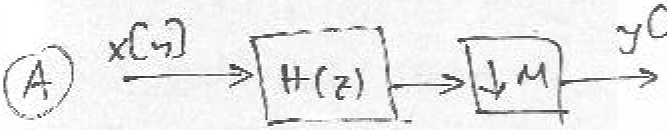
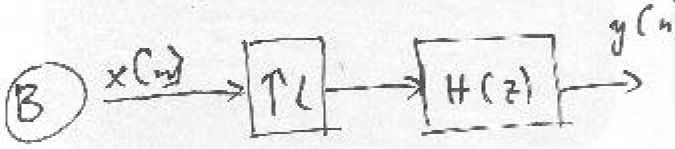
EFFICIENT IMPLEMENTATION OF MULTIRATE SYSTEMS

Consider a decimator and an interpolator



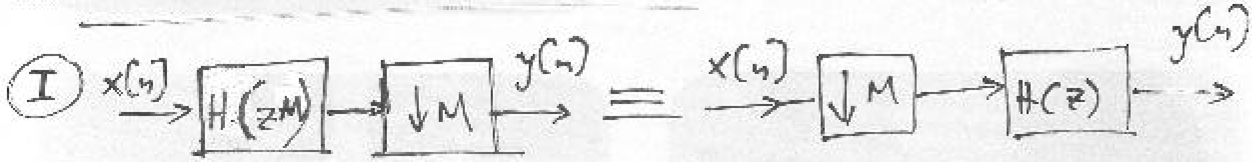


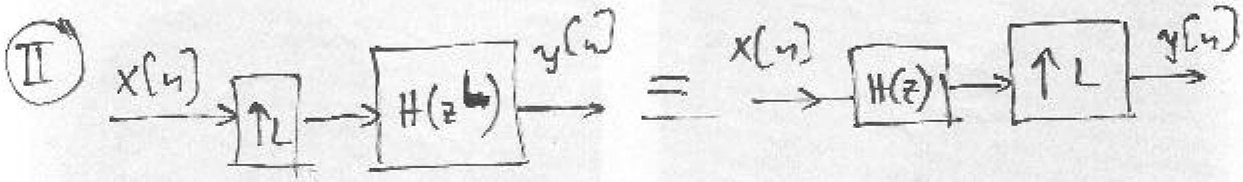
Inefficient designs since a lot of computations are done for nothing.

In (A) M-1 out of M computed values is thrown away
In (B) L-1 out of L inputs to the linear filter are Ø.

A better way to implement (A) and (B) that do not make unnecessary computations is based on the so called "Noble identities" and "Polyphase decomposition".

NOBLE IDENTITIES





where, $H(z^M) = \sum_{n=-\infty}^{\infty} h[n] z^{-nM} = \ldots + 0 + h(0) + 0 + .. + 0 + h[1] z^{-M} +$
$+ 0 + .. + 0 + h[2] z^{-2M} + 0 + \ldots$

So if the filter has particular structure then the noble identities can apply and provide a more efficient structure.

SIGNIFICANCE !!

Looking at (I) and (II), the implementations at the right is more efficient, because it operates at the lower sampling rate. The implementations at the left are wasteful since they process a lot of terms that are zero.

"Proof" of identities

Let us pick $H(z) = z^{-1}$ and show that (I) and (II) work.

In (I) at the right $y[n] = x[(n-1)M] = x[nM-M]$

at the left $\left.\begin{array}{l} y[n] = z(nM) \\ z[n] = x(n-M) \end{array}\right\} \Rightarrow y[n] = x[nM-M]$

etc.

QUESTION :

In case $H(z)$ does not have a special structure can we still utilize the noble identities to improve efficiency? The answer is "yes" by using "polyphase decomposition"

POLYPHASE DECOMPOSITION OF LINEAR FILTERS

Given $\underline{M}$ and a LSI Filter $H(z) = \sum_{n=-\infty}^{\infty} h[n] z^{-n}$

we can always write $H(z)$ as follows:

$$H(z) = \sum_{k=0}^{M-1} z^{-k} H_k(z^M)$$

where, $H_k(z^M) = \sum_n h(nM+k) z^{-nM}$.

