular speed of the power system and $\omega_i^{act}$ is the actual angular speed of generator $i$ (in radians per second). Let $E_i$, $\delta_i$, and $\dot{\omega}_i$ denote the derivatives of $E_i$, $\delta_i$, and $\omega_i$ with respect to time, respectively. Then, the third-order single-axis model for generator $i$ is represented as [18]

$$
\dot{E}_i = -\frac{1}{M_i}[E_i + (X_{di} - X_{di}^*)i_{di} - E_{di}]
$$

$$
\dot{\delta}_i = \omega_i
$$

$$
\dot{\omega}_i = \frac{1}{M_i}[-D_i \omega_i + P_{m,i} - P_{e,i}]
$$

where $E_{di}$ and $i_{di}$ are the value of the field excitation and the stator current of generator $i$, respectively. Further, the electrical power of generator $i$ is defined as [21]

$$
P_{e,i} = \sum_{k=1}^{N} |E_i||E_k| [G_{ik} \cos(\delta_i - \delta_k) + B_{ik} \sin(\delta_i - \delta_k)]
$$

where $G_{ik} = G_{ki} \geq 0$ is the Kron-reduced equivalent conductance between generators $i$ and $k$, and $B_{ik} = B_{ki} \neq 0$ is the Kron-reduced equivalent susceptance between generators $i$ and $k$, and $Y_{ik} = G_{ik} + \sqrt{-1}B_{ik}$ is the Kron-reduced equivalent admittance between generators $i$ and $k$. All of $Y_{ik}$, $G_{ik}$, and $B_{ik}$ are expressed in per unit values. Let $\phi_{ik} = \arctan(G_{ik}/B_{ik})$ and $P_{ik} = |E_i||E_k|/|Y_{ik}|$, then the electrical power of generator $i$ is also calculated as

$$
P_{e,i} = |E_i|^2 G_{ii} + \sum_{k=1, k \neq i}^{N} P_{ik} \sin(\delta_i - \delta_k + \phi_{ik}).
$$

Let $P_{a,i} = P_{m,i} - P_{e,i}$ denote the accelerating power of generator $i$, then the mechanical dynamics of a synchronous generator can be captured by investigating the swing equation model, which is a subset of (2), and can be represented as [22], [23]

$$
\dot{\delta}_i = \omega_i
$$

$$
\dot{\omega}_i = \frac{1}{M_i}[-D_i \omega_i + P_{a,i}].
$$

We next introduce a cyber-controller that enables the overall cyber-physical smart grid system to achieve transient stability in the face of severe disturbance.

### III. Parametric Feedback Linearization Control for Smart Grid

Typically synchronous generators are equipped with power control schemes (such as exciter and governor controls) that help to adjust a generator’s internal settings to respond to changes in the overall power grid. However, these local controllers, partly due to their decentralized nature requiring knowledge of only the local state $x_i$, often exhibit slow reaction to rapid systemwide changes and can be insufficient to address significant disruptions. Thus, in this paper, we consider the development of a local cyber-enabled controller at each generator that provides faster response time by using PMU measurements of its own agent and those of its neighbors. Due to the nonlinearity of the generator dynamics, we consider the use of distributed parametric feedback linearization (PFL) control that only requires knowledge of the local and neighboring states $\Xi_i$. To design for more aggressive stabilization, we do not assume the existence of other local generator controls that would aid in stabilization to provide a more conservative view of the stability problem during design.

Fig. 1(b) demonstrates the distributed control scenario in which the synchronous generator is equipped with a measurement device such as a PMU that obtains generator state readings including rotor speed and phase angle and passes the information to neighboring controllers. The controller then obtains the local and neighboring PMU readings to compute a signal that is injected into a local storage entity such as a flywheel. The flywheel then interfaces at the generator bus absorbing or injecting energy as needed.

The distributed external control can, therefore, be leveraged to achieve stability and the corresponding cyber-enabled physical dynamics can be described for generator $i$ as

$$
\dot{\delta}_i = \omega_i
$$

$$
\dot{\omega}_i = \frac{1}{M_i}[-D_i \omega_i + P_{a,i} + U_i]
$$

where $U_i$ is the power output of the flywheel such that a positive value of $U_i$ indicates that the controller of generator $i$ is injecting power into the corresponding generator bus and a negative value implies that power is being absorbed.

