A Network Shadow Fading Gain Model For Autonomous Cellular Networks

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Abstract

In traditional cellular networks the base stations (BSs) are more or less regularly deployed. In the channel model, correlation is only considered between a reference node (BS or terminal) and multiple nodes. Essentially, the correlation between two links with no common end is neglected. This may be a reasonable model for large cell sizes (or equivalently large inter-site distances) in traditional networks. In autonomous cellular networks, however, a large number of BSs are deployed in random positions according to the network traffic demand. With the dense irregular deployment of BSs, a more comprehensive correlation model should be developed which can capture correlation among links with no common end. In this paper, we re-visit the principles of the shadow fading phenomenon and discuss its essential properties. Accordingly a model is proposed which efficiently generates the channel gains. The proposed model is compared with earlier models in channel gain correlation, outage probability for large/small cells and aggregate interference.

Index Terms

Shadow fading, Correlation, Performance evaluation, Cellular networks
I. INTRODUCTION

The propagation of electromagnetic waves in a wireless communication system is studied from two main standpoints. The first class considers the fine structure of the multi-path propagation and is known as small-scale (or multi-path) fading. Multi-path fading studies the time and frequency variations of the channel impulse response due to differences in the delays, phases and amplitudes of the multiple reflections of the transmitted signal arriving from different directions at the moving/static receiver. It has been extensively studied and well-established analysis and simulation models have been developed. The second class deals with the signal power attenuation on a macroscopic level and is often referred to as large-scale (or shadow) fading.

Large-scale fading predicts the average received signal power level. The received power is not only a function of the distance from the transmitter but also highly dependent on the topographical properties of the environment. Due to lack of accurate propagation measurements and landscape information, the effect of the environment on signal propagation is modeled by a random process often referred to as shadow fading.

The shadow fading gain is widely assumed to be the result of a large number of multiplicative attenuating factors. Hence based on the Central Limit Theorem, the logarithm of the received power has a normal distribution. The log-normality of the shadow fading gain has been verified with various numerical measurements [1], [2]. In [3], an alternative additive model as a physical basis for shadow fading is proposed. This model also results in a log-normal distribution.

In a sense, shadow fading accounts for the topographical properties of the communication environment. Consider the link between a moving terminal and a fixed BS. Due to the natural continuity in the topography, the channel gain experienced by the terminal cannot change abruptly after having traveled a relatively small distance. In other words, the channel gain experienced by the terminal should be correlated along the line of travel. Gudmundson studies this scenario in [4] and proposes an exponentially decaying correlation function. This model is verified by
experimental results in [2] and has become the most widely used model in the literature.

The correlation discussed in time is similarly applicable in the spatial domain. Following the same arguments, the channel gains between a BS and two terminals (or two BSs and a terminal) should also be correlated. Mathematically, if we view the channel gains of all links in the network as random processes in time, the temporal and spatial correlations should be reflected in the auto- and cross-correlation functions respectively. The proposed model by Gudmundson can be easily modified to capture spatial correlations [5], [6]. In [7] an efficient way of generating shadow fading gains from one (reference) point to multiple points is proposed. In order to do so, a two-dimensional Gaussian field across the area of interest is generated. The value of the Gaussian field at each point is equal to the shadow fading gain to the reference point. This field is often referred to as a shadow fading map. Several techniques have been developed to generate these maps efficiently [8]–[11]. It should be noted that shadow fading maps can generate correlated channel gains between links with one common end. Hence, in order to generate channel gains from two BSs to a set of terminals, two separate shadow fading maps should be created.

In traditional cellular networks the BSs are more or less regularly deployed. In the channel model for such networks, correlated channel gains are only considered between a BS and multiple terminals (or one terminal and multiple BSs). This may be a reasonable model for large cell sizes (or equivalently the large inter-site-distances) in the traditional networks. In the future autonomous cellular networks, however, it is expected that a large number of BSs will be deployed in random positions. In this case, the correlation between links with no common end plays an important role for a realistic evaluation of the system performance.

