Student Name: Student Number:

University of Toronto
Faculty of Applied Science and Engineering

MIDTERM EXAMINATION
ECE431, Digital Signal Processing
March 3, 2003, 5-6 pm, HA 401
Examiner: D. Hatzinakos

Exam type A
Non-programmable Calculators are allowed

Pierre, a piano tuner, has asked for your help to tune a piano (apparently he suffers from a severe ear infection). You propose to employ frequency domain DSP operations. In this respect, two tones (frequencies) from two different pianos (one of the pianos is tuned and is used as reference to tune the other piano) are recorded and their distance is estimated based on frequency analysis of the recorded signal. Assuming the two frequencies are F_1 and F_2 where $F_2 = F_1 \pm \Delta F$ and $F_1, F_2 \leq 5$ kHz, the recorded continuous time signal takes the form xa(t)= $[\ \sin(2\pi F_1 t) + \sin(2\pi F_2 t)\ ]$, t=0,...,T sec. The objective is that $|\Delta F| \longrightarrow 0$. The recorded signal is ideally and uniformly sampled with a period of Ts secs and then a N-point DFT is computed and plotted to estimate the distance between the two frequencies.

a) Let T/Ts=L. Assume that 1/Ts has been chosen at least four times greater than the highest possible frequency so that aliasing is negligible. Also, assume that we require $\Delta F \leq 0.01$ Hz in order to decide that the piano has been properly tuned. What should be the minimum value of L (or the corresponding length T) so that our frequency estimator has sufficient resolution to decide whether tuning of the piano has been achieved? What are the normalized frequencies f_1 and f_2, corresponding to the true frequencies F_1 and F_2? (3 points)

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b) What is the minimum length of the N-DFT , X(k) , k=0,1,...,N-1 we should calculate? What is then the approximate range of values of k of the N-DFT corresponding to the true frequencies F_1 and F_2? (3 points)

c) To improve our decision process, we decide to zoom on the region of the DFT containing the two frequencies in order to examine it in more detail. Given that X(f) , is the DTFT and X(k), k=0, 1, ..., N-1 is the N-DFT of the discretized signal x(n), n=0,1,...,L-1, we would like now to compute (interpolate) N new samples of X(f) between $f = f_1 - \Delta f$ and $f = f_1 + \Delta f$. Describe a digital interpolation procedure to achieve this, by clearly identifying all the corresponding operations and parameters. (8 points)

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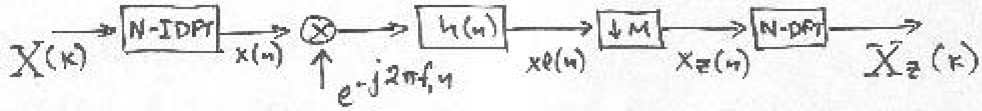
d) One possible system that can implement the zoom operation is shown in Figure 1. Assume that h(n) is an ideal low pass filter with cutoff frequency Δf.



What is the largest (possibly no integer) value of M that can be used if aliasing is to be avoided in the downsampler? (5 points)

e) Assume that the DTFT of x(n) is as shown in Figure 2. Using the values of N and M from the previous question, sketch the DTFTs of the intermediate signals xl(n) and xz(n) as well the N-DFTs X(k) and Xz(k) when $f_1 = 0.25$ and $\Delta f = 0.025$. (6 points)