In this paper, we assume $U_i$ represents a feedback control signal computed from PMU measurements (or estimates) of one or more state variables; accordingly, the feedback controller compares the measured value with a desired one and generates a control signal aimed to minimize the difference.

#### A. Transient Stability

We design the PFL controller to asymptotically drive the relative normalized rotor speed to zero after the occurrence of a disturbance in the power grid; specifically, we require that $\lim_{t \to \infty} \omega_i(t) = 0 \forall i \in \{1, \ldots, N\}$ is achieved after the activation of the distributed control.

In feedback linearization, the control signal aims to cancel out nonlinear terms of the system dynamics such that the closed-loop system exhibits (full or partial) linear dynamics. Thus, to cancel the nonlinear term of the swing equation (i.e., $P_{a,i}/M_i = (P_{m,i} - P_{e,i})/M_i$), we let

$$
U_i = -(P_{a,i} + \alpha_i \omega_i)
$$

where $(D_i + \alpha_i) > 0$ and $\alpha_i \geq 0$ is called the frequency stability parameter. Consequently, the swing equation of the interconnected power system (assuming exact knowledge of the system parameters), after implementing the PFL controller,
reduces to a decoupled linear equation of the form
\[ \dot{x}_i = A_i x_i \] (8)
where \( x_i \) is the state variable of generator \( i \). In this case
\[ x_i = \begin{bmatrix} \delta_i \\ \omega_i \end{bmatrix} \quad \text{and} \quad A_i = \begin{bmatrix} 0 & 0 \\ 0 & -\frac{1}{M_i} (D_i + \alpha_i) \end{bmatrix}. \] (9)

The eigenvalues of \( A_i \) are \(-1/M_i (D_i + \alpha_i)\) and 0. The zero eigenvalue produces an undetermined change of phase until the frequency stabilizes; we introduce a phase cohesiveness parameter later to address issues of phase trajectory. For \((D_i + \alpha_i) > 0\), \( \lim_{t \rightarrow \infty} \omega_i(t) = 0 \) [24, Th. 4.5]. Consequently, the power system is stable under the PFL controller. Because the frequency stability parameter directly affects the value of the nonzero eigenvalue, it is expected that higher values of \( \alpha \) will drive the rotor speed of the system generators to faster stability. However, higher values of \( \alpha \) implies that the PFL controller would need greater quantities of external power.

**B. Phase Cohesiveness**

The development of the PFL controller, as shown in (7), focuses on stabilizing the rotor speed of the system generators. However, for transient stability phase cohesiveness amongst system generators is also needed; specifically, the absolute difference between the phase angle of any two generators should be less than 100° [16, 25].

In order to simultaneously accomplish phase cohesiveness, the PFL controller can be modified as
\[ U_i = -((P_{a,i} + \beta_i (\delta_i - \delta^*_i) + \alpha_i \omega_i) \] (10)
where \( \beta_i \geq 0 \) is denoted the phase cohesiveness parameter and \( \delta^* = [\delta^*_1, \delta^*_2, \ldots, \delta^*_N]^T \) is the desired phase angle of the system generators. The \( \beta_i (\delta_i - \delta^*_i) \) term will drive the PFL controller to settle the phase angle of the system generators on \( \delta^* \). The values of \( \delta^* \) are selected such that
\[ |\delta^*_i - \delta^*_j| \leq 100°, \forall i, j \in \{1, \ldots, N\}. \] (11)

Consequently, phase cohesiveness is maintained during and after the controller’s active time. Substituting the PFL control (10) into (6) results in
\[ \dot{x}_i = A_i x_i + b_i \delta^*_i \] (12)
where \( x_i = [\delta_i, \omega_i]^T \), \( A_i = \begin{bmatrix} 0 & -\frac{\beta_i}{M_i} \\ -\frac{1}{M_i} (D_i + \alpha_i) \end{bmatrix} \), and \( b_i = \begin{bmatrix} 0, \frac{\beta_i}{M_i} \end{bmatrix}^T \).