In [12], an underlying Gaussian field, referred to as spatial loss field, is proposed to generate correlation between all links in wireless sensor networks. This field is different from a shadow fading map since the channel gains between all nodes in the network — whether they have a common end or not — are generated from the same field. The proposed method has the following drawbacks:
• The authors propose to generate the channel gain between any two points as a function of the line integral of the Gaussian field between the desired two points. Depending on which direction one takes to calculate the integral, two opposite values are derived for the shadow fading gain. This can be translated into a correlation coefficient of $-1$ between the up-link and down-link channel gains. This is in contrast to the reported measurements in the literature which suggest high correlation between the up-link and down-link directions [13], [14].

• Early studies in radio propagation report a constant variance for the shadow fading gain [15]. Accordingly, the shadow fading function in [12] has a constant variance. However, more recent findings suggest that the variance should change with distance which conforms by the general rule of continuity in all natural phenomena [16].

In this paper, we propose a shadow fading model for autonomous cellular networks. Section II provides the background material and motivates the need for a new model. Section III starts with three essential properties and accordingly the model is developed. In section IV, the effect of the proposed model on correlation between channel gains, outage probability in large/small cells and aggregate interference in an autonomous cellular network is studied. Section V concludes the paper with a summary of the work.

Notation: Bold upper case letters ($X$) denote matrices and bold lower case letters ($x$) are adopted for column vectors. The Hermitian transpose of $X$ and the expected value of random variable $X$ are denoted by $X^H$ and $E\{X\}$ respectively. $X \sim \mathcal{N}(\mu, \sigma^2)$ denotes a random variable $X$ with a Gaussian distribution, a mean of $\mu$ and a standard deviation of $\sigma$.

II. BACKGROUND MATERIAL

In free space with a line-of-sight link between a transmitter and a receiver, the Friis formula gives the received signal power level, $P_r$, as a function of the transmit power level $P_t$, the
transmitter antenna gain $G_t$ and the receiver antenna gain $G_r$, as follows [17]:

$$P_r = P_t G_t G_r \left( \frac{\lambda}{4\pi d} \right)^2,$$

(1)

where $\lambda$ is the wavelength of the transmitted signal and $d$ is the distance between the transmitter and the receiver.

Given a few parameters $G_t, G_r, \lambda$ and $d$, the received signal power can be predicted in free space. In terrestrial communication the situation is more complicated. The received signal power is not only a function of these parameters but it is also and more importantly a complex function of the communication environment and its topography. Two links of the same length but sufficiently far from each other will have different received signal power levels. This is due to the unique topographic characteristics each link experiences. The average received signal power is approximated by

$$P_r = c \cdot \frac{1}{d^\gamma} \cdot L,$$

(2)

where the constant $c$ is a function of $P_t, G_t, G_r, \lambda$ and other factors. The unknown effect of the topography on the signal is captured by $L$, a random variable also known as the shadow fading gain. Based on empirical results, $L$ has a log-normal distribution, i.e. $\log L \sim \mathcal{N}(0, \sigma_0^2)$.

Due to the inherent correlation in the topography of the communication environment, the $L$ values for different links should correspondingly be correlated. Let us consider the link from node A to node B and the link from node A to node C in Figure 1. Naturally links A – B and A – C should be correlated. In 1991, Gudmundson proposed a correlation model [4] which has become the most widely used in the literature. The proposed correlation function between links A – B and A – C is

$$R(d) = \sigma_0^2 e^{-d/d_c},$$

(3)

where $d$ is the distance between node B and node C and $d_c$ is the correlation distance.
According to the Gudmundson model, the channel gains between a given BS and a set of $K$ terminals are generated in the following four steps:

- **Step 1:** Generate a $K \times 1$ vector $\mathbf{v}$, where $v_i$’s are independent Gaussian random variables with zero mean and a standard deviation of unity.
- **Step 2:** Generate the $K \times K$ correlation matrix $\mathbf{R} = [r_{ij}]$ such that
  \[ r_{ij} = \sigma_0^2 e^{-d_{ij}/d_c}, \]  
  where $d_{ij}$ denotes the distance between terminal $i$ and terminal $j$.
- **Step 3:** Decompose the correlation matrix using the Cholesky factorization such that $\mathbf{R} = \mathbf{B}\mathbf{B}^H$.
- **Step 4:** The correlated channel gains between the BS and the $K$ terminals are stacked in the $K \times 1$ vector $\mathbf{Bv}$.

