SHORT TIME FOURIER TRANSFORM (OR WINDOWED FOURIER TRANSFORM

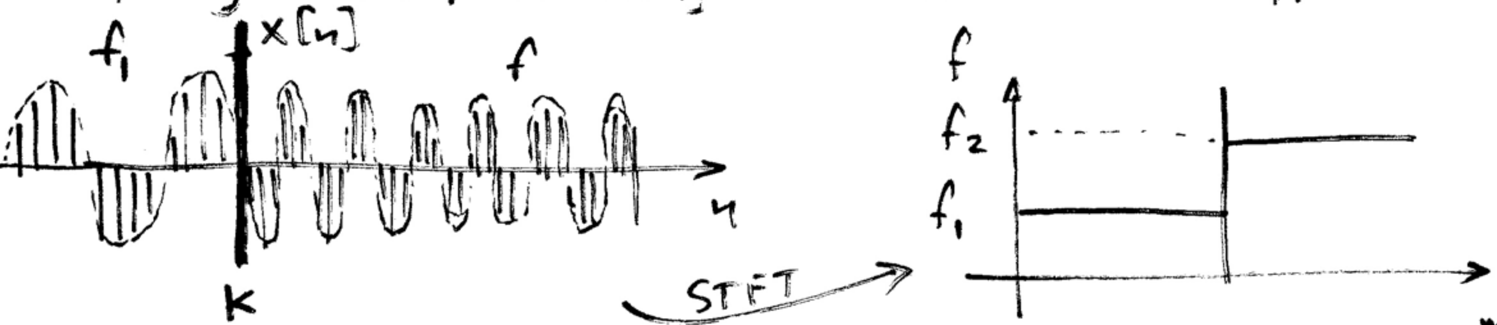
(I) The DTFT (Discrete time Fourier Transform $X(f)=\sum_{n=-\infty}^{\infty} x[n]e^{-j2\pi fn}$ looks at a "global" characteristic of the signal by summing over the entire time $n=-\infty,\dots,0,\dots\infty$. This is reflected by the fact that the basis function $e^{-j2\pi fn}$ are spread over the whole time axis. Thus $X(f)$ cannot analyze local behavior of the signal.

(II) The short time FT (STFT) or windowed FT (WFT)

$$X(m,f)=\sum_{n=-\infty}^{\infty} x[n]\cdot w[n-m]\,e^{-j2\pi fn}$$

$$=\mathrm{DTFT}\left[x[n]\,w[n-m]\right]$$

with $w[n-m]$ a window placed (or centered) around the variable m provides a better local description of the frequency content of $x[n]$ at time instant m.



Ideally we would like to have the representation shown in the picture.

The quantity $|X(m,f)|^2$ is called the <u>spectogram</u>. Usually, we choose a window (real) with unit energy that is $\sum_n |w[n]|^2=1$