It is straightforward to determine that the eigenvalues of \( A_i \) are
\[ \lambda_{1,2} = \frac{1}{2M_i} \left[ -(D_i + \alpha_i) \pm \sqrt{(D_i + \alpha_i)^2 - 4\beta_i M_i} \right]. \] (13)

For \((D_i + \alpha_i) > 0\) and \( \beta_i > 0 \), the eigenvalues lie in the left-hand complex plane resulting in global asymptotic stability under the proposed PFL controller [24, Th. 4.5].

The reader should note that in cases that the fault is cleared knowledge of \( \delta^*_i \) prior to the fault would allow a more aggressive stabilization back to the former equilibrium state. However, use of a target phase is optional and transient stabilization control alone along with governor control as we discuss next will also facilitate transient stabilization.

**C. Robustness Study**

In this section, we demonstrate the robustness of the PFL controller in the presence of measurement uncertainty and model error. Given that \( \delta^*_i \) represents a set point for the generator rotor angles, for mathematical convenience, we assume in this section that \( \delta_i \) is an incremental version of the rotor angle state variable with an isolated equilibrium at the origin that represents convergence of the rotor angle of generator \( i \) to \( \delta^*_i \).

Let the state variable measurements be denoted \( \hat{\omega}_i \) and \( \hat{\delta}_i \) that represent estimates of the normalized rotor speed \( \omega_i \) and rotor angle \( \delta_i \), respectively. We model uncertainty in the nonlinear component of the electromechanical dynamics using \( \hat{P}_{a,i} \). The overall relationships are represented as follows:
\[ \hat{\delta}_i = (1 + e_{\delta_i}) \delta_i \]
\[ \hat{\omega}_i = (1 + e_{\omega_i}) \omega_i \]
\[ \hat{P}_{a,i} = (1 + e_{P}) P_{a,i} \] (14)
where the parameters \( e_{\delta_i}, e_{\omega_i}, \) and \( e_P \) capture the degree of uncertainty in the phase angle, rotor speed, and accelerating power of generator \( i \), respectively.

The value of the feedback control signal in the presence of uncertainty is given by
\[ U_i = -\left( \hat{P}_{a,i} + \beta_i \hat{\delta}_i + \alpha_i \hat{\omega}_i \right) \] (15)
which leads to system dynamics of the form
\[ \dot{x}_i = \hat{A}_i x_i + \hat{f}_{NL}(x_i) \] (16)
where
\[ \hat{A}_i = \left[ \begin{array}{c} 0 \\ -\frac{\beta_i (1 + e_{\delta_i})}{M_i} - \frac{1}{M_i} [D_i + \alpha_i (1 + e_{\omega_i})] \end{array} \right] \] (17)
and
\[ \hat{f}_{NL}(x_i) = [0, -e_P P_{a,i}]^T. \] (18)

1) **Measurement Uncertainty:** We first focus on the effects of measurement error by neglecting model uncertainty; we assume \( e_P \ll 1 \). Thus, our dynamics can be approximated as
\[ \dot{x}_i = \hat{A}_i x_i. \] (19)

It is straightforward to show that the eigenvalues of \( \hat{A}_i \) are given by
\[ \lambda_{1,2} = \frac{1}{2M_i} \left[ -(D_i + \alpha_i) \pm \sqrt{(D_i + \alpha_i)^2 - 4\beta_i M_i} \right]. \] (20)

A sufficient condition to ensure that both eigenvalues lie in the left-hand plane is
\[ e_{\omega_i} > -\frac{D_i + \alpha_i}{\alpha_i} \quad \text{and} \quad e_{\delta_i} > -1 \] (21)
where we assume \( \alpha_i, \beta_i > 0 \) which is necessary for a stabilizing controller. Reformulating (21), we observe
\[ 1 + e_{\omega_i} = \frac{\hat{\delta}_i}{\delta_i} > \frac{\omega_i}{\hat{\omega}_i} \quad \text{and} \quad 1 + e_{\delta_i} = \frac{\hat{\delta}_i}{\delta_i} > 0. \] (22)
This implies that as long as the rotor speed and angle estimates \( \hat{\omega}_i \) and \( \hat{\delta}_i \) each have the correct sign as their ideal counterparts \( \omega_i \) and \( \delta_i \), stabilization will occur. In fact, in the case of rotor speed, even if the sign of \( \hat{\omega}_i \) is reversed, stabilization is possible as long as \( |\hat{\omega}_i| \) is bounded to be less than \( D_1/\omega_i|\omega_i| \). The reader is reminded that, as in the case of rotor angle, both \( \hat{\omega}_i \) and \( \hat{\delta}_i \) represent incremental rotor speeds where 0 corresponds to the utility frequency of 50 or 60 Hz.

