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EC431H1 Digital Signal Processing
FINAL EXAM
April 28, 2003, 2:00 p.m.
Instructor: D. Hatzinakos

Instructions:

1. Type A exam
2. Non-programmable calculators are allowed
3. Please solve all five problems. All problems are equally weighted.
4. All answers must be written in the examination booklet. Do not write any answers in this problem handout.

PROBLEM 1 (10 points)

let $\{x[n]\}_{n=0}^{1} = \{-2, 2\}$, $\{y[n]\}_{n=0}^{3} = \{0, -2, 2, 0\}$and $\{z[n]\}_{n=0}^{3} = \{0, -2, 0, 2\}$

(a) Calculate $X(\omega) = DTFT\{x[n]\}$ and sketch the magnitude and phase for $|\omega| < \pi$.

(b) Calculate $Y(\omega) = DTFT\{y[n]\}$ and calculate $Z(\omega) = DTFT\{z[n]\}$. Express the results in terms of $X(\omega)$.

(c) Calculate $Y(k) = 4 - DFT\{y[n]\}$ and Calculate $Z(k) = 4 - DFT\{z[n]\}$. Express the results in terms of $X(\omega)$.

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PROBLEM 2 (10 points)

(a) The bilinear transformation $s = 2(1 - z^{-1})/(1 + z^{-1})$ was applied to an analog prototype $H_a(s) = 1/(s^4 + 1/2)$ to design a digital filter. Calculate the (steady-state) response $y[n]$ of the digital filter for input $x[n] = 3cos(\frac{\pi}{3}n + \frac{\pi}{4})$. (5 points)

(b) Indicate which of the following statements are ”True” or ”False”.

(i) FIR filters always have generalized linear phase (1 point)

(ii) The bilinear transform $s = a\frac{1-z^{-1}}{1+z^{-1}}$ where a is an appropriate constant can map a high-pass analog filter to a Low pass digital filter. (1 point)

(iii) In windowing design of FIR filters, the rectangular window gives a shorter transition band than the Hamming window. (1 point)

(iv) In windowing design of FIR filters, the rectangular window gives lower ripples than the Hamming window. (1 point)

(v) Let $X(k)$ and $X_d(\omega)$ be 32-DFT and DTFT of a real-valued sequence $x[n]$ of length 32 samples. Then, $X(29) = X_d^*(\frac{3\pi}{16})$ where * denotes conjugation operation. (1 point)

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PROBLEM 3 (10 points)

(a) Derive explicitly and draw the signal diagram (butterfly structure) of a 4-point, radix-2 , decimation in time FFT algorithm.

(b) Let $\{x[n]\}_{n=0}^{2} = \{1, 2, 3\}$, $\{y[n]\}_{n=0}^{1} = \{1, 0, -1\}$

(i) Evaluate the linear convolution $x[n] * y[n]$ using the 4-FFT derived in part (a).

(ii) Evaluate the 4-point circular convolution between $x[n]$ and $y[n]$ using the 4-FFT derived in part (a).

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PROBLEM 4 (10 points)

Consider the following system, where $T = 0.001$ sec, and $H(\omega)$ is an ideal LPF with $\omega_c = \frac{\pi}{L}$. The $F_a(\Omega)$ is an analog compensation filter, picked such that the system from $y[n]$ to $y_a(t)$ functions as an ideal D/A converter, with any signal $x_a(t)$ bandlimited to 500π rad/sec. $F_a(\Omega)$ can have a transition band , in which its response can be arbitrary.

(i) Assuming $L = 1$, find the beginning and end of the transition band of $F_a(\Omega)$.

ii) Repeat (a) assuming $L = 5$

iii) Sketch the magnitude Frequency responce of the ZOH for an arbitrary L

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PROBLEM 5 (10 points)

The system equation of a LSI system is given below

$$y[n] - y[n-1] + \frac{1}{9}y[n-2] = x[n] - \frac{1}{2}x[n-1]$$

(a) Is this a linear phase system ?

(b) Draw the Cascade and Parallel realizations for this system using first order Direct II sections.

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