In [7], a two-dimensional spatial correlation model is proposed. The model generates correlated shadow fading gains from one BS to multiple terminals. For a given BS, a two-dimensional Gaussian random field with an appropriate spatial correlation is generated. A Gaussian field is a random process where each realization is a function from the plane to the real numbers. If we

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1The underlying spatial correlations are generated based on the Gudmundson model.
fix a point in the plane, the outcome is a Gaussian random variable. The shadow fading gain between the BS and any terminal in the field is the value of this random variable at the terminal.

Let us consider the links A – B and C – D where transmitters A and C are located close to each other and receivers B and D share the same topographic properties. Clearly the shadow fading gains of the two links should be correlated. However since the two links do not have a common end, the Gudmundson model does not capture this correlation.

The development of a model which can capture such correlations requires immediate attention. It is now widely accepted that in order to satisfy the ever-increasing demand for service in cellular networks, a significantly larger number of BSs should be deployed. The BS deployment pattern in such networks follows that of the traffic, i.e. many BSs in hotspots and few in rural areas. In such networks, the correlation between links similar to A – B and C – D becomes instrumental in a realistic evaluation of the network performance. In the next section, the new shadow fading model is presented.

III. NETWORK SHADOW FADING MODEL

Shadow fading is an important component in the channel gain of a wireless link. The model used to account for this effect in computer simulations can have a significant impact on the end-results. In the previous section, we briefly discussed the need for a model which would capture correlation between all links in the network. In this section we study the shadow fading phenomenon more closely and develop our model.

The shadow fading gain has three essential properties. These properties are as follows:

• **Property I (Consistency):** In a wireless communication network there exist multiple active links operating at the same time in the same environment. All links should be correlated in accordance with the topographic characteristics of the terrain.

• **Property II (Continuity):** As the length of a wireless link grows, the signal has to traverse a larger area with possibly more diverse topographic properties. Since no prior knowledge
of the topography is available, this should be reflected mathematically by a larger variance (or degree of uncertainty) in the shadowing random variable. Conversely as the link length decreases and eventually reduces to zero, the variance should grow smaller and converge to zero. This effect has been reported in [16].

- **Property III (Symmetry):** High correlation between up-link and down-link directions are reported in the literature [13], [14]. In this paper, we assume the shadow fading gain experienced in both directions is identical.

In order to satisfy consistency, we develop a universal reference based on which all shadow fading gains in the network are generated. We refer to this reference as the *potential field.* The value of this field at point A is referred to as the *potential level* of point A and is denoted by \( X_A \). Since all channel gains are generated from the same underlying reference, all links experience a level of correlation whether or not they share a common end.

Mathematically, the potential field is defined as a correlated two-dimensional Gaussian field across the network coverage area. The value of the field at each point is the potential level of that point. The model in (3) is adopted to generate correlation across the field. Figure 2 shows a realization of the potential field in an area of \( 300 \times 300 \) m\(^2\).

The shadow fading gains are generated based on the topographic properties reflected in the potential field. The shadow fading gain between node A and node B, \( L(A, B) \) is generated from the potential field \( \mathcal{P} \). The diagram in Figure 3 shows the process. The channel gain generator \( \Psi(.) \) produces the channel gain according to the location of the nodes A, B and the corresponding realization of the potential field. In the second stage \( \Phi(.) \) maps the generated channel gain to the linear scale.

*Special case (Log-normality):* The shadow fading gain is widely assumed to have a log-normal distribution. This, however, is purely based on empirical results and cannot be proven mathematically. Hence log-normality has not been considered as an essential property and is categorized as a special case. This property is stated as follows: Based on empirical results, the
Fig. 2. A realization of the network potential field in an area of 300 × 300 m² with $d_c = 50$ meters and $\sigma_0 = 6$ dB. Darker areas have a smaller potential level than lighter areas. The gradual change in shade illustrates the spatial correlation in the potential field.

Fig. 3. The shadow fading gain $L(A, B)$ is generated in two stages. The first stage, $\Psi(\cdot)$, generates the channel gain based on the location of node A, node B and the potential field $P$. In the second stage, $\Phi(\cdot)$ maps the channel gain into the linear scale.