2) Model Error: We next consider the effect of model error and assume that measurement uncertainty is negligible; that is, linear term \( \omega \) and angle \( \delta \) at specific periodic intervals. The measured readings can also be easily implemented in a step-wise manner. If the exact system parameters are unknown to the PFL controller, the governor can provide added robustness against the parameter error. In addition, the design of the proposed controller, the governor control aligns naturally with the PFL controller. Thus, during an interval, the value of \( \hat{\delta}_i \) is reversed, stabilization is possible as long as \( |\hat{\omega}_i| \) is bounded to be less than \( D_1/\omega_i|\omega_i| \). The reader is reminded that, as in the case of rotor angle, both \( \hat{\omega}_i \) and \( \hat{\delta}_i \) represent incremental rotor speeds where 0 corresponds to the utility frequency of 50 or 60 Hz.

Consider a Lyapunov function of the form
\[
V(x_i) = x_i^T P x_i,
\]
where \( P \) is a 2 \times 2 positive definite matrix. Taking the time derivative gives
\[
\dot{V}(x_i) = x_i^T P x_i + x_i^T P x_i \quad (24)
\]
\[
= (A_i x_i + \hat{f}_{NL}(x_i))^T P x_i \quad (25)
\]
\[
+ x_i^T P (A_i x_i + \hat{f}_{NL}(x_i)) \quad (26)
\]
\[
= x_i^T (A_i^T P + PA) x_i + 2 P \hat{f}_{NL}(x_i) x_i. \quad (27)
\]
Let \( Q = -(A_i^T P + PA) \) which can be shown to be positive definite given that \( A \) is Hurwitz. Moreover, we let \( R = 2 P \hat{f}_{NL}(x_i) \) to give
\[
\dot{V}(x_i) = -x_i^T Q x_i + R x_i \quad (28)
\]
\[
\leq -\lambda_{\min}(Q) \|x_i\|^2 + \|R\|_\infty \|x_i\| \quad (29)
\]
\[
= -\lambda_{\min}(Q) \left( \|x_i\| - \|R\|_\infty / \lambda_{\min}(Q) \right) \|x_i\| \quad (30)
\]
where \( \|x_i\| \geq 0 \) is the \( \ell_2 \)-norm of vector \( x_i \), \( \lambda_{\min}(Q) > 0 \) represents the minimum eigenvalue of the positive definite matrix \( Q \), and \( \|R\|_\infty > 0 \) is the maximum absolute value of the elements in \( R \). Thus, \( \dot{V}(x_i) < 0 \) for \( \|x_i\| > (\|R\|_\infty / \lambda_{\min}(Q)) \). Thus, we establish ultimate boundedness to a neighborhood that includes the origin. That is, the states are able to converge toward the origin up to this neighborhood.

We see that this neighborhood decreases in size for decreasing magnitudes of \( \epsilon_P \) as expected. Moreover, the neighborhood decreases as the minimum eigenvalue of \( Q \) increases, which can be controlled by increasing control gain \( \alpha_i \) and \( \beta_i \).

D. Integration With Governor Control

We assert that the PFL controller integrates naturally with the governor control commonly found in power systems. A schematic of an integration is shown in Fig. 2. The PFL controller aims to maintain the generator’s rotor speed within the stability margin. Although the governor has a similar goal, its response time is much slower.

One strategy to implement governor control is to slowly close the gap between the mechanical and electrical powers of the generator. Mathematically, let \( \hat{P}_{m,i} \) denotes the derivative of \( P_{m,i} \) with respect to time, then this implementation can be modeled for generator \( i \) as
\[
\hat{P}_{m,i} = \kappa_i (P_{e,i} - P_{m,i}) \quad (31)
\]
where \( \kappa_i \geq 0 \); a value of \( \kappa_i = 0 \) indicates that the governor control is not activated on generator \( i \).

