Array Processing

* Consider a linear array of M sensors (antennas) equally spaced in a line configuration. Denote by d the spacing between sensors. Then the total length of the array is $(M-1)\,d$.

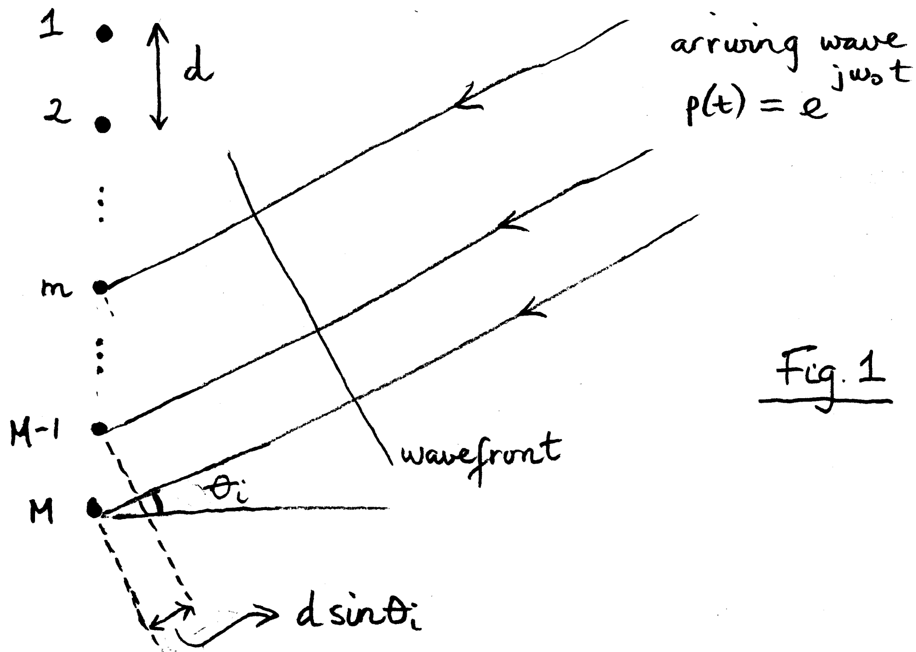

Fig. 1

* In Fig. 1, a plane wave of a single frequency ω_0 is shown arriving at the array at an angle θ_i.
* From the configuration shown, if an antenna measures $p(t)$ then an antenna below it measures $p(t - \tau(\theta))$, where $\tau(\theta)$ is the time required for the wavefront to travel from the top antenna to the one below. Evidently, 2 consecutive antennas exhibit a time delay of $\tau(\theta) = \dfrac{d \sin\theta_i}{c}$, where c is the speed of light.

In reference to antenna 1, an m^{th} antenna exhibits a delay of:

$$\tau(\theta) = (m-1)\, d \sin\theta_i / c \qquad (1)$$

* Let the excitation or weight for the antenna m at instant n be $w_m[n]$. Then the weighted sum of signals from all sensors / antennas is :

$$y(t) = \sum_{m=1}^{M} w_m[n]\, p(t - (m-1)\, d \sin\theta_i / c)$$

$$= e^{jw_0 t} \sum_{m=1}^{M} w_m[n]\, e^{-jw_0 (m-1)\, d \sin\theta_i / c} \qquad (2)$$

* Now make the substitution $c = \lambda f = \lambda w_0 / 2\pi$ to yield

$$y(t) = e^{jw_0 t} \sum_{m=1}^{M} w_m[n]\, e^{-j 2\pi (m-1)\, d \sin\theta_i / \lambda} \qquad (3)$$

$$= e^{jw_0 t}\, A(\theta_i, n) \qquad (4)$$

* Here $A(\theta_i, n)$ introduces a magnitude and phase change to the incoming wave $p(t) = e^{jw_0 t}$. By selecting the weights $w_m[n]$, one can thus change the radiation pattern $A(\theta_i, n)$.
* Note also that $A(\theta_i, n)$ can be interpreted as the DTFT of the finite-sequence consisting of the weight coefficients $w_m[n]$, and where the frequency variable is $\frac{d \sin\theta_i}{\lambda}$ with $-\pi/2 \le \theta_i \le \pi/2$.