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ECE 431H1S, Digital Signal Processing

Quiz 10 Solutions

March 28, 2005

The following table shows $x[n]$, $n = 0, 1, \dots, 15$ and its 16-DFT $X[k]$ in columns 2 and 3 respectively.

n/k	x[n]	X[k]	y[n]	z[n]
0	2	409.000 + 0.000i	6	98
1	1	36.127 + 111.469i	19	4
2	6	76.317 + 101.489i	17	61
3	19	-26.237 + 144.309i	2	6
4	17	-104.000 + 179.000i	1	19
5	2	-48.432 - 62.953i	59	17
6	1	-86.317 + 55.489i	16	2
7	59	-17.458 - 51.793i	11	1
8	16	-37.000 + 0.000i	55	59
9	11	-17.458 + 51.793i	25	16
10	55	-86.317 - 55.489i	6	11
11	25	-48.432 + 62.953i	8	55
12	6	-104.000 - 179.000i	83	25
13	8	-26.237 - 144.309i	98	6
14	83	76.317 - 101.489i	4	8
15	98	36.127 - 111.469i	61	83

The 4th column shows another signal $y[n]$ that is (somehow) related to $x[n]$. And the 5th column shows yet another signal $z[n]$ that is related to $y[n]$ (and thus, also to $x[n]$).

Now suppose the 16-DFTs of $y[n]$ and $z[n]$ are respectively $Y[k]$, $Z[k]$, $k=0, 1, \dots, 15$.

Compute the following DFT values:

1. $Y[2]$
2. $Z[2]$
3. $Y[16-2]$ and $Z[16-2]$

Hint: You may need to apply the sliding DFT technique more than once.

Solutions:

- The idea is to use the sliding DFT twice for the signal $y[n]$.
- For $z[n]$, an additional application of the circular-shift property of the DFT is needed.

Apply sliding DFT, with $N=16$, $k=2$, $X[2] = 76.317 + 101.489i$, $x[n-N]=2$, $x[n]=4$

$$Y_{\text{TEMP}}[2] = \exp(j*2*\pi*k/N) * (X[2] - x[n-N] + x[n]) = -16.385 + 127.142i$$

Apply sliding DFT again, with $x[n-N] = 1$, $x[n] = 61$

$$Y[2] = \exp(j*2*\pi*k/N) * (Y_{\text{TEMP}}[2] - x[n-N] + x[n]) = \underline{\underline{-59.062 + 120.744i}}$$

Note that $z[n]$ is circular-shifted version of $y[n]$ by 3 units, so with $k = 2$, $r = 3$,

$$Z[2] = Y[2] * \exp(-j*2*\pi*k*r/N) = \underline{\underline{127.142 - 43.615i}}$$

Since x, y, z are all real signals, $Y[16-2]$ and $Z[16-2]$ are **conjugates** of $Y[2]$ and $Z[2]$, respectively.