.

EC431H1 Digital Signal Processing
LAB 4.
Linear Array Processing: Beaforming, Bearing estimation and Spatial filtering.

Introduction

The purpose of this experiment is to become familiar with some aspects of modeling and operation of linear antenna arrays. It is recommended that you use MATLAB to program and execute the required steps of the experiment. Your lab report should include: a)printouts of your programs, b) printouts/plots of the obtained results, and c) answers to questions and brief but critical discussion of the obtained results.

Experiments

PART A. Linear array modeling and Beam-forming

A uniform linear array is a set of M sensors in a line configuration with spacing d between sensors (see Fig. 1). Thus, the total length of the array is $(M-1)d$. A SNAPSOT is a set of M sensor readings (outputs) at a given time instant. In array processing many SNAPSHOTS taken at different time instants are combined appropriately to produce a desired result.

Now consider a linear array of M equispaced sensors with intersensor spacing d. Taking as reference the sensor 1 and by denoting as $w_m[n]$ the excitation or weight signal at sensor m at time instant n, then the far field radiation pattern (or equivalently, the response of the array to an incident signal) at an angle θ is given by

$$A(\theta, n) = \sum_{m=1}^{M} w_m[n] \cdot e^{\frac{j2\pi(m-1)d\sin(\theta)}{\lambda}}$$

where λ is the wavelength of the excitation. Usually, we want to estimate $A(\theta)$ for $-90^o \leq \theta \leq 90^o$. Notice that this, at time instant n , is nothing else but the Fourier transform of the SNAPSOT signal at time instant n at frequencies $\frac{d\sin(\theta)}{\lambda}$.

1. Assume for the moment that $M = 12$ and all array sensors are equally weighted, that is,

   $$w_m[n] = 1, \qquad m = 1, \ldots, M, \qquad \forall n$$

   Calculate the Power radiation (or response) pattern of the array $P(\theta) = |A(\theta)|^2$ for $d = \frac{\lambda}{2}$ and $-90^o \leq \theta \leq 90^o$ in increaments of $\Delta\theta = 1^0$. Can you explain how to use FFT algorithms to calculate $A(\theta)$? Draw $P(\theta)$ vs θ.

2. Consider now to use unequal weighting at different sensors for example by applying a triangular distribution of weights instead of uniform (in similarity to Bartlet vs rectangular window. Calculate and draw $P(\theta)$ vs θ making the same assumptions as in item 1.

3. Consider now 'steering' the power pattern of the array to an angle $\phi \neq 0$. To achieve this choose $\phi = 30^o$, for example, and generate the weight vector

   $$w_m[n] = e^{\frac{j2\pi(m-1)d\sin(\phi)}{\lambda}}, \qquad m = 1, \ldots, M, \qquad \forall n$$

   Calculate and draw $P(\theta)$ vs θ making the same assumptions as in item 1.

The above procedure that shapes the power pattern of the array is called BEAMFORMING.

PART B. Linear array and Bearing Estimation

Let assume now that a number of narrowband radiating sources (such as a modulated sinusoidal carrier) , $s_i()$, $i = 1, ..., I$ located far from array at different angles θ_i as indicated in figure 1. Then, to find the directions of arrival of the radiating signal from each source and therefore detect their presence, we need to estimate the Power spectrum along SNAPSHOTS of the array and draw it in terms of $-90^o \leq \theta \leq 90^o$ as shown in Figure 2. This is what we call the "BEARING ESTIMATION PROBLEM". In practice noise will be alo present in addition to the signals. Thus, to increase the accuracy of our estimates the estimation will be repeated for many snapshots over n and an average estimate will be obtained. Furthermore, since the signals are real valued, to avoid disrtributing the power at both $\pm\theta$ due to the symmetry of the power spectrum , first we must compute the analytic signal for each sensor signal $x_m(n), \quad n = 1, \ldots, N$, as indicated in Figure 3.

1. Suppose you are given two narrowband radiating sources $s_1[t] = cos[2\pi \cdot 10^4 \cdot t]$ and $s_2[t] = 0.5 \cdot cos[2\pi \cdot 10^4 \cdot t + \frac{\pi}{6}]$ (that is of the of the same wavelength λ) radiating from angles $\theta_1 = 20^o$ and $\theta_2 = 40^o$, respectively, towards a linear array consisting of $M = 12$ sensors uniformly spaced at $d = \frac{\lambda}{2}$.
   Sample the combined signal with a sampling frequency $F_s = 4 \cdot 10^4$ and generate the combined signals sensed by the array sensors $m = 1, \ldots, 12$ over time, that is
   $$s_m[n] = cos[\frac{\pi}{2}n + \pi(m-1)\sin(\theta_1)] + 0.5 \cdot cos[\frac{\pi}{2}n + \frac{\pi}{6} + \pi(m-1)\sin(\theta_2)]$$
   for $m = 1, 2, \ldots, 12$, $n = 1, 2, \ldots, 512$.

2. For each sensor signal $s_m[n]$, $m = 1, \ldots, 12$ generate independently a noise sequence as a zero mean, unit variance Gaussian random sequence $q_m[n]$, $n = 1, ..., 512$ by using the corresponding MATLAB random generator. Then, generate
   $$x_m[n] = s_m[n] + G \cdot q_m[n], \qquad , n = 1, \ldots, 512$$
   where, G controls the corresponding SNR. That is,
   $$G = \sqrt{(\frac{1}{N}\sum_{n=1}^{512} |s_m[n]|^2)} \cdot 10^{-\frac{SNR}{20}}$$
   Assume that SNR=20 dB.

