THE CONCEPT OF FREQUENCY

A) SINUSOIDS

- **Continuous time** $(-\infty < t < \infty)$
- $x(t) = A\cos[\Omega t + \theta] = A\cos[2\pi F t + \theta]$
- F : cycles/sec
- Periodic with period $T = \frac{1}{F}$, i.e.,
  $x(t) = x(t+T) = x(t+\ell T)$, T: fundamental period
- F takes any value : $-\infty < F < \infty$
- Distinct sinusoids are obtained from any distinct frequencies in $-\infty < F < \infty$
- The highest rate of oscillation is achieved when $\Omega \to \infty$, or $F \to \infty$

- **Discrete time** $(n = 0, \pm 1, \pm 2, \ldots$
- $x(n) = A\cos[\omega n + \theta] = A\cos[2\pi f n + \theta]$
- f : cycles/sample
- Periodic with period N samples, i.e.
  $x(n) = x(n+N) = x(n+\ell N)$
  N: fundamental period
  $A\cos(\omega n + \theta) = A\cos(\omega(n+N) + \theta) \Rightarrow$
  iff
  $\Rightarrow \omega n = \omega(n+N) \pm 2k\pi \Rightarrow \boxed{f = \pm \frac{k}{N}}$
  i.e., f must be rational

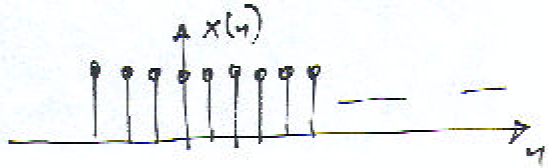
- Let $\omega_1 = \omega_2 + 2k\pi$: Then $\cos[\omega_1 n + \theta] = \cos[\omega_2 n + \theta]$
  Thus distinct discrete sinusoids can be obtained only in intervals of 2π, i.e. $-\pi \le \omega \le \pi$ or $0 \le \omega \le 2\pi$.
  (principal range) i.e. $-\frac{1}{2} \le f \le \frac{1}{2}$ or $0 \le f \le 1$
- The highest rate of oscillation for a discrete sinusoid is achieved when $\omega = \pi$ or $f = \frac{1}{2}$.

Ex. Let $x(n) = \cos \omega_0 n$

$\omega_0 = 0, \pi/8, \pi/4, \pi/2$

$f = 0, \frac{1}{16}, \frac{1}{8}, \frac{1}{4}, \frac{1}{2}$

a) $\omega_0 = 0$, $f = 0$, period $N = \infty$



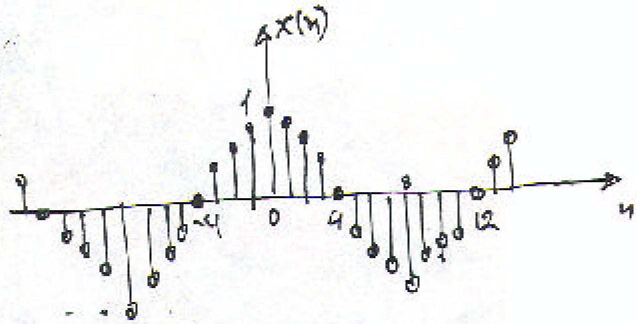
b) $\omega_0 = \frac{\pi}{8}$, $f = \frac{1}{16}$, period $N = 16$.

zeros

$\cos\left[\frac{\pi n}{8}\right] = 0 \Rightarrow \frac{\pi n}{8} = k\pi + \frac{\pi}{2} \Rightarrow n = 8k + 4$

$k = 0, 1, 2, 3, \ldots$

$n = 4, 12, 20, \ldots$



maxima

$\cos\left[\frac{\pi n}{8}\right] = \pm 1 \Rightarrow \frac{\pi n}{8} = k\pi \Rightarrow n = 8k$

c) ...

Highest possible rate of oscillation

d) $\omega_0 = \pi$, $f = 1/2$, $N = 2$

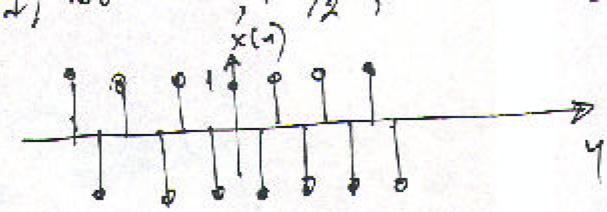
Zeros

$\cos[\pi n] = 0$

$\pi n = k\pi + \pi/2 \Rightarrow n = k + 1/2$

n : integer

no solution



Maxima $\pi n = k\pi \Rightarrow n = k$