E. Practical Considerations

The proposed parametric controller works in a practical setting as follows. First, PMU sensors scattered around the power grid measure the synchronous generator rotor speed and phase angle at specific periodic intervals. The measured readings are then transmitted over the cyber-communication network to the distributed controllers. Once the values of \( x_i = [\hat{\delta}_i, \hat{\omega}_i]^T \) are received, each controller calculates \( P_{e,i} \) according to (4). The value of \( U_i \) is then computed as shown in (7) or (10). If \( |U_i| \geq U_{\max} \), where \( U_{\max} \) is the maximum power that the flywheel can either inject or absorb to the power system, then \( U_i \) is limited to only \( U_{\max} \). Next, \( U_i \) is applied to generator \( i \) (through a fast-acting flywheel) for the entire time duration until the next reading of the system state variable is received by the controllers; thus during an interval, the value of \( U_i \) remains constant resulting in a step-wise control signal implementation.

F. Features of the Proposed Controller

Some of the advantages of the proposed stabilizing solution over previous work includes shorter transient stability time for the synchronous generators. The controller also integrates well with existing power system controllers; for example, the governor control aligns naturally with the PFL controller. Thus, if the exact system parameters are unknown to the PFL controller, the governor can provide added robustness against the parameter error. In addition, the design of the proposed controller is straightforward and is easy to implement. Further, the development of this stabilizing control provides a natural tool to demonstrate tradeoffs between the degree of available external power and the stability time of the synchronous generators. Moreover, the PFL controller does not need a continuous stream of real-time updates of the system state information to effectively stabilize the power system; as we demonstrate the controller can stabilize the power grid as long it obtains frequent and periodic updates from the system sensors though the smart grid communication network. The proposed controller can also be easily implemented in a step-wise manner.
Fig. 3. System performance when only PFL control is activated.

IV. NUMERICAL RESULTS

The New England 10-generator 39-bus-based smart grid system of Fig. 1 is simulated using the MATLAB environment. The values of \( M_i \)’s and \( X_{di}’s \) are found in [26] and [27] and \( D_i \) is set to 20 ms for all generators.

The power system is assumed to be running in normal secure state from \( t = 0 \) to \( t = 0.5 \) s. A three-phase fault is applied at bus 17 at \( t = 0.5 \) s. It is assumed that line 17–18 trips to clear the fault at \( t = 0.6 \) s (beyond the critical clearing time possibly due to a coordinated cyber-attack) and that the PFL controller is activated on all generators at \( t = 0.7 \) s.

Before the occurrence of the fault, load flow analysis of the power system is conducted to find the electrical power of the system generators. Because the power system is balanced and there are no transients (prior to the fault), using continuity arguments the mechanical power of each generator also equals the electrical power of that generator at the initial moment of disruption.

For the following numerical results, stability time of a generator is calculated by finding the controller’s active time after which the relative normalized rotor speed of the generator is restricted to a 2% threshold (i.e., the relative normalized rotor speed of the generator is limited to \( \pm 0.02 \) p.u.). In other words, stability time of generator \( i \) is the time it takes for the controller to consistently keep the rotor speed in the stability margin. For improved clarity and conservation of space, we show the performance results for the first four synchronous generators only. Readers should note that similar stability behavior is exhibited for the remaining generators.

A. Transient Stability

Fig. 3 displays the effect of implementing the proposed PFL controller on the performance of the power system. Both governor control and phase cohesiveness are not incorporated in this figure to emphasize the results of using the proposed controller in stabilizing the rotor speed of the system generators; in other words, \( \kappa_i = 0 \) and \( \beta_i = 0 \) in Fig. 3. The value of the frequency stability parameter \( (\alpha_i) \) in (7) is set to 1 for all generators.

It is noted that the power system achieves transient stability within a short time. For example, stability time of generator 1 is about 0.89 s. It is to be noted that because the mechanical power of the synchronous generators does not change in this case, the stabilizing controller compensates the difference between \( P_m \) and \( P_e \) in order to keep the power system stable.

