

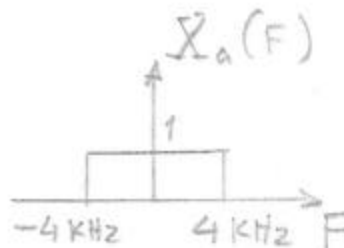
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ECE 431H1S, Digital Signal Processing

Quiz 5
January 31, 2005

A signal $x_a(t)$ has the Fourier Transform $X_a(F)$ shown in the figure.



0.7/ The signal is digitized with $F_{s1} = 8\text{KHz}$ and resolution $B_1 = 16$ bits/sample. How much can we reduce the resolution (B_2 bits/sample) if we increase the sampling frequency to $F_{s2} = 128\text{KHz}$?

System ① $x_a(t) \rightarrow \text{C/D} \rightarrow \text{QUANT. } B_1 \rightarrow x[n] + q_1[n]$
 $F_{s1} = 8\text{KHz}$
 $\sigma_{q_1}^2 = \frac{V_s^2}{3 \cdot 2^{2B_1}}$
 $\sigma_{q_1}^2 = \frac{V_s^2}{3 \cdot 2^{32}}$

System ② $x_a(t) \rightarrow \text{C/D} \rightarrow \text{QUANT. } B_2 \rightarrow x[n] + q_2[n] \rightarrow H(f) \rightarrow x[n] + q_2'[n]$
 $F_{s2} = 128\text{KHz}$
 $\sigma_{q_2}^2 = \frac{V_s^2}{3 \cdot 2^{2B_2}}$
 $\sigma_{q_2}^2 = \sigma_{q_1}^2 \cdot \frac{1}{16}$

Want: $\frac{\sigma_x^2}{\sigma_{q_1}^2} = \frac{\sigma_x^2}{\sigma_{q_2}^2} = 16 \frac{\sigma_x^2}{\sigma_{q_1}^2} \Rightarrow B_1 = B_2 + 2$

\therefore CAN REDUCE RESOLUTION TO $B_2 = 14$ bits/SAMPLE.

0.3/ What is the corresponding Signal to Quantization Noise Power Ratio (SQNR) in the two cases?

- will have same SQNR in both cases (by design).

- $SQNR = \frac{\sigma_x^2}{\sigma_{q_1}^2}$, $\sigma_{q_1}^2 = \frac{V_s^2}{3 \cdot 2^{20}}$

CAN ASSUME $x[n]$ IS UNIFORM: $[-V_s, V_s]$:

$\therefore \sigma_x^2 = \frac{(2V_s)^2}{12} = \frac{V_s^2}{3}$

$\therefore SQNR = 2^{20} = 2^{32}$

This is the 6 dB/bit rule: $10 \log_{10} SQNR = 10 \log_{10}(2^{20}) = 20 \text{ dB}$
 $= B_1(6.02) \text{ dB}$