The shadow fading gain in the logarithmic scale (dB) has a Gaussian distribution with zero mean and a standard deviation of $\sigma$ [17]. This translates into the exponential mapping function

$$\Phi_{LN}(x) = 10^{x/10}. \quad (5)$$

Although this special case is the only acceptable mapping for the time being, such classification allows for other possibilities in the future. In the next section, $\Phi_{LN}(x)$ is considered and a channel generator $\Psi(\cdot)$ is proposed.
A. Symmetrized potential difference function

Let us consider a cellular network with $J$ BSs and $K$ terminals. In the proposed channel generating function a total of $N = K + J$ potential levels are generated in the following four steps:

- **Step 1:** Generate an $N \times 1$ vector $v$, where $v_i$’s are independent Gaussian random variables with zero mean and a standard deviation of unity.

- **Step 2:** Generate the $N \times N$ correlation matrix $R = [r_{ij}]$ such that

  $$r_{ij} = \sigma_X^2 e^{-d_{ij}/d_c},$$

  (6)

  where $\sigma_X$ is the standard deviation of the potential field and $d_{ij}$ denotes the distance between point $i$ and point $j$. The appropriate value for $\sigma_X$ will be discussed later in this section.

- **Step 3:** Decompose the correlation matrix using the Cholesky factorization such that $R = BB^H$.

- **Step 4:** The potential levels of the $N$ points are stacked in the $N \times 1$ vector $Bv$.

The symmetrized potential difference (SPD) channel gain generator is defined as

$$\Psi_{\text{SPD}}(P, A, B) = \text{sgn}(X_A + X_B) \cdot |X_A - X_B|,$$

(7)

where $\text{sgn}(.)$ is the Signum function.

The proposed function clearly satisfies symmetry. Continuity and log-normality are verified by the following lemma.

**Lemma:** If $S$ and $T$ are two independent Gaussian random variables with zero mean and standard deviations of $\sigma_S$ and $\sigma_T$ respectively, the random variable $Z = \text{sgn}(S) \cdot |T|$ has a Gaussian distribution with zero mean and a standard deviation of $\sigma_T$.

**Proof:** Since $X$ is a zero-mean Gaussian random variable, $t = \text{sgn}(X)$ is a discrete random variable with $P(t = 1) = P(t = -1) = 0.5$. The cumulative distribution function of $Z$ is
\[ F_Z(z) = P(Z \leq z) \]
\[ = 0.5P(|T| \leq z) + 0.5P(-|T| \leq z) \tag{8} \]

or,

\[ F_Z(z) = \begin{cases} 
\frac{1}{2} P(|T| \leq z) + \frac{1}{2}, & z \geq 0 \\
\frac{1}{2} P(-|T| \leq z), & z < 0
\end{cases} \tag{9} \]

It is well-known that the random variable \( L = |T| \) has a half-normal distribution with probability distribution function

\[ f_L(l) = \sqrt{\frac{2}{\pi \sigma_T^2}} e^{-\frac{l^2}{2\sigma_T^2}}. \tag{10} \]

Hence when \( z \geq 0 \),

\[ F_Z(z) = \frac{1}{2} P(L \leq z) + \frac{1}{2} \]
\[ = \frac{1}{2} \left( 1 + \sqrt{\frac{2}{\pi \sigma_T^2}} \int_0^z e^{-\frac{l^2}{2\sigma_T^2}} dl \right) \]
\[ = \frac{1}{2} \left( 2 - \sqrt{\frac{2}{\pi \sigma_T^2}} \int_z^{+\infty} e^{-\frac{l^2}{2\sigma_T^2}} dl \right) \]
\[ = 1 - \frac{1}{\sqrt{2\pi \sigma_T^2}} \int_z^{+\infty} e^{-\frac{l^2}{2\sigma_T^2}} dl \]
\[ = \frac{1}{\sqrt{2\pi \sigma_T^2}} \int_z^{-\infty} e^{-\frac{l^2}{2\sigma_T^2}} dl \]

and for \( z < 0 \),
F_Z(z) = \frac{1}{2} P(-L \leq z)
= \frac{1}{2} P(L \geq -z)
= \frac{1}{\sqrt{2\pi\sigma_T^2}} \int_{-z}^{\infty} e^{-\frac{l^2}{2\sigma_T^2}} \, dl
= \frac{1}{\sqrt{2\pi\sigma_T^2}} \int_{-\infty}^{z} e^{-\frac{l^2}{2\sigma_T^2}} \, dl.