3. For each noisy sensor signal $x_m[n]$, $n = 1, \ldots, 512$ calculate the analytic signal $y_m[n]$, $n = 1, \ldots, 512$ as described in Figure 3.

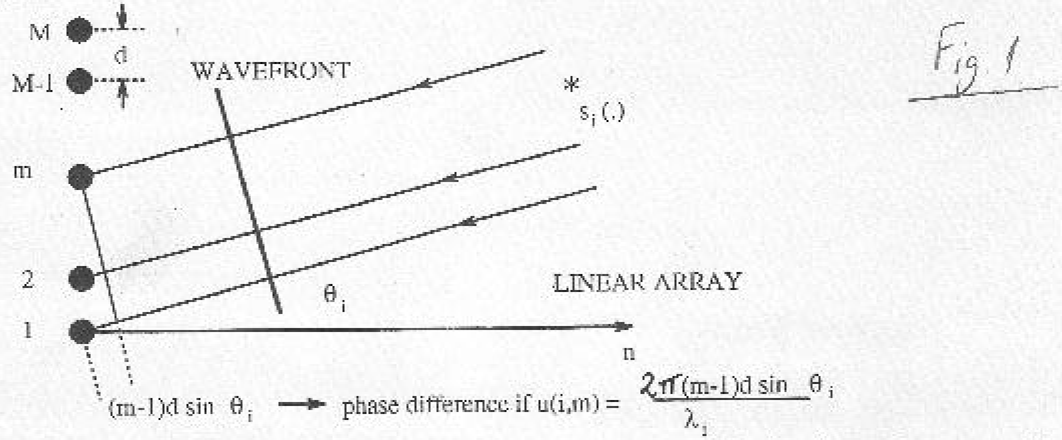
4. For $-90^o \leq \theta \leq 90^o$ in increaments of $\Delta\theta = 1^0$, calculate
   $$A(\theta, n) = \sum_{m=1}^{M} y_m[n] \cdot e^{\frac{j2\pi(m-1)d\sin(\theta)}{\lambda}}$$
   for $n = 1, \ldots, 512$.

5. Average over n to obtain
   $$P(\theta) = \frac{1}{512}\sum_{n=1}^{512} |A(\theta, n)|^2$$
   for $-90^o \leq \theta \leq 90^o$. Draw $P(\theta)$ vs θ. Observe the results and comment appropriately.

PART C. Linear array and Spatial Filtering

Assume now that one of the signals in Part B is unwanted interference (e.g., the signal at 20^o.). Design a process by combining Parts A and B to filter out the interference. Implement this process and calculate and draw the corresponding $P(\theta)$ vs θ.

■ 1-d ARRAY PROCESSING



d: spacing betwenn sensors, λ: wavelength of source $s_i(i)$, θ_i: bearing of source $s_i(.)$

■ MODEL EQUATION [I incoherent sources (i.e., with different $\frac{\sin\theta_i}{\lambda_i}$)]

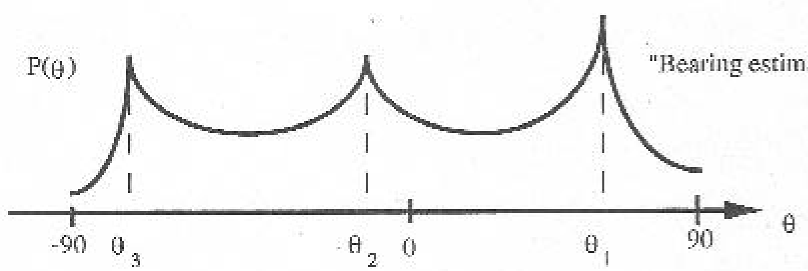
$$x_m(n) = \sum_{i=1}^{I} s_i[u(i,m) + \phi(i,n)] + w_m(n)$$

m^{th} sensor at time n phase difference random phase noise

Fig. 2

Estimate P(θ)~θ or equivalently find the distribution of power for $\{x_m(n)\}$ on the exponentials $\left\{e^{\frac{j2\pi(m-1)d\sin\theta_i}{\lambda}}\right\}$, $m = 1,2,\ldots,M$

- **Objective:** Given $\{x_m(n)\}$, n=1,...,N, m=1,...,M, estimate $\{\theta_i\}$, i=1,...,I



■ **DUALITY BETWEEN PSE and AP**

1) $P(f)$, $-\frac{1}{2} \le f \le \frac{1}{2}$ ↔ $P(\theta)$, $-90 \le \theta \le 90°$

2) Frequency, $f \leftrightarrow$ Angle, $\sin\theta$

3) Sampling period, $T \leftrightarrow$ sensor spacing, d

4) Aliasing $\left[f_o > \frac{1}{2T}\right] \leftrightarrow$ Aliasing $[\lambda<2d]$

■ COMPLEX ANALYTIC SIGNAL FOR SENSOR m

$$y_m(n) = x_m(n) + jH\left[x_m(n)\right]$$ where H[.] is the Hilbert transform

■ COMPUTATION USING FFT

Fig 3

Given $\{x_m(n)\}$, $n=1,2,\ldots,N$

1) Obtain: $X_m(k) = \mathrm{FFT}[x_m(n)]$, $k=1,\ldots,N$

2) Form: $$Y_m(k) = \begin{cases} X_m(k), & k = 2,3,\ldots,\frac{N}{2} \\ X_m(k)/2, & k = 1, \frac{N}{2}+1 \\ 0, & k = \frac{N}{2}+2,\ldots,N \end{cases}$$

3) Obtain: $y_m(n) = \mathrm{IFFT}\,[Y_m(k)]$, $n=1,\ldots,N$