Fig. 4 shows the effect of activating the phase cohesiveness parameter in the PFL controller on the performance of the power system. It is shown that the phase angle of the system generators is controlled as promised by (10). In this case, the stability time of generator 1 is around 2.95 s, which is slightly higher than that of the case when phase cohesiveness parameter is not activated. However, the extra time is needed by the PFL controller in order to achieve both desired phase cohesiveness and transient stability.

The performance of the power system when both governor and PFL controls are activated is shown in Fig. 5. The implemented governor slowly adjusts the value of the mechanical power of a generator in order to reduce the gap between \( P_m \) and \( P_e \) so that the change in rotor speed is slowed (and ultimately reversed back into stability). The implemented nonlinear governor utilizes \( \kappa_i = 1 \).

The stability time for generator 1 is found to be around 2.86 s in this case. With both controllers activated in the power system, the PFL controller quickly compensates for the difference between \( P_m \) and \( P_e \) in order to stabilize the power system fast. At the same time, the governor slowly adjusts the value of \( P_m \) to close the gap with \( P_e \). As a result, the external power used by the stabilizing controller declines slowly.

C. Comparison to Recent Work

For comparison, the performance of the PFL controller is measured against two recently proposed controllers,
specifically, flocking control (see [14]–[17]), and consensus proportional integral (CPI) control (see [28]–[30]).

Flocking control was proposed by Wei et al. [15], [16], [31] and Wei and Kundur [17] to address generator synchronization after a severe disturbance such as a fault or denial-of-service cyber-attack. This nonlinear control approach introduces a control input to shape the system dynamics to mimic that of a flock of stable bird-like objects (boids) where agent phase angle and rotor speed are analogous to boid position and velocity, respectively. The dynamics exhibit flock centering whereby boids (agents) remain in close proximity, collision avoidance where boids (agents) avoid colliding with neighbors, and velocity matching such that boids (agents) match the speed of neighbors [32]–[34]. The first two properties provide phase synchronization while the latter provides speed stabilization needed for transient stability [16]. The corresponding control input \( U_i \) is calculated as

\[
U = \Phi - G \delta - B \omega - D - c(\delta - \delta_0) \tag{32}
\]

where \( \Phi = [\Phi_1, \Phi_2, \ldots, \Phi_N]^T \) and \( \Phi_i \) is defined as

\[
\Phi_i = \sum_{k=1, k \neq i}^{N} \int_{t_0}^{t} \rho(\delta_i - \delta_k) \, dt \tag{33}
\]

where \( t_0 \) is the time to activate the flocking control, \( t \) is the time to calculate the value of the control, \( \rho \) is a control function, \( c \) is a navigation term, \( \delta_0 = [\delta_{01}, \delta_{02}, \ldots, \delta_{0N}]^T \) are the phase values at \( t_0 \), \( D = [D_1, D_2, \ldots, D_N]^T \), \( \delta = [\delta_1, \delta_2, \ldots, \delta_N]^T \), and \( \omega = [\omega_1, \omega_2, \ldots, \omega_N]^T \). Moreover, \( B \) and \( G \) are cyber-control matrices.

Andreasonn et al. [28]–[30] proposed the application of a CPI control strategy to affect the mechanical power of a generator for automatic frequency control that is applied at two levels. At the first level, the generator’s rotor speed is regulated against a reference speed. At the second level, the reference speed is updated to eliminate errors. Mathematically, the proposed CPI controller can be represented as [28]–[30]

\[
U_i = \alpha_i(\hat{\omega} - \omega_i)
\]

\[
\dot{\delta}_i = \beta_i \left( \omega_{i, \text{nom}} - \frac{1}{N} \sum_{j=1}^{N} \omega_j \right). \tag{34}
\]

The proposed PFL distributed controller aims to be more practical than the flocking-based approach by being amenable to integration with governor control. Furthermore, by design it aims to achieve faster stabilization through the use of tunable feedback linearization to control both rotor speed and phase angle. In contrast to the centralized CPI control that modulates the mechanical power of a designated generator, the PFL approach exploits cyber-enablement and the introduction of distributed storage to harness fast-acting external power sources in a distributed manner thus reducing communication overhead.