Example: Let $H(z) = 1 + 3z^{-1} + 2z^{-2} + 6z^{-3} + z^{-4} + 3z^{-5}$

and let $M=2$, Then:

$$H(z) = \underbrace{(1 + 2z^{-2} + z^{-4})} + z^{-1}\underbrace{(3 + 6z^{-2} + 3z^{-4})}$$

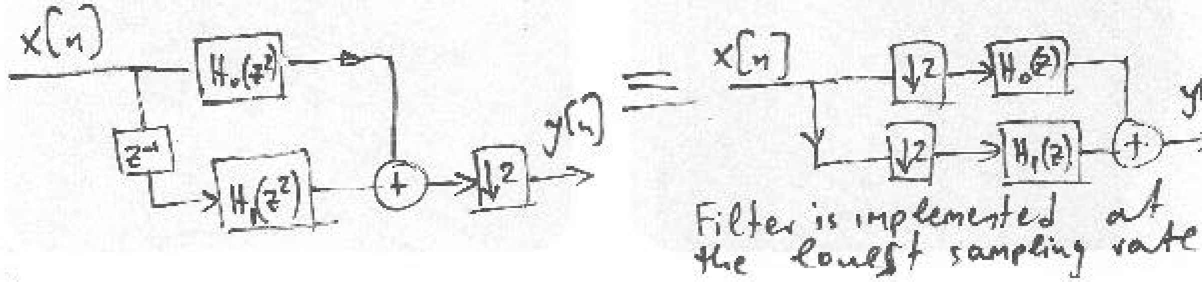
$$= H_0(z^2) + z^{-1} \cdot H_1(z^2)$$

Also, for $M=3$

$$H(z) = \underbrace{(1 + 6z^{-3})} + z^{-1}\underbrace{(3 + z^{-3})} + z^{-2}\underbrace{(2 + 3z^{-3})} =$$

$$= H_0(z^3) + z^{-1} \cdot H_1(z^3) + z^{-2} \cdot H_2(z^3)$$

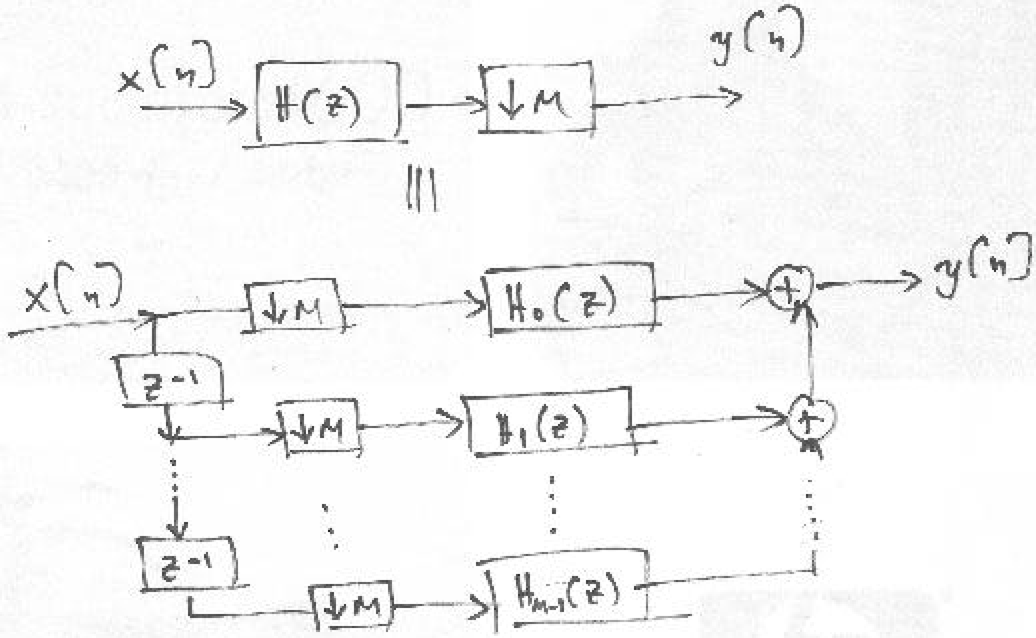
Implementations: Let $x[n] \rightarrow \boxed{H(z)} \rightarrow \boxed{\downarrow 2} \rightarrow y(n)$



Filter is implemented at the lowest sampling rate

General cases:

(A)



(B)