Hence, \( Z \sim \mathcal{N}(0, \sigma_T^2) \).

Let \( S \) and \( T \) take the values of \( X_A + X_B \) and \( X_A - X_B \) respectively. We have

\[
E\{ST\} = E\{(X_A + X_B)(X_A - X_B)\} = 0,
\tag{11}
\]

thus \( S \) and \( T \) are independent as they are both zero-mean Gaussian random variables.

Based on the lemma, the proposed shadow fading gain is Gaussian with variance

\[
\sigma^2 = E\{(X_A - X_B)^2\} = 2\sigma_X^2 - 2E\{X_AX_B\}
= 2\sigma_X^2 \left(1 - e^{-d_{AB}/d_c}\right),
\tag{12}
\]

where \( d_{AB} \) is the distance between points A and B and \( \sigma_X^2 \) is the variance of the potential levels \( X_A \) and \( X_B \).

The proposed shadow fading gain function satisfies continuity. The variance approaches zero as \( d_{AB} \to 0 \) and with \( d_{AB} \to \infty \), it converges to \( 2\sigma_X^2 \). As the link length grows, the variance should converge to that reported in the literature, hence \( \sigma_X^2 \) is chosen to be equal to \( \frac{1}{2}\sigma_0^2 \). Thus

\[
\sigma^2 = \sigma_0^2 \left(1 - e^{-d_{AB}/d_c}\right).
\tag{13}
\]
It is important to note that the variance of the Gudmundson model is not a function of distance. This violates the continuity property at $d$ equal to zero. By introducing the potential field and the proposed channel gain generating function, we are able to maintain this important property.

**B. Comparison with [12]**

In [12] a similar approach for generating correlated shadow fading gains is proposed. A *spatial loss field* is defined which is essentially the potential field in this paper. The shadow fading gain between any two nodes in the spatial loss field is defined as the line integral of the field along the line connecting the two ends

$$
\Psi_{[12]}(P, A, B) = \frac{1}{\sqrt{d_{AB}}} \int_{A \rightarrow B} P(x, y) dx dy,
$$

(14)

where $P(x, y)$ is the potential level at a point with coordinates $(x, y)$.

The integral is calculated along the line connecting nodes $A$ and $B$. Firstly, this results in $L(A, B) = -L(B, A)$ which contradicts the symmetry property. Secondly, the proposed line integral translates into the need for a database of the potential levels of all possible points across the network coverage area. This is in contrast with the model proposed in this paper as (7) requires the potential levels only at the end points of a link. Finally, the proposed channel gain in [12] has a constant variance which does not satisfy the continuity property.

**IV. NUMERICAL ANALYSIS**

In this section the effect of the proposed model on three measures is investigated. Section IV-A examines how the proposed model compares with the Gudmundson model in the correlation between two links sharing a common end. In section IV-B, the outage probability of large and small cells is studied. Finally in section IV-C the aggregate interference in a Poisson field of interfering BSs is reviewed. Throughout this section a standard deviation of $\sigma_0 = 6$ dB and a correlation distance of $d_c = 50$ meters is assumed.
A. Correlation between two links with a common end

Let terminals $k_1$ and $k_2$ be located at a distance $D$ from a BS as illustrated in Figure 4. The distance $d$ between the two terminals is equal to $2D \sin \frac{\theta}{2}$. In this section we study the correlation between the two links as a function of $(D, \theta)$ and compare the properties of the Gudmundson and the proposed models. While the correlation between the two links is an exponential function of $d$ according to the Gudmundson model, the proposed model suggests a more complicated behavior.

