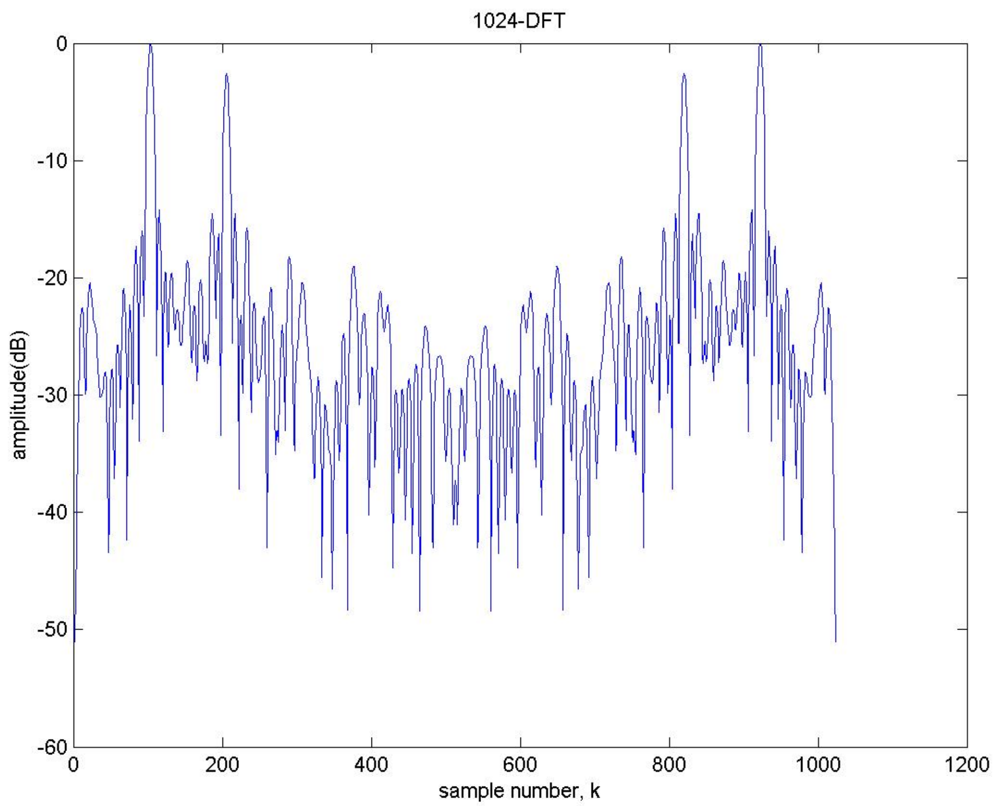
ECE1511, 2009
PROBLEM SET 1 (2 problems)

PROBLEM 1.
The plot below is the 1024-DFT of a set of data made up of 1024-samples taken with sampling period T=0.001 sec. The horizontal axis is labeled with the index *k*, the sample number of the DFT entries.



a) What length is the time interval from which these samples are taken? What is the Nyquist frequency for this system? At what frequency in (Hz) are the two dominant peaks located?

b) Assuming that a rectangular window of length less than 1024 T sec was applied to the signal prior to sampling, what would be the shortest such window you could use which would provide sufficient spectral resolution to distinguish the two dominant spectral peaks?

c)Using the given 1024-DFT describe how to obtain an estimate of the corresponding signal at time 10.5 msec.

d) Assuming that the signal is well matched to the dynamic range of the quantizer, what is the minimum number of bits per second required for the transmission of this signal with signal-to-Quantization noise ratio at least 30 dB ?

e) By observing the given DFT spectrum, we realize that we are wasting bandwidth with the applied discritization process (why is that?). Therefore, in addition to the given signal $x_1[n]$, we have been asked to transmit or store a second digital signal $x_2[n]$ with similar spectral properties to those of $x_1[n]$. To achieve this, the following two candidate procedures have been proposed:

1. Form and store or transmit the signal $y[n] = x_1[n] + (-1)^n x_2[n]$.
2. Form and store or transmit the signal $z[n]$ where $z[2n] = x_1[2n]$ and $z[2n+1] = x_2[2n]$.

Are the described procedures reversible? Draw approximately the corresponding frequency content of the y[n] and z[n] and comment appropriately. Describe the process to reconstruct $x_1[n]$ and $x_2[n]$ in each case by means of a block diagram.

f) Assuming that the same quantizer has been used for both signals in part (e), what is the resulting Signal-to-Quantization noise ratio per signal in the two cases considered? Assuming that we tolerate no loss of important signal's frequency content, how much could you have improved the Signal-to-Quantization noise ratio by low pass filtering each individual signal before forming any combination of the signals?

PROBLEM 2.

Let $x(n) = (0.9)^n u(n)$.

(*a*) Determine $x(n) * x(n)$ analytically, and plot its first 101 samples.

(*b*) Truncate $x(n)$ to the first 51 samples. Compute and plot the convolution $x(n) * x(n)$, using the `conv` function.

(*c*) Assume that $x(n)$ is the impulse response of an LTI system. Determine the `filter` function coefficient vectors `a` and `b`. Using the `filter` function, compute and plot the first 101 samples of the convolution $x(n) * x(n)$.

(*d*) Comment on your plots. Which MATLAB approach is best suited for infinite-length sequences and why?