The reader should note that for more consistent comparison in simulations we have implemented the CPI controller as the input to fast-acting flywheel storage opposed to that of controlling mechanical power of the generator, which would exhibit more sluggish behavior. In this way, the gains achieved by the controller design alone can more objectively be assessed.

\[ \text{D. Summary Results} \]

Five fault case studies are considered in this paper as shown in Table II. The power system is assumed to be running in normal state from \( t = 0 \) to \( t = 0.5 \) s. A fault (three-phase fault for case study 1–4 and line-to-line asymmetrical fault in case study 5) occurs at the faulted bus at \( t = 0.5 \) s, then the triped line is removed to clear the fault at \( t = 0.6 \) s. Finally, the controller is activated on corresponding generators at \( t = 0.7 \) s.

Tables III and IV detail the average stability time and control power, respectively, of generators 1–9 for the different controllers (governor, flocking, CPI, and proposed PFL) under the five different case studies. The values of \( \alpha_i \) and \( \beta_i \) are set to 0.25.

It is evident from the results of Table III that the PFL controller (with and without phase cohesiveness) outperforms the flocking and CPI controllers. It is obvious that activating the phase cohesiveness parameter increases the stability time slightly. Further, it is shown that savings in system stability can be accomplished by activating both PFL and governor controls at the same time. Consequently, the integration of the PFL controller with the governor control yields more robustness.

The reader should note that the generator models for controller derivation and simulation are distinct. For tractability, PFL control assumes a swing equation model of a synchronous generator widely used in the literature that is well suited for transient stability analysis. In contrast, empirical results make use of the third-order single-axis model of (2) that additionally accounts for the time-vary nature of the internal voltage and electrical dynamics within a synchronous generator. Thus, we believe that the simulations represent a flavor of the performance of the controller within a real system.

\[ \text{V. PERFORMANCE UNDER PRACTICAL LIMITATIONS} \]

This section investigates the performance of the PFL controller against some practical limitations. Further, the relation between the stability time and the controller parameters is considered. For the following numerical studies, case study 1 is considered where we set \( \alpha_i = 1, \beta_i = 0, \) and \( \kappa_i = 0. \)

\[ \text{A. Cyber-Limitations} \]

The performance of the proposed controller is evaluated against the physical limits of the external power source, the sampling frequency of the measurements, the signal-to-noise ratio, and the effects of cyber-attacks on the PFL controller.
ratio (SNR), and the communication latency between the control center and sensors.

1) Limits on External Power Source: The PFL controller relies on a fast-acting external power injection and absorption source, \( U_i \), to contribute to the power system dynamics. However, \( U_i \) will have practical physical limitations, specifically, \( |U_i| \leq P_{\text{max},i} \); this power limit can be a result of the number and capacity of the storage batteries employed, for example. In this paper, \( P_{\text{max},i} \) is set to a percentage of the mechanical power of each generator \( P_{m,i} \) before the occurrence of the fault. In other words, \( P_{\text{max},i} = \gamma/100 \ P_{m,i} \), where \( 0 < \gamma \leq 100 \) is the power limit percentage.

The effects of the external power limit are shown in Fig. 6(a). It is noted that the controller performs well when the external power supply does not have highly restrictive limits. For reasonable values of external power limit the PFL controller can effectively stabilize the New England power system within few seconds.

2) Measurement Sampling Frequency: The digital nature of PMU sensors requires that measurements be sampled prior to communication. As the value of the sampling period \( T_s \) is decreased, the PFL controller obtains more frequent updates of the system state variable \( x \). To account for the effect of sampling period, \( U \) is implemented in a step-wise manner; specifically, it can be considered a function of \( \delta(nT_s) \) and \( \omega(nT_s) \) for the duration \( nT_s < t < (n+1)T_s \), where \( n \geq 1 \) is a positive integer.

Fig. 6(b) shows the performance of the PFL controller against \( T_s \). As the value of \( T_s \) increases, the stability time of the system generators increases; however, even when the sampling period is approximately 200 ms, the different generators can be stabilized within few seconds.