![Common end simulation model](image)

Fig. 4. Common end simulation model

The normalized correlation $\frac{\mathbb{E}(g_1g_2)}{\sigma_0^2}$ based on the Gudmundson model is provided in Figure 5. For a fixed $D$, a larger $\theta$ increases $d$ and hence reduces the correlation level between the two links. Increasing $D$ would result in a larger $d$ which further reduces the correlation level. For a sufficiently large $d$, the correlation converges to zero as expected. The normalized correlation of the proposed model is provided in Figure 6. Based on this model, increasing the distance between the terminals also reduces the correlation between the channel gains. However unlike the Gudmundson model, the correlation plateaus at a non-zero value with increasing $d$. This is due to the fact that although the terminals are far apart the two links still share a common end and hence should remain correlated. As an example let us consider the case where the BS is located on a tall structure. With high probability there exists a line-of-sight path between the BS and the
two terminals, which in turn translates into high channel gains on both links. On the other hand, if the BS is located in a vicinity with many obstructions (e.g. in the middle of tall buildings), regardless of the distance between the terminals, both terminals would experience weak channel gains. These arguments suggest that regardless of the distance between the terminals, $g_1$ and $g_2$ should be correlated.

![Normalized correlation](image)

Fig. 5. Normalized correlation $\frac{\langle g_1 g_2 \rangle}{\sigma_1^2}$ as a function of $\theta$ (Gudmundson model)

**B. Outage probability**

The outage probability for a link in a wireless network is defined as follows:

$$P_{\text{out}}(d) = \text{Prob} \left( P_r \leq P_{\text{min}} \right) = \text{Prob} \left( \frac{C}{d^q} \cdot L \leq P_{\text{min}} \right) ,$$

(15)

where $P_{\text{min}}$ is the minimum required received power.

Let us define the cell outage radius $R$ as the distance (from the BS) where the outage probability is equal to 5%. The cell outage radius is calculated as
Fig. 6. Normalized correlation $\frac{\sigma(g_1g_2)}{\sigma_0}$ as a function of $\theta$ (Proposed model)

$$
P_{out}(R) = 0.05 \Rightarrow R = \left(\frac{c}{P_{min}} \cdot 10^{-\frac{\sigma_0}{10000} \text{erfc}^{-1}(0.1)}\right)^{\frac{1}{\gamma}},$$

where erfc(.) is the complementary error function. By increasing the transmit power $c$, the cell outage radius grows and hence a larger cell is formed. In a sense, this definition transforms the transmission level of a BS into a physical distance which can be viewed as the cell radius.

Using (16), the outage probability can be written as a function of the cell outage radius as follows:

$$
P_{out}(d) = \text{Prob}\left(\frac{c}{d^\gamma} \cdot L \leq P_{min}\right)$$

$$= \text{Prob}\left[L \leq \left(\frac{d}{R}\right)^\gamma \cdot 10^{-\frac{\sigma_0}{10000} \text{erfc}^{-1}(0.1)}\right]$$

$$= \frac{1}{2} \text{erfc}\left(-\frac{10 \log_{10}\left(\frac{d}{R}\right)^\gamma - \sigma \sqrt{2} \text{erfc}^{-1}(0.1)}{\sigma \sqrt{2}}\right),$$

(17)
In Figure 7 the outage probability of a cell with an outage radius of $100d_c$ (large cell) is plotted as a function of the normalized distance $(d/d_c)$. The outage probability according to the Gudmundson model is also provided for comparison. The Figure shows that both models predict the same outage probability for a large cell. Figure 8 depicts the outage probability for a cell with an outage radius of $d_c$ (small cell). Unlike the large cell, the outage probability of the proposed model is lower than that of the Gudmundson model for a small cell. This behavior can be explained by the smaller variance of the shadow fading gain for short distances.

![Fig. 7. Outage probability as a function of the normalized distance $(d/d_c)$ for large cells: A large cell with $R = 100d_c$ is considered. The Gudmundson and the proposed models predict similar outage probabilities for large cells.]

### C. Aggregate interference

In this section, the effect of the proposed model on the aggregate interference in an autonomous cellular network is studied. The BSs are modeled by a Poisson random field across the network coverage area, modeled by a circle of radius $100d_c$. A minimum distance of $0.5d_c$ between BSs is maintained. The channel gain model in (2) is employed with a normalized $c = 1$ and
Fig. 8. Outage probability as a function of the normalized distance $\frac{d}{d_c}$ for small cells: A small cell with $R = d_c$ is considered. Within the cell radius ($d \leq d_c$), the outage probability for the proposed models is less than that of the Gudmundson model. This is due to the smaller variations in the shadow fading gain for small distances.

