Student Name: Student Number:

University of Toronto
Faculty of Applied Science and Engineering

MIDTERM EXAMINATION
ECE431, Digital Signal Processing
February 28, 2003, 9-10 am, BA2175
Examiner: D. Hatzinakos

Exam type A
Non-programmable Calculators are allowed

In a frequency coding scheme, symbols (letters, numbers, etc.) are represented by a combination of two frequencies F_1 and F_2 where $F_1 \neq F_2$ and each taking one out of 10 possible values, e.g., 250 Hz, 750 Hz, 1250 Hz, 1750 Hz, 2250 Hz, 2750 Hz, 3250 Hz, 3750 Hz, 4250 Hz and 4750 Hz. In the transmitter continuous time signals xa(t)= $[\ \sin(2\pi F_1 t) + \sin(2\pi F_2 t)\ ]$, t=0,....T sec are formed and transmitted consequently every T secs. In the receiver detection of the transmitted symbols is based on discrete signal processing techniques. In other words assuming perfect synchronization is possible, T sec long signal segments are extracted from the incoming signal, each segment is ideally and uniformly sampled with a period of Ts secs and then a N-point DFT is computed and plotted to detect the underlying two frequencies and from those the transmitted symbol.

a) Let T/Ts=L. Write an expression for the discrete signal x(n)=xa(n Ts), n=0,...,L-1. Choose an appropriate value for Ts and then calculate the normalized frequencies f_i, $i = 1,2,...,10$ corresponding to the real frequencies F_i, $i = 1,2,...,10$. (3 points)

b) What should be the minimum value of L (or the corresponding length T) so that our frequency detector has sufficient resolution to detect any two of the frequencies that form the signal? Assuming that L and Ts have been properly chosen, draw approximately the DTFT of a signal segment containing the first two frequencies.(3 points)

c) What is the relation between the DTFT $X(f)$ and the N-DFT $X(k)$, k=0,1, ...,N-1of x(n), n=0,1,...,L-1? In your opinion, what should be the value of N so that we achieve best detection of the frequencies that form the signal? What are the corresponding values of k of the N-DFT that correspond to the frequencies to be detected? (4 points)

d) As N in part (c) increases, one observes that the maximum values of the N-DFT do not necessarily correspond to those values of k that represent the true locations of the sinusoidal frequencies . Furthermore the estimated sinusoids seem to be of different power. Why does this happen? (3 points)

e) Given x(n), n=0,1,...,L-1, describe a DSP procedure to calculate only those values of $X(f=250Ts+k500Ts)$, k=0,1,...,9 corresponding to the 10 possible frequencies in the signal. Do you think that this is a more efficient way to estimate and detect the true frequencies in the observed signal? Please explain explicitly. (6 points)

f) Suppose that in an effort to improve detection (i.e. decide the presence or not of a frequency component), you try an averaging technique as follows:

$$X_{avg}(k)=\alpha X(k-1)+X(k)+\alpha X(k+1)$$

where α is a constant of amplitude less than 1. Averaging in this manner is equivalent to multiplying the signal x(n) by a new window w(n) before computing the DFT. Derive a simple formula for w(n). (6 points)