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MIDTERM EXAMINATION
ECE431H1S, Digital Signal Processing
Time allowed: 60 minutes
February 21, 2005
Examiner: D. Hatzinakos

Exam type A
Non-programmable Calculators are allowed

A DC signal generator of nominal output 10 Volts is subject to a small sinusoidal error, that is, the generated signal is $x(t) = 10 + 0.5\cos[10\pi \cdot t]$, $t = 0,\dots,\infty$. The signal is observed over T sec. and sampled with sampling period T_s.

1. What is the Nyquist rate for this system? Assuming that you sample the signal at above the Nyquist rate, and obtain $x[n], n = 0,1,\dots, N-1$ what is the minimum number of samples N you need in order to achieve sufficient spectral resolution for detecting the sinusoidal component?

HIGHEST FREQ. COMPONENT : 5 Hz

∴ NYQUIST RATE: $\boxed{10\ Hz}$

① USING $F_s = 10$

$\Delta F = 5$ Hz, NEED $\frac{1}{N} \leq \frac{\Delta F}{F_s} \Rightarrow N \geq \frac{F_s}{\Delta F}$ GET $\boxed{N \geq 2}$

∴ $\boxed{N_{min} = 2}$ ①

2. Can you describe a process or design a filter to completely remove only the DC component from the $x[n]$?

① DC COMPONENT ⇔ AVERAGE VALUE

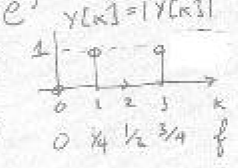
1. Calculate average: $\frac{1}{N}\sum_{n=0}^{N-1} x[n]$
2. Subtract average from $x[n]$.

Student Name: Student Number:

3. Using $T_s = 0.05$ sec, and assuming that the DC component has been removed, obtain $y[n] = 0.5\cos[10\pi \cdot n \cdot T_s]$, n=0,1,2,3 and calculate the 4-DFT $Y(k)$. Plot the magnitude $|Y(k)|, k = 0,1,2,3$. Does the DFT provide a good estimate of the real frequency F and the magnitude A of the sinusoidal component? (Hint: To find A recall that the corresponding value at the DFT domain is $0.5 \cdot A \cdot T / T_s$). If not what is the problem?

$y[n] = \{0.5, 0, -0.5, 0\}$, $Y[k] = \sum_{n=0}^{3} y[n] e^{-j\frac{2\pi k n}{4}} = 0.5 - 0.5 e^{j\pi k}$

$$= \begin{cases} 0 & K=0 \\ 1 & K=1 \\ 0 & K=2 \\ 1 & K=3 \end{cases}$$



Have a peak at K=1 (k=3 is an image)

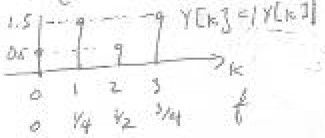
This CORRESPONDS TO $F = F_s/4 = 5$ Hz WHICH IS REAL FREQUENCY OF SINUSOID.

ALSO, SINCE $T/T_s = 4$, $0.5\frac{AT}{T_s} = 2A$, AND SINCE DFT AT K=1 IS 1 MEANS WE ESTIMATE A=0.5 WHICH IS TRUE MAGNITUDE OF SINUSOID.

4. Repeat the previous step with 6 samples of $y[n], n = 0,\ldots,5$. and calculate the 4-DFT $Y(k), k = 0,1,2,3$. Plot the magnitude $|Y(k)|, k = 0,1,2,3$. Compare the estimates of the sinusoidal component in the two cases and comment about the influence of aliasing, spectral leakage or any other factor that affects the estimation.

$y[n] = \{0.5, 0, -0.5, 0, 0.5, 0\}$, TIME ALIAS $\Rightarrow \{1, 0, -0.5, 0\}$

$\therefore Y[k] = 1 - 0.5 e^{j\pi k} = \{0.5, 1.5, 0.5, 1.5\}$



AGAIN, HAVE PEAK AT F = 5Hz, WHICH IS CORRECT, AND SINCE $T/T_s = 6$, $0.5\frac{AT}{T_s} = 3A$, SO HAVE CORRECT ESTIMATE OF SINUSOID MAGNITUDE.

THERE IS ALIASING AND SPECTRAL LEAKAGE IN BOTH CASES, BUT STILL ABLE TO ESTIMATE F AND A CORRECTLY IF THE PEAK VALUE OF 4-DFT IS USED. } IN TERMS OF THE DTFT

IN PART 3, HOWEVER THERE IS NO ALIASING OR SPECTRAL LEAKAGE IN TERMS OF 4-DFT

IN PART 4 THERE IS ALIASING AND SPECTRAL LEAKAGE BUT DO NOT AFFECT THE ESTIMATION OF THE SINUSOID

Student Name: Student Number:

ALTERNATIVELY:

$$\begin{tikzcd} Y(k),\ k=0,1,2,3 \arrow[r] \arrow[d, leftrightarrow] & \boxed{\uparrow 256} \arrow[r] & \otimes \arrow[r] & Y(k),\ k=0,1,\ldots,1023 \arrow[d, leftrightarrow] \\ y[n],\ n=0,1,2,3 + 256 \text{ images} \arrow[rrr] & & & y[n],\ n=0,\ldots,1023 \end{tikzcd}$$

256 (into $\otimes$)

5. Given the 4-DFT of step 3 and properties of the DFT calculate the 1024-DFT of $y[n] = 0.5\cos[10\pi \cdot n \cdot T_s]$, $n=0,1,\ldots,1023$, where T_s is as before.

NOTICE THAT IN 3, THE PERIODIC EXTENSION OF THE FOUR POINTS IS THE SINUSOID ITSELF, AND THE 4-DFT HAD ONLY TWO NON-ZERO COMPONENTS, BUT THE PERIODIC EXTENSION OF THE SIX POINTS (USING A PERIOD OF 4) IS NOT THE SINUSOID AND THE 4-DFT HAS NO ZERO COMPONENTS. IN THIS QUESTION, THE 1024 pts WILL BE PERIODICALLY EXTENDED WITH PERIOD 1024 AND THIS WILL GIVE THE SINUSOID AGAIN. SO IT IS EXPECTED, AND CAN BE VERIFIED, THAT THE 1024-DFT WILL HAVE ONLY TWO NON-ZERO COMPONENTS, BOTH OF $\boxed{\text{HEIGHT: } 256}$, INDICES $\boxed{K = 256 \text{ AND } K = 768}$

6. Assuming that the sampled generator output $x[n]$ is quantized with a uniform quantizer, how would you choose the dynamic range and number of bits of the quantizer so that the sinusoidal component is removed from $x[n]$? What is the signal to quantization noise ratio, SQNR, in this case?

SINUSOID CAUSES VARIATIONS OF DC COMPONENT AROUND 10 (i.e FROM 9.5 to 10.5 Volts).

SO, TO REMOVE SINUSOID, MAKE QUANTIZER CLIP ALL SIGNALS HIGHER THAN 9.5 V, [SO CAN HAVE A DYNAMIC RANGE FROM −9.5 to 9.5 FOR EXAMPLE], AND USE JUST ONE BIT TO ENCODE. AS LONG AS QUANTIZER ALLOWS FOR POSITIVE VOLTAGES, ONE BIT IS SUFFICIENT TO GET THE DC SIGNAL AND REMOVE THE SINUSOID. IN THIS CASE THERE IS NO NOISE, SO

SQNR = ∞.

IN GENERAL CHOOSE DYNAMIC RANGE AND NUMBER OF BITS SO THAT

$$\Delta = \frac{V_{span}}{2^B} > 1$$