$\gamma = 4$. First, we study the distribution of the aggregate interference at a terminal in section IV-C1. Subsequently, the correlation between the aggregate interference levels at two terminals is studied in section IV-C2.

1) Single link: Let us consider ten interfering BSs inducing interference on terminal 0 located at the origin (See figure 9). The aggregate interference at the terminal is written as

$$I_0 = \sum_{j=1}^{10} I_{0j},$$

(18)

where $I_{ij} = p_j h_{ij}$, $p_j$ is the transmit power of BS $j$ and $h_{ij}$ is the channel gain between terminal $i$ and BS $j$.

Figure 10 illustrates the cumulative distribution function (CDF) of the aggregate interference $I$ for the proposed and Gudmundson models. The two models show similar levels of interference at the terminal.
Fig. 9. Single link simulation model for aggregate interference analysis: A terminal (blue square) is located at the center of 10 interfering BSs (red circles).

Fig. 10. Cumulative distribution function of the aggregate interference level at a terminal: The Gudmundson and the proposed models predict similar interference levels.
2) Two links: In this section, terminal 0 and terminal 1 separated by a distance $d$ at the center of the network coverage area are considered (See Figure 11). The correlation between aggregate interference levels $I_i$ and $I_j$ is defined as

$$\rho_{ij} = \frac{\text{cov}(I_i, I_j)}{\sigma_i \sigma_j},$$

where $\text{cov}(X,Y)$ denotes the covariance between $X$ and $Y$.

Figure 12 illustrates the correlation between the two interference levels as a function of distance. Due to the fact that the Gudmundson model does not consider correlation between two links with no common end, the correlation between the two interference levels are largely under-estimated specifically for small distances.

![Fig. 11. Two links simulation model for aggregate interference analysis: Two terminals (blue squares) are located at the center of 10 interfering BSs (red circles)](image)

V. SUMMARY

We have discussed three essential properties of the shadow fading gain. First, due to the inherent spatial correlation of the topography, all channel gains in the network should be cor-
Fig. 12. Correlation coefficient of aggregate interference levels at two terminals separated by $d$ as a function of the normalized distance $\frac{d}{d_c}$. The Gudmundson model largely under-estimates the correlation between the two terminals especially when they are close (small $d$). Both models predict similar correlation levels for large distances.

related (consistency). Second, shorter links experience less variations due to the topography of the environment (continuity). In other words, as the link length increases more variations in the channel gain is expected due to the larger distance the signal has to travel. This should be translated into a varying variance for the channel gain which increases with distance. And third, high correlations between up-link and down-link channel gains have been reported in the literature (symmetry). In this paper, we have assumed the shadow fading gain experienced in both directions is identical. A general formula for the shadow fading gain is given and the well-known log-normal model is categorized as a special case.

A shadow fading gain generator is developed. All channel gains in the network are generated from one common reference referred to as the shadow fading potential field. The field is essentially a correlated two-dimensional Gaussian random field. Given a realization of the potential field, the channel gain generating function calculates the shadow fading gain between
any two nodes in the network coverage area. Due to the fact that all channel gains are generated from the same potential field, the proposed model satisfies consistency. In addition, it is shown that the proposed channel generating function satisfies continuity, symmetry and has a log-normal distribution.

In the numerical results, we first study the correlation between two links with a common end. Unlike the Gudmundson model, the proposed model predicts a non-zero correlation between the two links when the two uncommon ends are sufficiently far apart. When one end is common there always exists a degree of correlation between the two links. The outage analysis shows that the proposed model predicts the same outage probabilities for large cells when compared to earlier models but predicts a smaller probability of outage for small cells. This is in turn due to the smaller variance of the channel gain in short distances. In the last part, a study on aggregate interference is provided. A Poisson field of BSs models an autonomous cellular network. The cumulative distribution function (CDF) of the aggregate interference at a terminal located in the center of the network coverage area is studied. Compared to earlier models, a similar CDF for the aggregate interference is observed. Subsequently, the correlation between the aggregate interference levels of two terminals is analyzed. The study shows that the Gudmundson model significantly under-estimates the correlation between the aggregate interference levels of two terminals located in the same proximity. For large distances, the proposed and the Gudmundson models both predict similar diminishing levels of correlation.

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