The reader should note that the robustness of the proposed PFL controller to a broad range of sampling periods \( T_s \) has other implications. Specifically, our paradigm makes use of fast-acting storage technologies to stabilize the system. Questions naturally arise as to the performance of flywheels and batteries in providing sufficient tracking of the control signal \( U_i \). We assert that given the system is robust to sampling periods of up to 200 ms and that current reports on response times of utility scale energy storage systems are in the order of 0.35–20 ms [35] with flywheel technology reported to have the potential of 0.5 MW [36], our paradigm has the potential to perform well by leveraging a variety of new storage technologies.

Moreover, current standards for synchrophasor measurements for power systems such as the IEEE 37.118 standard specify sampling rates in the range 10–120 samples/s that correspond to sampling periods of 8.3–100 ms. Thus, the PFL controller has the potential to perform well within current and expected future standards.

3) Sensor Noise: SNR (in units of dB) is defined as \( \text{SNR} = 10 \log_{10}(E_S/E_N) \) where \( E_S \) is the energy of the original uncorrupted signal, and \( E_N \) is the energy of the associated noise computed as the deviation between the original signal at the source and the version received at the controller. SNR is a measure of the quality of the received sensor signal at the controller; the higher the value of SNR, the higher the fidelity and accuracy of the readings employed to compute control.

Fig. 6(c) displays the effect of SNR on the performance of the PFL controller. It is noted that the controller is robust to noise given that a relatively modest value of SNR (achievable by most modern smart grid sensors) is required by the controller to stabilize the power system.

4) Communication Latency: Communication latency can occur in the cyber-infrastructure due to sampling and quantization time, encryption, channel propagation, and queueing delays. Latency can be variable or fixed for each data transmission. The case of fixed latency is considered in this paper; this means that all data packets sent by the sensors to the controller experience the same amount of delay. To reflect the effect of latency on the controller, \( U(t) \) is a function of \( \delta(t - \tau) \) and \( \omega(t - \tau) \) where \( \tau \) is the communication latency.

Fig. 6(d) displays the effect of communication latency on the controller performance. It is noted that the PFL controller is robust to reasonable communication delays. However, when the communication latency is higher than 175 ms,

### Table III

<table>
<thead>
<tr>
<th>Case</th>
<th>Governor Control</th>
<th>Flocking Control</th>
<th>CPI Control</th>
<th>PFL Control</th>
<th>PFL with Phase Coheresiveness</th>
<th>PFL with Phase Coheresiveness &amp; Governor Controls</th>
</tr>
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<tbody>
<tr>
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### Table IV

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<th>PFL with Phase Coheresiveness</th>
<th>PFL with Phase Coheresiveness &amp; Governor Controls</th>
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</table>
the synchronous generators cannot be stabilized by the PFL controller within the simulation run time. Further, Fig. 7 provides a performance comparison between the PFL, flocking, and CPI control schemes versus latency.

**B. Controller Parameters**

Fig. 8(a) displays the stability time of the power system as a function of the frequency stability parameter ($\alpha_i$), where $\beta_i = 0$ in this case. It is shown in (9) that the term $(D_i + \alpha_i)/M_i$ controls the exponent of decay of $\omega_i$; thus, the relative normalized rotor speed of the generators approaches zero with a rate that depends on the frequency stability parameter. Consequently, higher values of $\alpha_i$ lead to faster decay and shorter stability times as confirmed by the results of this figure.

The relation between the stability time and phase cohesiveness parameter is shown in Fig. 8(b). Compared to the case of no phase cohesiveness (i.e., $\beta_i = 0$), it is observed that the stability time is higher when $\beta_i > 0$. However, for values of $\beta_i > 1$, the stability time is slightly decreasing.

**VI. CONCLUSION**

This paper proposes a stabilizing controller for smart grid systems under severe fault or malfunction of protection devices. The proposed parametric controller relies on feedback linearization theory. System state information is collected by sensors and transmitted through a communication network to distributed controllers. Based on the received data, a PFL control is applied using fast-acting flywheels situated near the synchronous generators to balance the swing equation and drive the power system to stability.

System performance is investigated when the proposed controller is applied to the New England 39-bus 10-generator test system. Further, the performance is studied when both proposed and governor controls are activated. Results demonstrate the effectiveness of the proposed controller in stabilizing the power grid and achieving more resilience to disturbances.

**REFERENCES**


